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Decomposition of MT impedance tensors in 1D electric anisotropic media

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Summary

The study on distortion and decomposition of MT impedance tensors lays the fundament for inversion and interpretation of MT data. The existence of small-scale, near-surface conductivity anomalies would cause galvanic distortion, which will lead to distortion of observed MT impedance tensors. Therefore, the distortion decomposition of MT impedance tensors is required to eliminate local distortion effect and restore regional impedance elements for the true regional tectonic structure in anisotropic media.

Introduction

Due to the complexity of crustal electrical-conductivity structures, the observed MT impedance tensors can be distorted by shallow small-scale inhomogeneities or topographic relief, which is called galvanic distortion (Chave & Smith, 1994). Decomposition techniques of MT impedance tensors have been widely used in isotropic conductivity media, but they have not been applied in anisotropic media. In this presentation, we present primarily results of decomposition of MT impedance tensors in electric anisotropic media. The effect of galvanic distortion is simulated by adding random noise and distortion matrix to impedances of 1D anisotropic media, along with 1D/2D forward modeling. We adopt the approach of constrained multifrequency Groom-Bailey decomposition and BFGS Quasi-Newton algorithm with constraints of phase tensor analysis to obtain the optimal solution.

Constrained Multi-frequency GB Decomposition

Galvanic distortion can be described by a real and frequency-independent 2×2 distortion tensor C (Chave & Smith, 1994). For the case of 1D anisotropic Earth, the effect of which on MT impedance tensors can be expressed as:

$$\mathbf{Z}_{obs} = \mathbf{C}\mathbf{Z}_{1-Da}$$

The distortion tensor C is factorizable in the product of one scalar and three matrices:

$$\mathbf{Z}_{obs} = g\mathbf{TSAZ}_{1-Da} = g\begin{bmatrix}1 & -t\\ t & 1\end{bmatrix}\begin{bmatrix}1 & s\\ s & 1\end{bmatrix}\begin{bmatrix}1+a & 0\\ 0 & 1-a\end{bmatrix}\mathbf{Z}_{1-Da}$$

where g,T,S,A respectively represent site gain, twist tensor, shearing tensor and anisotropy tensor. The complex functions α_i are defined as (Garcia & Jones, 2002)

$$\alpha_0 = Z_{xx,obs} - Z_{yx,obs}, \quad \alpha_1 = Z_{xx,obs} + Z_{yx,obs}$$
$$\alpha_2 = Z_{xy,obs} - Z_{yy,obs}, \quad \alpha_3 = Z_{xy,obs} + Z_{yy,obs}$$

The above equations can be written as follows

$$\alpha_0 = (1 - st - t - s)(1 + a)Z_{xx}(\omega) + (s - t - st - 1)(1 - a)Z_{yx}(\omega) + \varepsilon_0(\omega)$$
$$\alpha_1 = (1 - st + t + s)(1 + a)Z_{xx}(\omega) + (s - t + st + 1)(1 - a)Z_{yx}(\omega) + \varepsilon_1(\omega)$$
$$\alpha_2 = (1 - st - t - s)(1 + a)Z_{xy}(\omega) + (st + 1 - s + t)\,(1 - a)Z_{xx}(\omega) + \varepsilon_2(\omega)$$
$$\alpha_3 = (1 - st + t + s)(1 + a)Z_{xy}(\omega) + (t - s - st - 1)\,(1 - a)Z_{xx}(\omega) + \varepsilon_3(\omega)$$

where $\varepsilon_i(i = 0,1,2,3)$ are the errors between model parameters(a, t, s, Z_{xx}, Z_{xy}, Z_{yx}) and observations. Since the single frequency solution is ill-posed, the constrained multi-frequency GB Decomposition is adopted, and in this way at n frequencies there are 3+6n unknows and 8n knows, which is an overdetermined problem. The objective function is

$$f = \sum_{i=1}^{n}\sum_{j=0}^{3}\{real(\varepsilon_j)^2 + imag(\varepsilon_j)^2\}$$

The fitting error for evaluation is:

$$\varepsilon^2 = \frac{\sum_{i=1}^{2}\sum_{j=1}^{2}\left|Z'_{ij} - Z_{ij}\right|^2}{\sum_{i=1}^{2}\sum_{j=1}^{2}\left|Z_{ij}\right|^2}$$

BFGS Quasi-Newton Algorithm with Constraints

We adopt BFGS Quasi-Newton algorithm as a solution to the optimization problem. Quasi-Newton condition

$$\mathbf{d}_k = -\mathbf{H}_k\mathbf{g}_k$$

is utilized to determine the search direction. Optimal step-size is determined by 1D Armijo-Goldstein search criterion:

$$f(\mathbf{x}_0 + \delta^m\mathbf{d}_k) \le f(\mathbf{x}_0) + \sigma\delta^m{\mathbf{g}_k}^T\mathbf{d}_k$$

