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"Numerical Models for the Interpretation of EM Deep Sounding Measurements over Two Dimensional Structures"

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This paper discusses the second assumtion in CAGNIARD's (1953) paper concerning the magneto-telluric (MT) determination of the earth's resistivity. It is that of a homogeneous, horizontally-layered conductivity structure. Although there has been much discussion on the first assumption of a homogeneous source field, little has been said concerning the conductivity structure. With the increasing number of profiles or whole arrays of MT soundings, the need for another method of interpretation has been recgnised. The method must be capable of allowing for a non-homogeneous conductivity structure. A method of direct inversion of the field data, as proposed by SCHMUCKER (1967), is one such method. However, often the structure can be stipulated as two dimensional and another indirect approach can be used.

Most of the studies for two dimensional problems have been concerned with the fault (or coastline) effect, in which two quarter spaces of different resistivity form the half space representing the earth. This is of course, a very restricted two dimensional model and a more general solution is sought.

For $\vec{H}$ polarised along the strike direction, the only field components are

$$\vec{H} = (H_x, 0, 0) \quad \text{and} \quad \vec{E} = (0, E_y, E_z) \ .$$

Thus, $\vec{H}$ may be treated as a scalar H_x and the two components of $\vec{E}$ deduced from it. The boundary conditions to be applied are the usual ones of continuity of tangential $\vec{E}$ and $\vec{H}$ and normal $\vec{H}$ and $\vec{J}$ at any boundary. For $\vec{H}$ polarised, this implies that H_x at $z = 0$ is constant for all y. This Dirichlet condition can be used as the source for this polarisation.

Similarily for $\vec{E}$ polarisation, the only field components are,

$$\vec{E} = (E_x,0,0) \quad \text{and} \quad \vec{H} = (0,H_y,H_z) \ .$$

Now $\vec{E}$ may be treated as a scalar E_x and the components of $\vec{H}$ deduced from it. The boundary conditions are the same as before, but the simple z = 0 condition is no longer true. This complication in the source condition has impeded the proper solution for $\vec{E}$ polarised in two dimensional structures.

It is proposed that a numerical procedure, based on a transmission line (TL) analogy, be used to calculate the MT fields. The equation to be solved is

$$\nabla^2 V - k^2 V = 0 \ , \qquad (1)$$

where V is a scalar representing the polarised component. In their general form the TL equations and Maxwell's equations are shown in Fig. 1. Their similarity is more apparent when they are written in the two dimensional form as in Fig. 2. This figure also shows the associations that must be made for the $\vec{H}$ and $\vec{E}$ polarised cases. The Z and Y are the impedance and admittance in the TL analogy. These associations are valid for any continuous system.

When the system is discretised, for solution at certain points in a mesh, the parameters Z and Y must be replaced by the lumped parameters. The form of these is shown in Fig. 3 along with the unit cell for the mesh. Now it is possible to build up a network, simly by combining unit cells (all of which may be different). Hence, any arbitrary cross section can be produced; however, for convenience in programming, a network limited to ten vertical and ten horizontal sections, has been studied.

The network must be solved for the node voltages. This is done by writing Kirchhoff's current law at each node, in terms of the node voltages. That is, for points like (i,j) in Fig. 3, the equation is

$$(1/Z_{i-1})(V_{ij}-V_{i-1,j})+(1/Z_{j-1})(V_{ij}-V_{i,j-1})+(1/Z_{i+1})(V_{ij}-V_{i+1,j})$$

$$+(1/Z_{j+1})(V_{ij}-V_{i,j+1})+ YV_{ij} = 0. \qquad (2)$$

For a uniform network after making the associations, equation (2) reduces to the finite difference form of (1).

The TL analogy is useful in the study of the source and boundary conditions that should be applied to the mesh. Along the sides of the mesh, Dirichlet (using Cagniard values) or homogeneous Neumann conditions have been used. Ideally, the mesh should extend to very large distances before either of these conditions is applied. (In fact the Neumann condition is never mathematically correct). As the computer has limited storage space and the fields are sought only in the region of the non-homogeneity, it is desirable to apply a boundary condition close to the non-homogeneity. The condition must simulate the response of a homogeneous line extending to infinity, where the voltage is that corresponding to a Cagniard field. Such a condition is the impedance boundary condition of electrical engineers. The characteristic impedance is

$$Z_c = \sqrt{Z/Y} .$$

This condition can be imposed much closer to the non-homogeneity than the previous, without seriously distorting the true field values. More of this will be said in a later section.

