# CSEM modelling of different sources in the RMT frequency band including displacement currents

**S. Schöttle[1]*, M. Smirnova[1], M. Smirnov[2], A. Grayver[1], P. Yogeshwar[1], B. Tezkan[1]**

[1]Institute of Geophysics and Meteorology, University of Cologne [2]Division of Geosciences and Environmental Engineering Luleå University of Technology
*Contact: stefan.schoettle@uni-koeln.de, www.geomet.uni-koeln.de

Deutsche Forschungsgemeinschaft

University of Cologne

## Introduction

Radio-magnetotellurics (RMT) is a passive electromagnetic (EM) technique often used for shallow environmental and geotechnical applications. The method uses remote radio antennas broadcasting in a range of around 10-1000 kHz as transmitters. Due to the natural limitations of RMT (depth of investigation and dependency on remote radio transmitters), the technique has been extended in the past decade to use controlled sources in a wide frequency range. Here, we term this extension 'CS/RMT' measurements, as it combines the source near and intermediate field zone (CSEM) with the plane wave RMT range. The CS/RMT measurement frequency range extends from 1-1000 kHz. Up to now mostly the far-field has been considered. However, considering the source near and intermediate zone provides several advantages over the prior approach: (i) easier logistics, as far-field conditions do not have to be maintained (i.e., the source does not need to be placed far away from the receivers); (ii) better signal-to-noise ratio; (iii) combined resolution of CSEM and RMT, among others. In comparison to lower frequency CSEM, three aspects must be considered when modelling CS/RMT: (i) given technical limitations, precise synchronization of the source current with the receiver can not be established; (ii) like in RMT (e.g., Kalscheuer et al. 2008 [1]), displacement currents must be considered in CS/RMT, especially in regions of high resistive bedrock; (iii) fast varying fields need a highly accurate 1D background solution to stabilize the 3D forward solver. To overcome (i) we use full impedances instead of E/I, B/I-fields, even in the near & intermediate zone. Here, we show the effects & difficulties of fulfilling (i – iii) by modelling full impedances on a resistive subsurface including displacement currents.

## Impedance tensor in Tensor-CSEM

Like Li and Pedersen 1991 [5] we consider the full impedance tensor by measuring two independent source polarisations:

$$\begin{pmatrix} E_x^X & E_x^Y \\ E_y^X & E_y^Y \end{pmatrix} = \begin{pmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{pmatrix} \begin{pmatrix} H_x^X & H_x^Y \\ H_y^X & H_y^Y \end{pmatrix},$$

with: superscript denoting the source polarization and subscript the field component. The 1D primary field solutions included in our 3D code *MR3DMod.py* (based on [2], [3], [4]) handles bipoles and extended inductive sources. Even so, for the sake of simplicity, we only consider perfectly x- and y-polarised horizontal point dipoles in this work.

## Influence of displacement currents on Impedances

We show the influence of displacement currents for a highly resistive 1D model reasonable for the Fennoscandian Shield (Figure 1a) by comparing the full solution and the quasistatic solution of $\mathrm{Z}$. The source field is generated by two electric dipole sources. By using a receiver on the diagonal, $\mathrm{Z}$ becomes symmetrical [5]. For this reason, we show only $Z_{xx}$ and $Z_{xy}$.

**Frequencies < 2500 Hz;** *near field (DC) zone*, negligible influence of displacement currents ($\sigma/(\omega\varepsilon) > 230 \gg 1$):

- $\mathrm{Z} \propto 1/(\sigma r)$, with $\sigma$: apparent conductivity and *r:* source receiver separation
- $\mathrm{Z}$ independent of frequency

**2500 Hz < Frequencies < 600 kHz;** *intermediate field zone:*

- $Z_{xy}$ component: minor deviations between quasistatic solution and full solution
- $Z_{xx}$ component: both cases show sign reversal in complex and real part of $Z_{xx}$:
  - large differences in extrema of $Z_{xx}$, faster increase and decrease of $Z_{xx}$ in full solution
  - frequency range between first maximum and last minimum is "squeezed" in full solution

**Frequencies > 600 kHz;** *far field zone:*

- $Z_{xx}$ component: quasistatic solution converges to quasistatic far field solution, whereas the full solution diverges with $|\Re(Z_{xx})| \approx |\Im(Z_{xx})|$

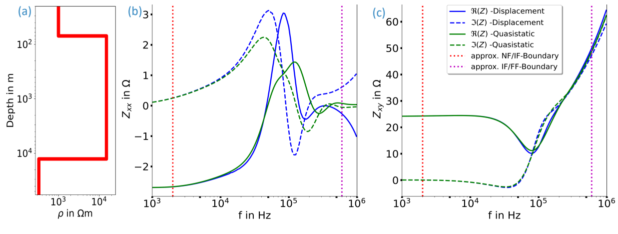

**Figure 1:** *(a)* ***Simple 1D model****, reasonable for the Fennoscandian Shield. (b)* ***Modelled $Z_{xx}$*** *and (c)* ***modelled $Z_{xy}$*** *in the quasistatic case (green) and the case including displacement currents (blue) for a receiver 400 m diagonally away from a x- and y- polarized HED with the given model in Figure 2. The modelled response including displacement currents was generated via quadrature with extrapolation based on Key 2012 [6]. We used a quadrature order of 1200.*

## Conclusions

Like in RMT, displacement currents must be considered for resistive structures in high frequency CS/RMT. Especially in the intermediate zone, the extrema of the diagonal components of $\mathrm{Z}$ increase by at least a factor of two in comparison to the quasistatic solution. Although this increase is visible in all integral solver techniques, the shown solutions deviate more than an error floor of 1% in higher frequencies. Thus, high quadrature orders are necessary. This expands the time needed for calculating the 1D background field in a secondary field approach. Still, the background solution has not to be calculated in every iteration step during an inversion, which is why the high calculation time might be acceptable.

