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Some remarks on the behaviour of the magnetotelluric phase

I. Introduction

For many of those working with the MT, AMT, or VLF sounding techniques the behaviour of the phase remains somewhat mysterious (cf., e. g., Fischer et al. 1983), even though it can be deduced from theory and its observed response is often used with advantage when field data are inverted. Two simple questions may serve to illustrate the elusiveness of the phase response in electromagnetic sounding:

1) When a monochromatic electromagnetic (EM) plane wave is incident from vacuum on a uniform good conductor, it is well known that the phase of the electric field $E(\omega)$ is ahead of the phase of the magnetic field $H(\omega)$ by 45° at the interface. However, far above the surface of the conductor, in the postulated vacuum half-space, the electric phase is leading by 90°, as suggested by Fig. 1. Can this be understood ?

2) If the uniform good conductor is replaced by a two-layer structure, everyone knows that the phase lead of E will usually differ from 45°. If the top layer is a better conductor than the bottom half-space ($\rho_1 < \rho_2$) and its thickness h is less than its skin-depth δ_1, then the phase will drop below 45°. If the bottom layer is the better conductor the phase will exceed 45°. Can this behaviour be explained in simple terms ?

In what follows we propose to look into these two questions with the hope of providing answers which are conceptually easy to understand, thereby also clarifying the behaviour of the EM fields in the vicinity of the interface between conductor and vacuum.

II. The Phase in the Medium of Incidence

Let us assume a uniform or layered conductor in the half-space of positive z coordinates. At the z = 0 interface the ratio of electric to magnetic fields is given by the surface impedance $Z(\omega)$, which for a uniform conductor of resistivity ρ reduces to

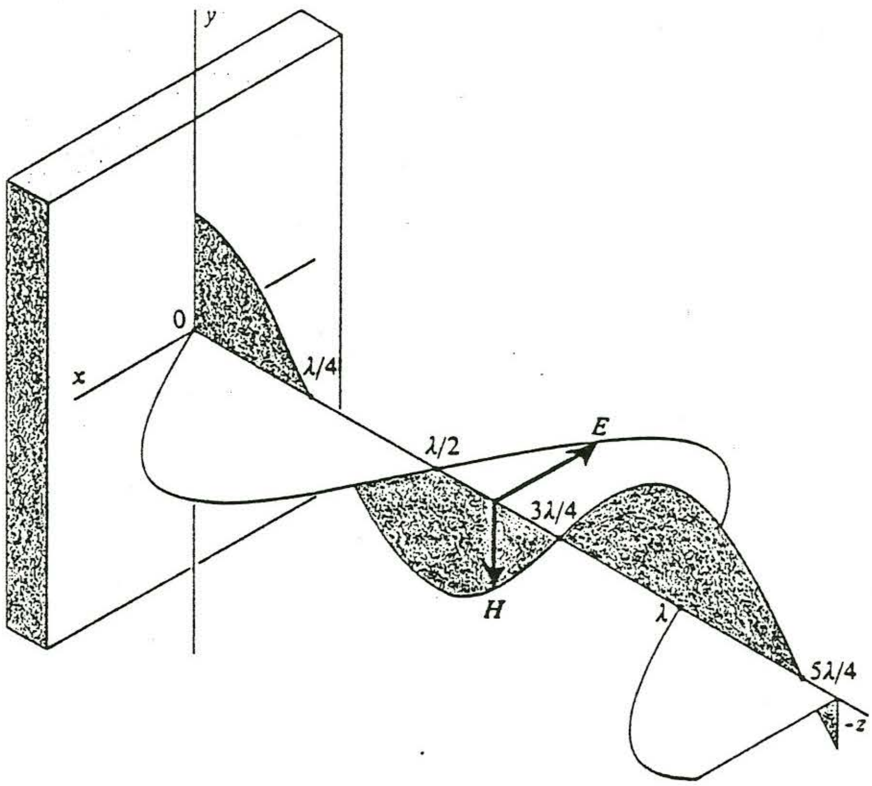

Fig. 1. Standing-wave pattern resulting from the reflection of an electromagnetic wave at the surface of a good conductor. The nodes of E and H are spaced λ/4 apart as shown. (This figure is reproduced from Lorrain and Corson 1970)

$$E(\omega)/H(\omega) = Z(\omega) = \sqrt{\omega \mu_o \rho} \quad \exp(i\pi/4) \quad . \tag{1}$$