Similarily along the bottom edge, a characteristic impedance corresponding to a half-space is used for the boundary condition.

The source conditions for the two cases are different, as has already been discussed. For the $\vec{H}$ polarised case, the source condition is just that of a constant voltage (for constant H_x) imposed along the edge z = 0. For $\vec{E}$ polarisation, a source of constant H_y (and thus, E_x) at some height in the ionosphere is required. This must be far enough away from the non-homogeneity in the earth, that the field near the source is always undisturbed. In the TL analogy an air layer with a constant voltage at the top is placed above the ground to simulate the source. It is known that, as the conductivity can safely be assumed to be zero, the field obeys Laplace's equation in air. In the TL analogy, if the conductivity is zero, the values of Y for the air layer are also zero. Hence, no current flows to ground in the air layer or there are no sources or sinks and the voltage is conservative or obeys Laplace's equation as desired.

The above conditions fully specify the network problem and it is now possible to procede to solve the set of equations of the form (2). Then, the associations can be made to obtain the MT field components. For a mesh of m x n points, there is a system of m.n x m.n linear equations to be solved. In practice, the values of m and n are around 30 or 40, so that a very large coefficient matrix results. However, the matrix is sparce and can be inverted by using a matrix analogue (developed by GREENFIELD (1965)) to the well-known alogarithm for the solution of tri-diagonal equations.

These methods were tested on the fault problem of d'ERCEVILLE and KUNETZ (1962). The numerical results agree exactly, to within the four figure accuracy of the calculation, with the analytical results. Both are represented by the solid lines in Fig. 4 . Using the law of similitude, one curve can be used to represent the whole range of y and T values through the scaled horizontal axis. The discontinuity at y = 0, the fault, is $(\rho_2/\rho_1)^2$ or 10^4 as given by the theory.

This example provides a good study for the influence of the boundary conditions. The dots on the low resistivity side of the H polarised curve, represent the deviations in the field component introduced by the impedance boundary condition. In all cases, for the periods corresponding to the value of the horizontal axis, the points are on the boundary only one grid unit (2 km) from the fault. It is seen that the errors introduced are always small, around 2%, even when the apparent resistivity itself changes by more than one order of magnitude from the Cagniard (true) value. Hence, the impedance condition can be applied close to non-homogeneities, for a large range of periods, and still not adversely affect the field values.

The values of the field components can be represented for all periods by the single curve of figure 4 because the geometry of the cross section is infinite. That is, the dimensions of the profile can be scaled in terms of the skin depth of one of the media. For a finite-sized structure, such as the Rhine graben, where the size and conductivity values must be the same for all periods, this simple representation can no longer be used.

It is suggested, that for a finite-sized structure, the method of plotting as shown in Fig. 5 be used. This is a contour map of apparent resistivity plotted against the y distance along the cross section and the period (on a logarithmic scale). Now the normal apparent resistivity profile for any point on the cross section can be obtained by taking the section through the point parallel to the period axis. Similar plots for the H polarised phase, the E polarised apparent resistivity and the E polarised phase can be constructed. These can all be used together to obtain an interpretation for a given MT profile.

References:

CAGNIARD, L.: The basic theory of the magneto-telluric method of geophysical prospecting. Geophysics, 18, 605-635, 1953.

d'ERCEVILLE, I. and G. KUNETZ: The effect of a fault on the earth's natural electromagnetic field. Geophysics, 27, 651-665, 1962.

GREENFIELD, R.: Two dimensional calculations of magnetic micropulsations resonances. Unpublished Ph. D. thesis, Department of Geology and Geophysics, MIT, 1965.

SCHMUCKER, U.: Anomalies of gemagnetic variations in the southwestern United States. Manuscript in press, 1967.

