Propagation of signal-to-noise ratio into semi-airborne EM inversions

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Introduction

Semi-airborne EM (sAEM) transfer functions are estimated by regression over Fourier coefficients over a (small) number of time windows and binned frequencies. The estimation error is determined from the prediction error. However, Fourier coefficients themselves are afflicted with noise and thus erroneously estimated. Here, we make an attempt to incorporate the noise level as weighting coefficients in a weighted least squares regression (WLS).
Reliable error estimates are crucial for inverting data; we show 2D inversion results with new error estimates for a test data set using a scalar magnetometer (OPM).

Approach

Magnetic transfer function:
$\mathbf{B}(\omega) = T(\omega)\,\mathbf{I}(\omega)$
→ univariate problem [4]: $\mathbf{d}=\mathbf{G}m$

From ordinary least squares (OLS)
$m^{est} = [\mathbf{G}^T\mathbf{G}]^{-1}\mathbf{G}^T\mathbf{d}^{obs}$
$[\text{cov } m^{est}] = \sigma_d^2[\mathbf{G}^T\mathbf{G}]^{-1}$

To weighted least squares (WLS)
$m^{est} = [\mathbf{G}^T\mathbf{w}_e\mathbf{G}]^{-1}\mathbf{G}^T\mathbf{w}_e\mathbf{d}^{obs}$
$[\text{cov } m^{est}] = \sigma_d^2[\mathbf{G}^T\mathbf{w}_e\mathbf{G}]^{-1}$

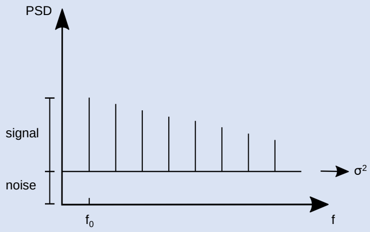

Fig. 1 Sketch: theoretical line spectrum at fundamental frequency f_0 and harmonics superimposed on a noise floor. The variance for each Fourier coefficient is given by the noise at that point.

Time series
Magnetic flux density (TMI) contaminated with noise

↓ STFT

Frequency domain
Extract noise floor as data variance σ^2 for $f=f_0$ and harmonics

↓ binning

Transfer function estimation
$m^{est} = [\mathbf{G}^T\,\mathbf{w}_e\,\mathbf{G}]^{-1}\mathbf{G}^T\,\mathbf{w}_e\,\mathbf{d}^{obs}$
Univariate regression using WLS with $\mathbf{w}_e = [\text{cov } \mathbf{d}]^{-1}$

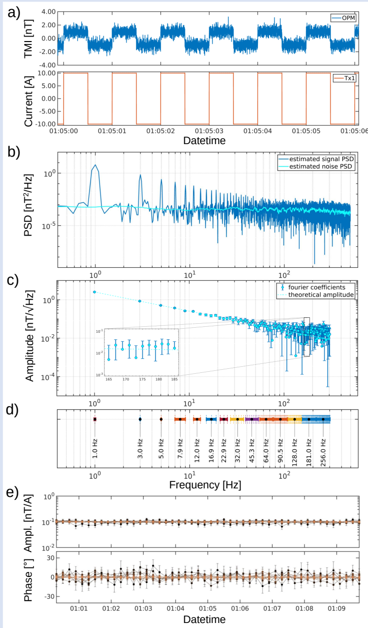

Fig. 2 Updated processing flow for sAEM data using an OPM. Illustrated for synthetic data.
a) Time series: 10A input current (red), 1nT measured amplitude (blue) with Gaussian noise (σ=0.5nT).
b) Estimated PSD for one time window (blue) with peaks at f_0 and harmonics. Estimated noise PSD (turquoise) by performing a moving average on signal PSD.
c) Fourier coefficients with errors estimated from the noise floor in b). Dashed line (turquoise) marks the theoretical values at the absence of noise.
d) Bandsetup for the frequency binning of Fourier coefficients.
e) Magnetic transfer function of the synthetic time series from a) shown as amplitude (top) and phase (bottom) for all estimation frequencies from d).

Case study – Namibia Hope ore deposit

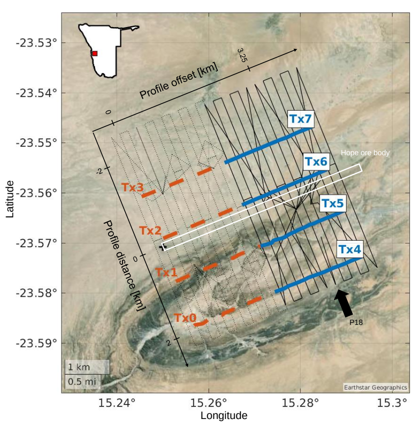

Fig. 3 Sitemap of the sAEM survey area in Namibia (adjusted from [2]).
The hope ore deposit indicated by the white line is located north of the prominent amphibolite belt and crops out to the surface at the map origin.
Flightlines for transmitters Tx0-Tx3 (orange) were flown in 2021 and Tx4-Tx7 (blue) in 2023, respectively.
The black arrow points on profile 18, which is used as data example here.

Inversion results

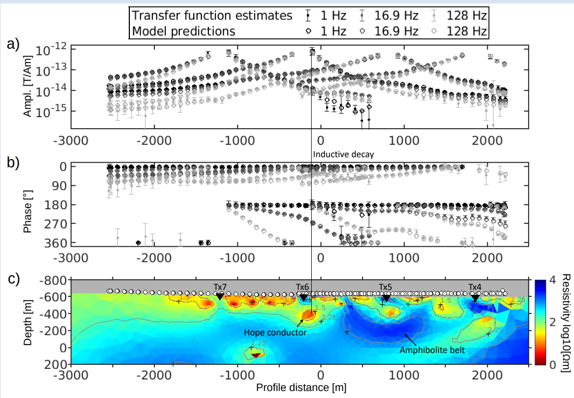

Fig. 4 Transfer function and inversion results for profile 18 (see fig. 3, 3.25 km offset) with the WLS approach using MARE2DEM [3].
a) Amplitude and **b)** phase of the complex transfer function for three frequencies. The circles depict the model responses of the inversion and fit the data to an RMS of 1.1.
c) Final inversion result without additional error floors added to the data prior to the inversion. Hope conductor and resistive amphibolite belt can be resolved.

Conclusion

We assigned **individual error estimates** to observed magnetic flux data prior to the sAEM transfer function estimation, which was achieved in a **weighted least squares** manner in order to take **measurement noise** into account.
We found that
- the impact of noisy time windows can be reduced.
- less masking is required for inversion.
- the main features of the test survey can be resolved by the inversion without adding additional error floors.

References
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