For a layered conductor $Z(\omega)$ can be expressed in terms of an apparent resistivity $\rho_a(\omega)$ and a phase $\psi(\omega)$ between 0 and 90° [with the assumed time-dependence of $\exp(+ i\omega t)$]:

$$Z(\omega) = \sqrt{\omega\mu_o \rho_a(\omega)} \quad \exp[i \psi(\omega)] \quad . \tag{2}$$

For a uniform or layered conductor we can make the usual assumption that the magnetic field $H(\omega)$ is uniform over the conductor in the entire volume $0 < - z << \lambda$, where λ is the vacuum wavelength. Under these conditions the electric field gradient is independent of the conductor and from Maxwell's equations is found to be (cf. Fischer 1983):

$$\frac{\partial E(z,\omega)}{\partial z} = - i \omega \mu_o H(\omega) \quad , \quad z < 0 \quad . \tag{3}$$

Above the surface of the conductor, i.e. for $z < 0$, the electric field is therefore given by

$$E(\omega) = H(\omega)\ [Z(\omega) - i\omega\mu_o z] \quad , \quad 0 \leq -z << \lambda \quad . \tag{4}$$

It is easily seen from this equation that the phase of $E(\omega)$ switches from $\psi(\omega)$ for $z = 0$ to $+ 90°$ for large values of $-z$. For a uniform conductor in particular it is easy to give an analytical expression, because $Z(\omega)$ is related to the classical skin-depth $\delta = \delta(\omega)$:

$$Z(\omega) = \frac{1+i}{2}\omega\mu_o \sqrt{2\rho/\omega\mu_o} = \frac{1+i}{2}\ \omega\mu_o\ \delta(\omega) \quad . \tag{5}$$

Equation (4) thus takes the form

$$E(z,\omega) = \omega\mu_o H(\omega)[\delta + i(\delta-2z)] \quad , \quad z \leq 0 \quad . \tag{6}$$

The behaviour of the phase is therefore

$$\psi(z,\omega) = \text{arc tan}\ [1 - 2z/\delta(\omega)] \quad , \quad z \leq 0 \quad . \tag{7}$$

As can be seen, the phase varies fairly quickly as one moves away from the surface of incidence:

$$\begin{aligned} &\text{for } z = -\delta/2 \quad , \quad \psi = 63.4° \quad , \\ &\text{for } z = -\delta \quad , \quad \psi = 71.6° \quad , \\ &\text{for } z = -3\ \delta \quad , \quad \psi = 81.9° \quad . \end{aligned} \tag{8}$$

The behaviour of phase and modulus of $E(z,\omega)$ could play a role when sounding at high frequencies, as for example in airborne VLF soundings.

A further consequence of equation (4), which we believe is worthy of attention, concerns the methods of two- and three-dimensional (2-D) and (3-D) modelling. Over a 1-D structure and for a given primary excitation, characterized entirely by its magnetic surface field $H(\omega)$, the response $E(\omega) = Z(\omega)\cdot H(\omega)$ will be different for different layerings. Since the vertical gradient depends only on $H(\omega)$ the fields $E(z,\omega)$ at any height $-z << \lambda$ over the surface of incidence will exhibit the same difference. To render boundary conditions more tractable most modelling methods work with structures which, at large distances from a geological accident, taper off toward simple 1-D layering. If this layering is different in different directions it cannot be assumed that the electric field is uniform at the top boundary. Taking the example of a 2-D structure, if the model is different at the left- and right-hand limits it is incorrect to postulate a uniform electric field at the upper border, as some modelling programmes do (cf., e.g., Kisak and Silvester 1975). This point will receive further attention in a paper by Weaver et al. (1984).