## Acknowledgment

The project "Tensor 3D interpretation of CSRMT data with novel high-frequency sources considering displacement currents and anisotropy" was funded by the German Research Foundation (DFG) under grant number 444968617.

## Instability of $Z_{xx}$

To reach the result shown in Figure 1 b) a high quadrature order of nQuad=1200 was used. In 3D modelling this is not feasible (lack of speed, high memory usage). Therefore, digital linear filters (DLF) are the reasonable choice. Figure 2 shows the relative error (relError) of $Z_{xx}$ between the solution in 1b) and several faster options.

- **Near field zone:** relative error $< 1\,\%$ for all shown cases
- **Intermediate field zone and far field zone**:
  - Frequencies < 50.000 Hz: relError $< 1\,\%$ for all given cases (except nQuad = 25)
  - Frequencies < 100.000 Hz: close to the maximum of $Z_{xx}$; not even nQuad = 600 reaches relError $\leq 1\%$ → high precision calculation will be necessary!
  - Frequencies > 100.000 Hz:
    - relError of nQuad = 300 & nQuad = 600 $< 1\,\%$ for all frequencies → Lower quadrature order is sufficient
    - Both checked DLF reach errors comparable to nQuad = 25 (mostly $> 1\,\%$). → No literature or designed DLF (via [7]) reaches $< 1\,\%$

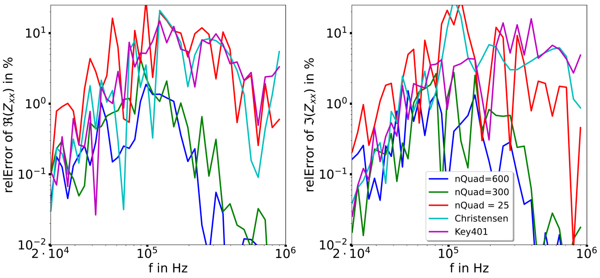

**Figure 2 :** ***(low rolling average of) relative Error to solution calculated with nQuad = 1200,*** *we calculated solutions via different lower quadrature numbers of nQuad = 600, nQuad = 300 , nQuad = 25 and DLF including a 1000 point long filter given by Christensen 1990 [8] and a filter by Kerry Key 2009 [9].*

## Comparison of electrical and horizontal magnetic dipole

Like for the electrical dipole sources (HED), horizontal magnetic dipole (HMD) sources can also be used to get full impedances. The results are shown in Figure 3.

- **In low frequencies:** $\vec{E}_{\text{horizontal}} \approx 0$ for HMDs, thus the components of $\mathrm{Z}$ vanish.
- **Intermediate frequencies:** Intermediate zone for HMD shifted to lower frequencies and broader than for HED → caused by field behaviour of $\vec{B}$ and $\vec{E}$
- **In high frequencies**: Even for nQuad=2000, $Z_{xx}$ of HMD oscillates → physical ?

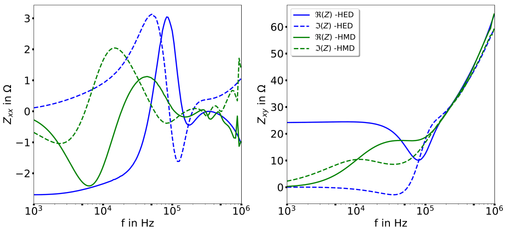

**Figure 3:** ***Modelled $Z_{xx}$*** *and* ***modelled $Z_{xy}$****: the solution given by two HED (blue) and by a HMD (green). The result was generated by solving the Hankel integrals with nQuad= 1200*

## References

[1] **Kalscheuer, T.; Pedersen, L.B.; Siripunvaraporn, W., 2008**. Radiomagnetotelluric two-dimensional forward and inverse modelling accounting for displacement currents. Geophysical Journal International, Vol. 175, pp.486-514
[2] **Løseth, L. O. ; Ursin, B.;** 2007. Electromagnetic fields in planarly layered anisotropic media. Geophysical Journal International, Vol. 170, No. 1, pp.44–80
[3] **Streich, R.;** 2009. 3D finite-difference frequency-domain modeling of controlled-source electromagnetic data: Direct solution and optimization for high accuracy**.** Geophyiscs, Vol. 74, No. 5, pp.95-105
[4] **Cherevatova, M.; Egbert, G.D.; Smirnov, M.Yu**. 2018. A multi-resolution approach to electromagnetic modelling. Geophysical Journal International, Vol. 214, No. 1, pp.656-671
[5] **Li, X., and L. B. Pedersen**; 1991. Controlled source tensor magnetotellurics. Geophysics, 56, pp.1456–1461, doi: 10.1190/1.1443165.
[6] **Key, K.** 2012. Is the fast Hankel transform faster than quadrature?. Geophysics*, Vol. 77, No. 3, pp.F21-F30*
[7] **Werthmüller, D., Key, K. and Slob, E.C.**, 2019. A tool for designing digital filters for the Hankel and Fourier transforms in potential, diffusive, and wavefield modeling. Geophysics, 84(2), pp.F47-F56.[7] Li, X., and L. B. Pedersen; 1991, Controlled source tensor magnetotellurics, Geophysics, 56, 1456–1461, doi: 10.1190/1.1443165.
[8] **Christensen, N.B.**, 1990. Optimized fast Hankel transform filters 1. Geophysical prospecting, 38(5), pp.545-568.
[9] **Key, K.,** 2009. 1D inversion of multicomponent, multifrequency marine CSEM data: Methodology and synthetic studies for resolving thin resistive layers. Geophysics, 74(2), pp.F9-F20.