The approximate inverse of Hessian matrix is:

$$\mathbf{H}_k = \mathbf{H}_k + \left(1 + \frac{{\mathbf{y}_k}^T\mathbf{H}_k\mathbf{y}_k}{{s_k}^T\mathbf{y}_k}\right)\frac{\mathbf{s}_k{\mathbf{s}_k}^T}{{\mathbf{s}_k}^T\mathbf{y}_k} - \left(\frac{\mathbf{s}_k{\mathbf{y}_k}^T\mathbf{H}_k + \mathbf{H}_k\mathbf{y}_k{\mathbf{s}_k}^T}{{\mathbf{s}_k}^T\mathbf{y}_k}\right)$$

The phase tensor is defined as:

$$\mathbf{\Phi} = \mathbf{X}^{-1}\mathbf{Y}$$

where X and Y represent the real and imaginary parts of Z, respectively. Due to

$$\mathbf{\Phi} = \mathbf{X}^{-1}\mathbf{Y} = (\mathbf{C}\mathbf{X}_0)^{-1}(\mathbf{C}\mathbf{Y}_0) = {\mathbf{X}_0}^{-1}\mathbf{C}^{-1}\mathbf{C}\mathbf{Y}_0 = {\mathbf{X}_0}^{-1}\mathbf{Y}_0 = \mathbf{\Phi}_0$$

the phase tensor is independent of galvanic distortion. In this way the phase tensor analysis is utilized to provide initial values for the above optimization algorithm.

References:
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Synthetic Examples

For the case of 1D electric anisotropy, galvanic distortion can be simulated. Given a distortion matrix

$$\mathrm{C} = \begin{bmatrix}1.261 & 0.5316\\ 0.5874 & 0.7596\end{bmatrix}$$

where the site gain g, distortion anisotropy a, twist t and shear s are 1.0, 0.2, -0.0875 (-5°) and 0.577(+30°) respectively. The Gaussian noise of 3% is added.

Table.1 Parameters of 1D anisotropic geoelectric model (GSLsz in northwestern Canada).

Layer	Thickness(km)	$\rho_x(\Omega m)$	$\rho_y(\Omega m)$	$\rho_z(\Omega m)$	α_s
Upper crust	10	10000	10000	10000	N/A
Lower crust	30	300	10000	300	30
Lithospheric mantle	160	30	1000	30	60
Asthenosphere	N/A	30	30	30	N/A

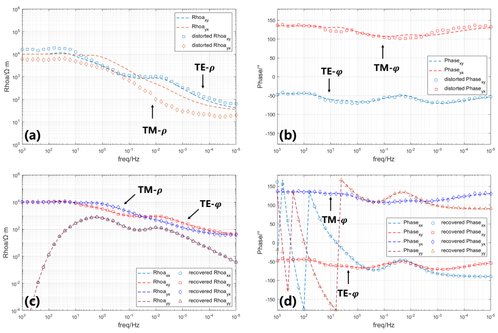

Fig.1 Synthetic(a-ρ, b-φ) and recovered(c-ρ, d-φ) data

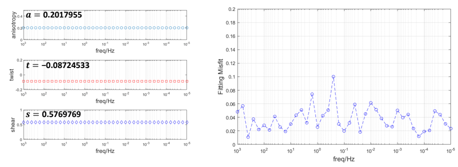

Fig.2 Recovered distortion parameters(a, t, s) and fitting error of impedances

A small-scale near-surface conductivity anomaly is embedded into the 1D anisotropic GSLsz model given in Table 1. For the 2D/1D geoelectric model, synthetic data are obtained by the 2D MT anisotropy unstructured finite element method.

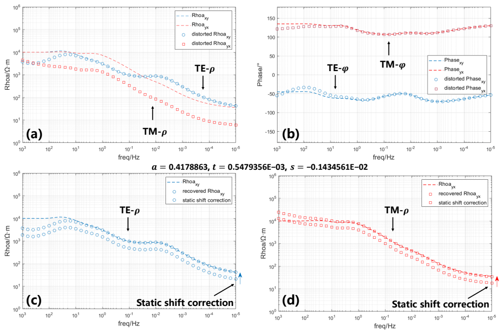

Fig.3 Synthetic(a-ρ, b-φ) and recovered(c-TE-ρ, d-TM-ρ) data of a measuring point right above the anomaly

Fig.3a indicates that the apparent resistivity curves are shifted vertically, which is called static effect. The commonly used methods for static shift correction are: spatial filtering, curve translation, conversion of impedance phase, rotation of electric principal axes. In addition, there are still some issues needed to be addressed, such as induction distortion in high-frequency band.

Conclusions

Distortion decomposition of MT impedance tensors in 1D electric anisotropic media is realized by using the constrained multi-frequency GB decomposition, with BFGS Quasi-Newton algorithm as an optimization method based on phase tensor analysis.