III. The E-Polarization Phase over a Graben Structure

The behaviour of the phase at the top of a graben under E-polarization excitation bears some resemblance to its behaviour at the top of a two-layer structure, but is conceptually simpler to understand. This is the reason for looking first at this situation.

If we assume that the graben width and depth are small against its own skin-depth $\delta = \sqrt{2\rho/\omega\mu_o}$, an E-polarization electric field over the graben cross-section will be almost identical with the field amplitude $E(\omega)$ over the unperturbed matrix surface, for reasons of continuity of the tangential field components. If the graben is a good conductor, as suggested in Fig. 2, it will carry a much larger current density than the adjacent matrix areas. The small graben will therefore carry an excess current ΔI which we can easily estimate:

$$\Delta I \cong E(\omega)\ wd[1/\rho - 1/\rho_o] \cong E(\omega)\ wd/\rho \quad . \tag{9}$$

The extra current creates an anomalous magnetic field vortex around the graben. This anomalous field $H_a(\omega)$ can be estimated crudely by considering the graben as a simple conducting wire (a more exact treatment is unwarranted as it would merely distract from the present argument). At the top of the graben center the anomalous field is therefore of the order of (cf. Lorrain and Corson 1970)

$$H_a(\omega) \cong \Delta I/\pi d \cong (\omega/\pi\rho)\ E(\omega) \quad . \tag{10}$$

The important point is that the anomalous magnetic field $H_a(\omega)$ is strictly proportional to the electric field $E(\omega)$. Whereas over the matrix, away from the graben, the surface impedance $Z_m(\omega)$ is given by the ratio of the normal fields [cf. equation (1)],

$$Z_m(\omega) = E(\omega)/H(\omega) = \sqrt{i\omega\mu_o\rho_o} \quad , \tag{11}$$

and has thus a phase of 45°, the same ratio over the graben center

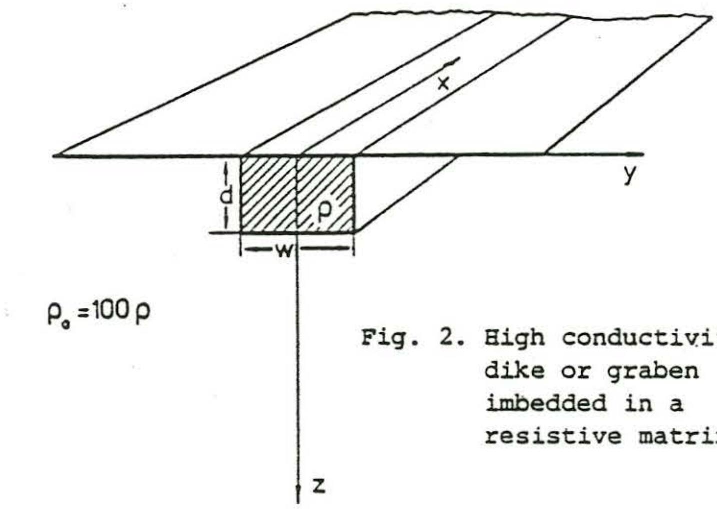

Fig. 2. High conductivity dike or graben imbedded in a resistive matrix.

is given by an expression of the form

$$Z_g(\omega) \cong \frac{E(\omega)}{H(\omega)+H_a(\omega)} \cong \frac{E(\omega)}{H(\omega)+\alpha E(\omega)} \quad , \tag{12}$$

where α is a real <u>positive</u> constant. Clearly, adding to the denominator a quantity proportional to the numerator always reduces the argument of $Z_g(\omega)$. In fact with $\alpha \to \infty$ the argument of $Z_g(\omega)$ would totally vanish.

This chain of arguments can obviously be reversed when the graben is more resistive than the matrix. The reduction of current in the graben will correspond to an effective negative anomalous magnetic field vortex. Over the graben top an equation like (12) will again apply, but with a real <u>negative</u> α, leading therefore to an argument for $Z_g(\omega)$ which will be larger than 45°. This E-polarization response of the graben is neatly confirmed by the field measurements and model calculations of Fischer et al. (1983).

