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RELIABILITY ANALYSIS OF RIVER EMBANKMENTS-- USING ANALYTICAL METHODS AND FINITE ELEMENTS --

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ABSTRACT: The presented probabilistic analysis for river embankments enables a comparative consideration of the reliability of dike stretches taking into account all relevant hydraulical and geotechnical input parameters and their uncertainties. For a case study at the Elbe river in Germany, a reliability analysis has been performed and results have been compared to dike failure statistics. A reliability water level and a reliability freeboard are designated as useful indicators to judge the degree of protection of a dike section. An existing model for slope instability is extended toward a probabilistic Finite-Element analysis which accounts for the effect of a transient seepage on the dike stability. The so-called FORM-ARS approach is shown to be an efficient probabilistic procedure to incorporate results of deterministic numerical analyses into a reliability analysis and to satisfy the need for an acceptable computational effort.
For further project information, the reader is referred to an accompanying paper (Merkel and Westrich, 2008).

Key Words: Reliability Analysis; River dike stability; Failure modes; Finite-Element-Method.

1. INTRODUCTION

The aim of a risk analysis in flood protection is to get a systematic judgement of the flood risk under cost-benefit aspects. The flood risk is defined as the product of failure probability and the allocated vulnerability of the flooded area. As the input data for the risk analysis itself contains statistical uncertainties, one cannot fully rely on its results. For sure, results cannot be communicated as an absolute flood risk and potential danger to the public. However, dike sections can be compared and those sections can be identified where flood protection should be improved first. Comparing the risk due to different failure modes to each other, one can furthermore indicate the most cost-efficient measures to increase safety. The project PCRiver which contributes to the German national research activity RIMAX – Risk Management of Extreme Flood Events has the aim to extend the existing software PC-Ring for the determination of the failure probability mainly of sea dikes to river dikes in Germany.

In recent years, risk analysis has been introduced successfully into the design practice of river dikes in Germany (Mueller, 2007). The governmental authorities are enforcing the establishment of maps that indicate the flood plains with respect to the return period of the flood all over the country (Merz and Gocht, 2003). The concepts shall now be extended to the European Union (Directive 2007/60/EC). However, only a failure of the dike due to overflow is considered.

Since the 1990s, the Dutch have taken into account also other relevant failure modes for the reliability analysis of sea dikes. In order to determine the degree of safety of the Dutch ring dike system, the software PC-Ring has been developed which takes into account the failure mechanisms overflow / wave overtopping, uplift / piping, slope instability and damage of the revetment with erosion of the dike body. Each failure mode is formulated by a corresponding limit state equation (Steenbergen et al., 2004). PC-Ring 4.0 allows for the calculation of a failure probability taking also the combination of the failure mechanisms and the effect of the length of dike sections into account (Vrouwenfelder, 2006). Therefore, it provides information about the degree of safety behind the dike ring.

Paragraph 2 presents the application of PC-Ring to the Elbe river in Germany and compares the results to dike failure statistics from a severe flood in 2002. Paragraph 3 deals with a probabilistic Finite-Element analysis which takes the effect of transient seepage, impermeable cores and drainage systems on the dike stability into account. The First Order Reliability Method with Adaptive Response Surface (FORM-ARS) is used as probabilistic calculation technique. The paper finally draws conclusions and gives an outlook to the adapted software PCRiver for priorisation of flood protection measures in Germany.

2. CASE STUDY AT THE ELBE RIVER

2.1 Hydraulical and geotechnical data base

The Elbe river is one of the major German rivers, having its origin in the Czech Republic, flowing across Eastern and Northern Germany into the North Sea. It was severely hit by two major floods in 2002 and 2006. The hydraulical boundary conditions are defined by a set of discharges, Q, with respect to a return period, T. A so-called workline is defined by equation 1 (Westrich et al., 2007) where a and b are the coefficients of the logarithmic function to be determined from historical discharge statistics.

[1] $Q(T) = a \cdot \ln T + b$

The exceedance duration line which can also be derived from historical discharge statistics enables to determine a corresponding duration of the flood wave N. Furthermore, a hydrodynamic-numerical flow model needs to be set up to deliver water levels h at a dike section also for extreme discharge conditions and a variability of these water levels. For details the reader is referred to the accompanying paper. (Merkel and Westrich, 2008).