Transmission Line Analogy for Maxwell's Equations

Transmission Line	Maxwell
$\vec{\nabla} V = -Z\vec{I}$	$\vec{\nabla}\times\vec{H} = \sigma\vec{E}$
$\vec{\nabla}\cdot\vec{I} = -YV$	$\vec{\nabla}\times\vec{E} = -i\mu\omega\vec{H}$

Units

V (volts)	
$\vec{I}$ (amps/m)	rationalized mks
Y (mhos/m^2)	
Z (ohms)	

Fig. 1

Equations for One Vector Polarized along the Strike of a Two Dimensional Structure

$\vec{H}$ Polarized		$\vec{E}$ Polarized
H_x, E_y, E_z	V, I_y, I_z	E_x, H_y, H_z
$\frac{\partial H_x}{\partial y} = -\sigma E_z$	$\frac{\partial V}{\partial y} = -ZI_y$	$\frac{\partial E_x}{\partial y} = i\mu\omega H_z$
$\frac{\partial H_x}{\partial z} = \sigma E_y$	$\frac{\partial V}{\partial z} = -ZI_z$	$\frac{\partial E_x}{\partial z} = -i\mu\omega H_y$
$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -i\mu\omega H_x$	$\frac{\partial I_y}{\partial y} + \frac{\partial I_z}{\partial z} = -YV$	$\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = \sigma E_x$

Associate

H_x	V	E_x
E_z	I_y	$-H_z$
$-E_y$	I_z	H_y
σ	Z	$i\mu\omega$
$i\mu\omega$	Y	σ

Fig. 2

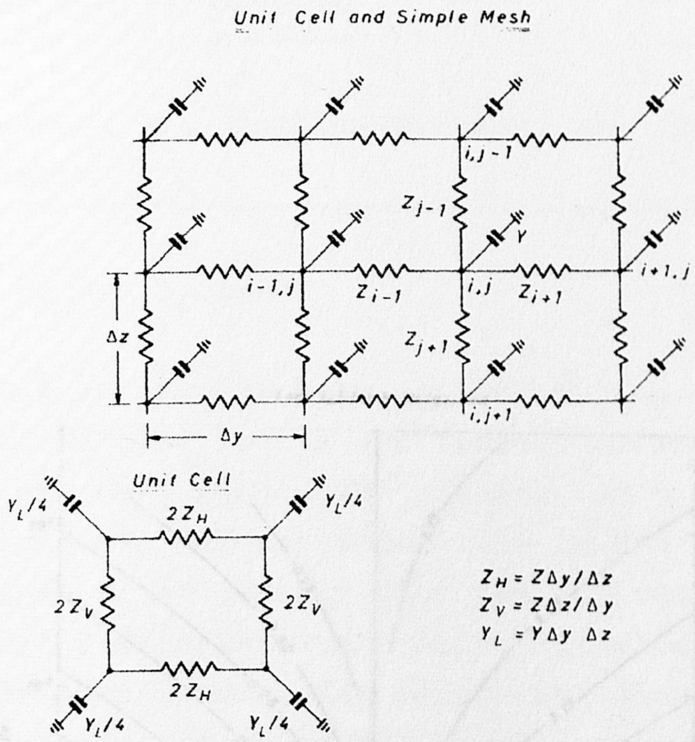

Unit Cell and Simple Mesh
i,j-1
Zj-1
Y
i-1,j
Zi-1
i,j
Zi+1
i+1,j
Δz
Zj+1
i,j+1
Δy
Unit Cell
YL/4
2ZH
2ZV
ZH = ZΔy/Δz
ZV = ZΔz/Δy
YL = YΔy Δz