IV. Behaviour of the Phase over a Two-Layer Structure

Let us now consider a structure consisting of a basement of resistivity ρ_o covered with a thin overburden of much lower resistivity ρ. We shall assume the overburden to be thin against its proper skin-depth, but shall see later that this requirement can be greatly relaxed. At the top of the basement the relation between electric and magnetic fields, respectively $E_b(\omega)$ and $H_b(\omega)$, is identical with equation (11):

$$Z_b(\omega) = E_b(\omega)/H_b(\omega) = \sqrt{i\omega\mu_o\rho_o} \quad . \tag{13}$$

While $E_b(\omega)$ still leads $H_b(\omega)$ by 45°, it must be pointed out that $H_b(\omega)$ refers to the magnetic field at the top of the basement; in general it will have a phase which is different from the field $H_t(\omega)$ at the top of the structure. Since the overburden is thin the electric field at the top face, $E_t(\omega)$, is not much different from the field at the bottom face, $E_b(\omega)$, again for reasons of continuity. However, because of its low resistivity the overburden carries a large current density and therefore an extra current ΔI in phase with the electric field. This current ΔI creates a large horizontal magnetic field vortex around the top and bottom faces of the overburden. Consequently, at the top of the overburden the surface impedance $Z_t(\omega)$ will be strongly influenced by this anomalous extra magnetic field $H_a(\omega) \cong \alpha E_b(\omega) \cong \alpha E_t(\omega)$, with α a real <u>positive</u> constant:

$$Z_t(\omega) = \frac{E_t(\omega)}{H_t(\omega)} \cong \frac{E_b(\omega)}{H_b(\omega)+\alpha E_b(\omega)} \quad . \tag{14}$$

Here again the denominator is increased by a quantity which has the same argument as the numerator and the phase of $Z_t(\omega)$ is therefore diminished below the 45° of the uniform basement.

Conversely, in a thin overburden which is more resistive than the basement conductor the current is depressed by subtraction of an amount in phase with the electric field. This pushes the phase above the 45° value.

The range of top layer thicknesses h for which the reasoning we have given applies depends on the resistivity ρ of the overburden. While our argument can be understood best when the overburden thickness is much smaller than the overburden skin-depth, this restriction is far less severe. As is well known (cf., e.g., Keller and Frischknecht 1966), the phase relationships we have explained extend to low frequencies $\omega/2\pi$, or large periods T, from the last cross-over point given by

$$h/\delta(\omega) \leqq \pi/2 \quad . \tag{15}$$

V. Conclusions

The behaviour of the electromagnetic phase in the medium of incidence over a good conductor has been analyzed. We have shown how the phase returns from its surface value to the characteristic standing wave phaseshift of 90° over a distance of a few skin-depths of the conductor, much shorter than the length of the incident wave. At the surface of a graben or of a two-layer conductor it has been shown that in both situations the basement can be looked upon as the matrix. If the graben or the overburden is a better conductor it will carry an extra current density in phase with the electric field, thus giving rise to an additional magnetic field also in phase with the electric field. This drives the argument of the surface impedance toward smaller values. For a more resistive graben or overburden the current is depressed, corresponding to a reduction of the surface magnetic field by an amount in phase with the electric field. This effect drives the phase of the impedance to values above the characteristic 45° of a uniform conductor.

Acknowledgments

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References:

Fischer G., Le Quang B.V. and Müller I.: VLF ground survey, a powerful tool for the study of shallow two-dimensional structures. Geophysical Prospecting, 31, 977-991, 1983.

Fischer G.: Current channelling, the consequences for electromagnetic soundings. Geophysical Research Letters, 10, 1152-1155, 1983.

Keller G.V. and Frischknecht F.C.: Electrical methods in geophysical prospecting. Pergamon Press, Oxford, 1966.

Kisak E. and Silvester P.: A finite element program package for magnetotelluric modelling. Computer Physics Communications, 10, 421-433, 1975.

Lorrain P. and Corson D.: Electromagnetic Fields and Waves. W.H. Freeman & Co., San Francisco, 1970.

Weaver J.T., Le Quang B.V. and Fischer G.: Remarks on the comparison of analytical and numerical model calculations. In diesem Protokollband, 1984.