A 6,5 km long river dike stretch in Eastern Germany near the city of Torgau is considered for a case study. The dikes were mostly constructed out of a silty loam which is available in the vicinity of the river. Also the upper layers of the dike foundation down to a depth of 6 metres consist of loam. Beneath, more permeable sand and gravel layers can be found. At distances of approximately 400 metres along the dike axes, the soil was directly investigated. At greater distances, data about standard penetration tests is available. At selected locations, clay classifications and shear tests were performed. The available cross sections and longitudinal sections are used for a geostatistical evaluation. First, scales of fluctuation are determined with a semivariogram technique (Baker and Calle, 2006). These are used for a Point Kriging procedure which determines mean values and standard deviations in the centre points of the dike sections from spatially distributed point data (Moellmann et al., 2008).

2.2 Failure probabilities for the significant failure modes and combined failure probability

After a validation of the computational results for the design water level with a return period of 100 years, comparisons can be made between dike sections and failure modes as listed in table 1. Here, the inverse of the annual failure probability, i.e. the return period of failure per year, is presented because those numbers can be understood more easily than the failure probability. It appears from table 1 that overflow / wave overtopping is the governing failure mode for most dike sections. However, one would overestimate the safety of most dike sections if other mechanisms were disregarded. Indeed, there is a significant

Table 1: Comparison of the return periods of the significant failure modes and the combined failure probability for the case study at the Elbe river

Return period of failure per mechanism [years]		Overflow / Wave overtopping	Uplift / Piping	Slope instability	Damage of the revetment	Combined failure probability
Dike stretch A	Section 1	324	476	> 5000	> 5000	223
	Section 2	429	2865	> 5000	> 5000	398
	Section 3	383	157	> 5000	> 5000	127
	Section 4	322	> 5000	487	> 5000	226
	Section 5	351	> 5000	> 5000	> 5000	351
	Section 6	595	389	> 5000	> 5000	263
Dike stretch B	Section 1	463	> 5000	1709	> 5000	461
	Section 2	448	> 5000	3759	> 5000	448
	Section 3	331	476	> 5000	> 5000	238
	Section 4	302	725	> 5000	> 5000	242
Dike stretch C	Section 1	746	> 5000	> 5000	> 5000	746
	Section 2	610	> 5000	> 5000	> 5000	610
Dike stretch D	Section 1	121	> 5000	> 5000	> 5000	121
	Section 2	143	> 5000	> 5000	> 5000	143
	Section 3	218	> 5000	> 5000	> 5000	218
	Section 4	234	1529	> 5000	> 5000	213
	Section 5	309	> 5000	> 5000	> 5000	309
	Section 6	208	> 5000	> 5000	> 5000	208
Dike stretch E	Section 1	3521	1045	> 5000	> 5000	847
	Section 2	1111	> 5000	> 5000	> 5000	1111

contribution of uplift / piping and slope instability to the combined failure probability. Damage of the dike revetment is usually a significant failure mode for sea dikes, but it is quite unlikely to occur for river dikes due to the negligible wave heights for these river parts. Considering the various dike sections, one observes that section 3 of dike stretch A and sections 1 and 2 of dike stretch D are the dike parts which should get priority for a future dike strengthening, because they have the smallest return periods of failure. It is also observed that the different stretches have different characteristics. For dike stretch D, an increase of the dike crest height will improve the flood protection. On the other hand, dike stretch A should be strengthened by inhibiting the subsoil erosion e.g. by a sheet pile or mixed-in-place wall.

2.3 Comparison to dike failure statistics at the rivers Elbe and Mulde in 2002

The results of the PC-Ring analysis can be compared to dike failure statistics at the rivers Elbe and Mulde for the flood in 2002 (Horlacher, 2005). Both layered and non-layered dike profiles were considered. For non-layered profiles, 38 dike failures were analyzed with respect to the governing failure mode. Table 2 shows the relative frequency of occurrence of the various failure modes during the flood in 2002 and a relative comparison of the failure probabilities for those sections that have a return period of failure of lower than 250 years. The calculated results derived from Table 1 show a similar tendency to the dike failure statistics. Overflow / wave overtopping is the governing failure mode but uplift / piping and slope instability have a significant impact.