Fig. 3

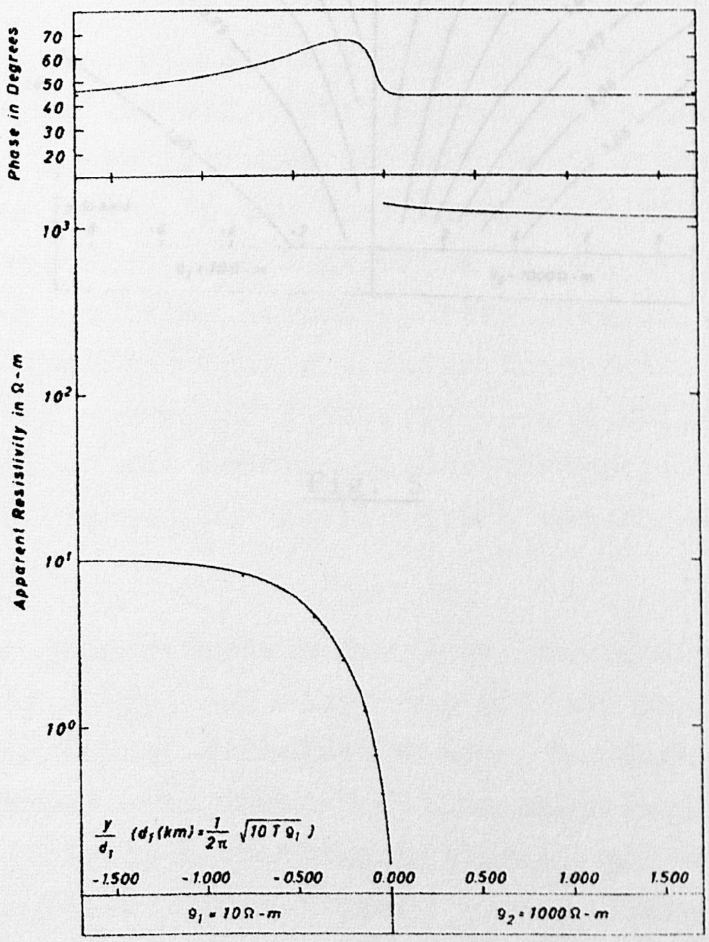
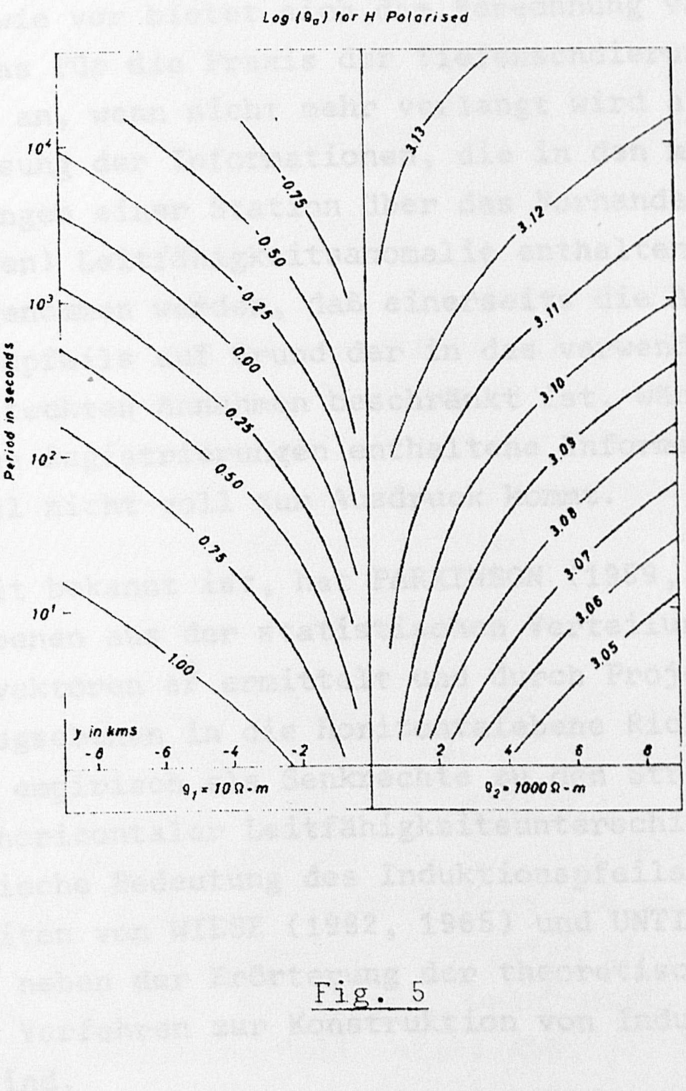

Phase in Degrees
70
60
50
40
30
20
Apparent Resistivity in Ω-m
10³
10²
10¹
10⁰
y/d1 (d1(km) = 1/2π √(10 T ϱ1))
-1.500
-1.000
-0.500
0.000
0.500
1.000
1.500
ϱ1 = 10 Ω-m
ϱ2 = 1000 Ω-m

Fig. 4



Log (ρa) for H Polarised
Period in seconds
10⁴
10³
10²
10¹
-0.75
-0.50
-0.25
0.00
0.25
0.50
0.75
1.00
3.13
3.12
3.11
3.10
3.09
3.08
3.07
3.06
3.05
y in kms
-8
-6
-4
-2
2
4
6
8
ρ1 = 10 Ω-m
ρ2 = 1000 Ω-m

Fig. 5