Table 2: Comparison of the dike failure statistics of the flood at the rivers Elbe and Mulde in 2002 to the relative failure probability for the significant failure modes determined with PC-Ring

Failure mode	Dike failure statistics (Horlacher, 2005)	Relative failure probability for "weak" dike stretches (PC-Ring)
External erosion (Overflow / wave overtopping)	47%	66%
Subsoil failure and internal erosion (Uplift / Piping)	24%	28%
Macro instability (Slope instability))	29%	6%

2.4 Designation of a reliability water level and a reliability freeboard

The above-mentioned results can be used to define a useful measure for the degree of protection as provided by a particular dike stretch. Knowing about the reliability index β and the corresponding α-factor as a measure for the sensitivity of the water level h on the failure probability, a standard-normalized water level u_h can be defined according to equation 2. The standard-normalized water level u_h can be assigned to a return period T through the standard-normalized probability density function shown in equation 3.

[2] $$u_h = \alpha_h \cdot \beta$$

[3] $$T = \left(1 - \Phi(\beta)\right)^{-1} = \left(1 - \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{u_h} e^{-\frac{1}{2}x^2} dx\right)^{-1}$$

Using the workline in equation 1, the corresponding discharge Q can be calculated and the hydraulic flow model enables to derive a water level. The so-determined reliability water level as indicated in figure 1 is the most probable water level at failure. In addition to that, a reliability freeboard can be defined which indicates the gap between the design water level and the reliability water level. In contrast to the frequently used freeboard, the reliability freeboard not only considers the dike failure due to overflow but also due to all other relevant failure modes and it therefore gives a good indication of the degree of protection by the dike. The reliability freeboard for the Elbe river stretches are shown in figure 2. Depending on the contribution of the remaining failure modes, the reliability freeboard can be smaller than the usual freeboard and it can even become negative if the reliability water level goes below the design water level. It should be mentioned that the definition of a reliability freeboard is not the major aim of a reliability analysis as it is just a semi-probabilistic approach, but it can serve as a well-suited indicator for practitioners.

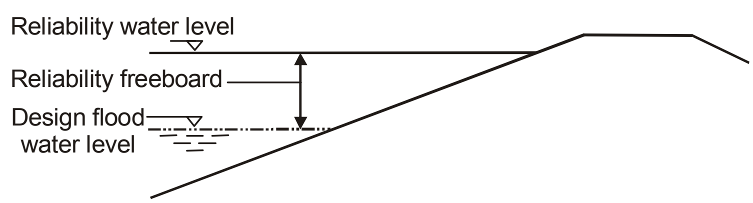

Figure 1: Designation of a reliability water level and a reliability freeboard

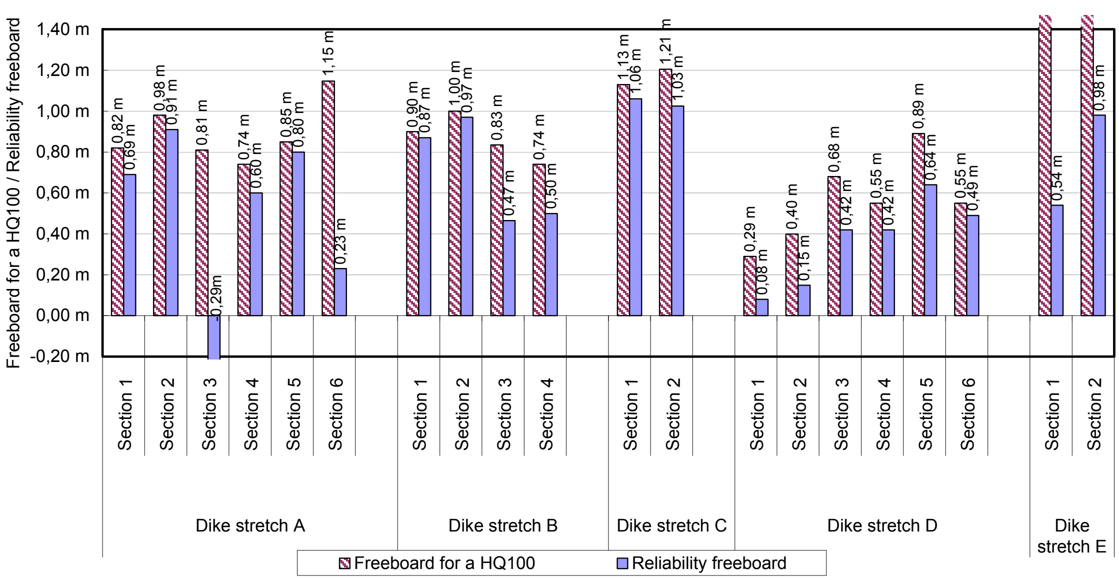

Figure 2: Freeboard for a 100-year flood and reliability freeboard for the case study at the Elbe river

3. FINITE-ELEMENT ANALYSIS OF DIKE STABILITY

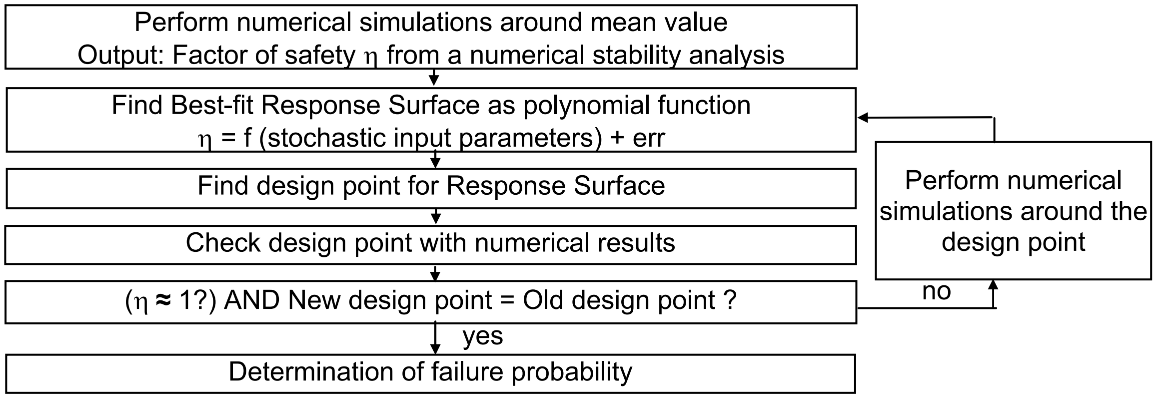

Figure 3: Scheme for the determination of the failure probability using FORM-ARS

The existing model for slope instability performs a slip circle analysis for a given pore water pressure distribution using the effective friction angle φ' and the cohesion c' as stochastic parameters. It is to be extended to a probabilistic Finite-Element analysis of dike stability. This numerical approach has the advantage that the effect of transient seepage on the dike stability can be taken into account. As a consequence, the benefit of a flood retention at upper river areas can be quantified. The First Order Reliability Method with Adaptive Response Surface (FORM-ARS) proves to be an efficient probabilistic calculation technique to satisfy the need for accuracy and an acceptable computational effort (Waarts, 2000). The results of a set of deterministic numerical simulations are replaced by a polynomial function, the Response Surface, for which a First Order Relia-bility Method is performed and the failure probability estimated according to the scheme in figure 3.

3.1 Correlation of water level and duration of the flood wave

For a transient seepage analysis of the dike, the variation of the river water level during the flood over time and its effect on the dike stability is to be modeled. The relationship between water level h and the exceedance duration N is adopted from PC-Ring calculations. Both are correlated via the discharge Q by the exceedance duration line and the hydrodynamic-numerical flow model already mentioned in paragraph 2.1. The shape of the discharge Q over the duration t of the flood is assumed to follow a log-normal function as shown in equation 4:

[4] $$Q(t) = \frac{f_0}{\sigma_{\ln t} \cdot t\sqrt{2\pi}} e^{-\frac{1}{2}\frac{(\ln t - \mu_{\ln t})}{\sigma^2_{\ln t}}}$$

The coefficients f_0 and $\mu_{\ln t}$ are determined in such a way that the discharge fits its exceedance duration N according to the exceedance duration line for two discharges. One discharge complies to a water level at the waterward dike toe and another discharge applies to an extreme water level. The parameter $\sigma_{\ln t}$ is a characteristic for the assumed shape of the log-normal relationship and was selected to be 0,328, according to discharge measurements during a flood at the river Iller in Southern Germany in 2005. Although the hydrodynamic-numerical flow model has been calibrated for steady-state flow conditions, the relationship of water level and discharge is assumed to fit for the unsteady flow conditions as well. The relationship can usually be approximated quite well with the logarithmic function 5 where f_1 and f_2 are determined with a least squares approach according to the location of the dike and according to the calculated water levels for various discharges.:

[5] $$h(Q) = f_1 \cdot \ln Q(t) + f_2$$

3.2 Finite-Element-Model

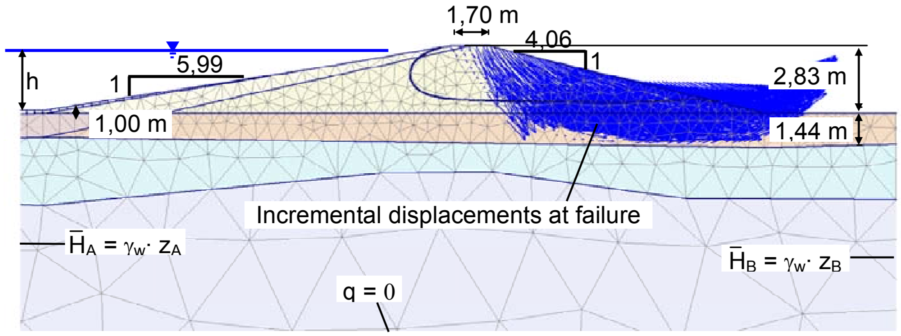

Figure 4: Example dike at the Elbe river for the probabilistic Finite-Element analysis

Table 3: Distributions and statistical moments for stochastic input parameters φ', c' and k

Stochastic input parameters		Mean value μ	Standard deviation σ	Coefficient of variation
Friction angle φ'	normally distributed	17,1°	1,756°	10,3 %
Cohesion c'	ln-distributed	4,65 kN m⁻²	3,72 kN m⁻²	80 %
Permeability k	ln-distributed	5,0E-08 m s⁻¹	2,5E-08 m s⁻¹	50 %

The geometry of a dike located in the area of the case study at the Elbe river is used for the application of the Finite-Element analysis. Three independent stochastic soil parameters are considered for the proabilistic approach, the effective friction angle φ', the cohesion c' and the isotropic permeability k of the dike body and the underlying loam layer. Their statistical moments and their coefficient of variation are shown in table 3. A variance reduction function takes into account the reduction of the variance of the shear strength parameters from a point to the dimensions of the slip surface (Baker and Calle, 2006). Underneath, a cohesionless sand layer with a constant permeability k = 10^{-5} m/s and a friction angle φ' = 32,5° is modeled. In order to prevent a possible failure of the dike due to a rapid drop of the water level, a thin layer with a high shear strength covers the waterside surface of the dike and subsoil.

The boundary conditions of the transient seepage analysis are chosen according to figure 4. The water level is set to the level of the waterward dike toe at the beginning of the numerical simulation. A van-Genuchten approach for the degree of saturation and the relative permeability is used to account for the seepage in the unsaturated zone. A more detailed analysis of the unsaturated behaviour is beyond the scope of this paper. The Finite element mesh has been refined until there is no change in the results for a finer mesh recorded. The time step for the total duration of the flood of about 10 days is chosen to about 2 hours. For the safety analysis, failure of the dike is defined according to the Mohr-Coulomb criterion.

3.3 FORM-ARS application

A transient seepage analysis was performed with the code PlaxFlow 1.5 (Brinkgreve et al., *PlaxFlow*, 2006). First, the river water level was increased up to the maximum water level and later reduced to the waterward dike toe. A safety analysis using the commercial Finite Element code Plaxis 8.6 (Brinkgreve et al., *Plaxis – 2D Version*, 2006) computes the factors of safety η for seven different pore water pressure distributions according to the rise and drop of the water level. The minimum safety factor is adopted for the iteration scheme according to figure 3. For steady-state seepage conditions, the accuracy of the determination of the failure probability has been checked by a set of 300 Monte-Carlo-Simulations for modified statistical moments of the input parameters. The failure probability according to the FORM-ARS-scheme deviates about 3% from the one determined by Monte-Carlo-Simulation (Jin, 2007). Furthermore, the order of the Response Surface Function on the accuracy and the computational effort has been studied. A linear function has been found to be superior to a quadratic Response Surface as it shows a faster convergence of the determination of the design point while producing the same results. A linear

Table 4: Results of the probabilistic Finite-Element analysis with FORM-ARS for three different maximum water levels

Iteration	Number of FE-calculations	α_1	α_2	α_3	Friction angle φ'	Cohesion c'	Permeability k	Factor of safety η	Reliability index β
Maximum water level at 2,83 m			Coefficients of water level shape: μ_{lnT} = 2,4, f_0 = 45000, f_1 = 3,0574, f_2 = 64,371						
Iteration 1	16	0,9564	0,2909	-0,0252	10,4°	1,62 kN m⁻²	4,7E-08 m s⁻¹	1,060	3,94
Iteration 2	32	0,6840	0,7294	0,0017	13,2°	0,69 kN m⁻²	4,5E-08 m s⁻¹	0,985	3,25
Iteration 3	32	0,6679	0,7443	0,0004	13,3°	0,69 kN m⁻²	4,5E-08 m s⁻¹	0,995	3,19
Iteration 4	32	0,6627	0,7489	-0,0043	13,4°	0,68 kN m⁻²	4,5E-08 m s⁻¹	0,996	3,18
Maximum water level at 2,40 m			Coefficients of water level shape: μ_{lnT} = 2,5, f_0 = 42000, f_1 = 3,0574, f_2 = 64,371						
Iteration 1	16	0,6895	0,7243	-0,0048	12,9°	0,64 kN m⁻²	4,5E-08 m s⁻¹	0,996	3,40
Iteration 2	32	0,6843	0,7272	-0,0540	13,0°	0,64 kN m⁻²	4,9E-08 m s⁻¹	0,996	3,41
Maximum water level at 1,20 m			Coefficients of water level shape: μ_{lnT} = 2,6, f_0 = 31000, f_1 = 3,0574, f_2 = 64,371						
Iteration 1	16	0,7630	0,6455	-0,0339	11,4°	0,53 kN m⁻²	4,8E-08 m s⁻¹	failed	4,25
Iteration 2	32	0,7581	0,6519	-0,0156	11,7°	0,58 kN m⁻²	4,6E-08 m s⁻¹	1,004	4,01
Iteration 3	32	0,7524	0,6587	-0,0055	11,8°	0,57 kN m⁻²	4,5E-08 m s⁻¹	1,003	4,01

Response Surface according to the limit state equation Z with the coefficients b_0 to b_3 has been adopted for the transient seepage analysis:

[6] $$Z = \eta - 1 = b_0 + b_1 \cdot \varphi' + b_2 \cdot c' + b_3 \cdot k - 1$$

For each iteration step, 16 or 32 FE-calculations for different values of φ', c' and k are performed, varying the parameters randomly around the design point of the former iteration step. The design point of the last iteration step for the upper maximum water level has been taken as startpoint of the iteration for the lower maximum water level. The results of the analyses are shown in table 4. For three different water levels, the reliability index β and the α-factors of the three stochastic parameters are determined and used as input of PC-Ring-calculations, which computes the annual failure probability. About 240 numerical calculations in total are required to determine the reliability indices for the three maximum water levels.

The reliability index of a failure by slope instability has been determined to $\beta = 4,06$ by PC-Ring which corresponds to an annual failure probability of $2,4 \cdot 10^{-5}$ 1/a. The results are compared to a reliability analysis of the slope instability using the method of slices. For steady-state seepage conditions, a reliability index of $\beta = 3,15$ is computed which corresponds to an annual failure probability of $8,2 \cdot 10^{-4}$ 1/a. Looking at the α-factors, the friction angle φ' and the cohesion c' are dominating the failure. The consideration of the variability of the permeability which is shown to be of minor importance for the dike reliability in this study will greater affect the dike reliability when dikes with impermeable cores or drainage systems are considered. Therefore, the benefit of these measures for the flood risk reduction can be quantified.

4. CONCLUSIONS AND OUTLOOK

The presented reliability analysis which takes into account various failure modes and the uncertainties of the dike resistance is the basis for a reliable flood risk management. For the case study at the Elbe river, a comparable tendency with the dike failure statistics from the flood in 2002 has been found. A so-called reliability water level has been introduced as well as a reliability freeboard. The values give a simple and useful indication of the degree of protection of a particular dike section.

The reliability analysis for slope instability is extended toward a probabilistic FE-analysis which can take the effect of a transient seepage and a zoned dike structure on the dike stability into account. As both water level and the duration of the flood wave depend on one another, their relationship has been studied. The FORM-ARS was shown to be an efficient calculation technique to incorporate results of determistic numerical analyses into a reliability analysis.

The probabilistic FE-analysis is to be extended to zoned dikes and coupled with other failure modes. This should result in an extended software PCRiver. It will provide a tool to water authorities as well as consulting companies in order to take risk-based decisions in river flood protection.

5. ACKNOWLEDGEMENTS

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