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About possible geophysical causes of the decade fluctuations in the length of day

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Fluctuations in the length of day (Δlod) can be caused by temporal variations of the inertia tensor described by the excitation function and by disturbances of the torque balance between core and mantle. Recent models of the atmospheric excitation of the variations of the lod certainly failed with respect to longer periods (e.g., at about 70 years), but seem to be responsible for the annual period, the 22 years period and for a part of the nearly 30 years period. So, another geophysical phenomenon is needed which is responsible for the remaining part of unexplained lod variations. Previous studies of the geomagnetic core-mantle coupling were re-examined using lod values from which atmospherically excited parts were removed. The remaining part of the 30 years period could be explained by core-mantle coupling. Additionally, the torque balance was realized by assuming lower values of the electrical conductivity at the bottom of the mantle. It was concluded that the consideration of geophysical processes within atmosphere and hydrosphere will be important for future refinements of the core-mantle coupling models of the decade fluctuations in the lod.

Key words: Earth – length of day – geomagnetic core-mantle coupling

AAA subject classification: 081; 084

1. Introduction

We will demonstrate that the consideration of the atmospheric excitation of the Δlod results a refinement of the parameters of the model of the core-mantle coupling and will discuss some other possibilities of this kind. As well-known, the atmospheric excitation is certainly responsible for annual variations in the lod, but failed for the explanation of decadal variations (Munk and Revelle 1952; Lambeck 1980; etc.). Bullard et al. (1950) suggested that processes within the outer earth's core are responsible for decade fluctuations in lod, and Rochester (1960) first explicitely worked out the theory of the geomagnetic core-mantle coupling, which was refined by e.g., Stix and Roberts (1984), and Greiner-Mai (1989). Otherwise, the assumed electrical conductivity of the lowermost mantle seems too high compared with laboratory experiments, which was, among other things, the reason for the forced investigations of the topographic coupling (e.g., Le Mouël et al. 1985; Jault and Le Mouël 1989). Contrary to the magnetic coupling, the topographic coupling produces torques, which are too high by two orders of magnitude. So it becomes evident that the process of the excitation of the decade variations of the earth's rotation can not be described by only one mechanism, but by a more complicated geophysical phenomenon, i.e., it must be described by a system of exciting processes. We shall discuss some of them below by using literature or, with respect to magnetic coupling, our own results, respectively. The latter were derived from time series of the geomagnetic field (Hodder 1981; IAGA News 1985), the Δlod (McCarthy and Babcock 1986) and air pressure variations (Voose et al. 1992), beginning in 1900.

2. About atmospheric excitation

In this section, we will give a short review about a former publication (Jochmann and Greiner-Mai 1995). The basic equation for the excitation of Δlod is the third component of the Liouville equation. The atmospheric influence on the lod was described by density variations causing temporal variations of the products of inertia and by changes in the relative angular momentum due to mass motions within the atmosphere (winds). Since wind observations are not very reliable during the above mentioned period of time and density values were not available, both quantities were usually calculated using air pressure data and the geostrophic approximation. The yielding of sea level due to air pressure variations is taken into account by 'inverse barometer', and related parts were considered by land and ocean functions according to Munk and MacDonald (1960) and Lambeck (1980).

Reviving the results of Lambeck and Cazenave (1976), the atmospheric excitation of the lod variations (Δlod) was re-examined by Jochmann and Greiner-Mai (1995) with air pressure data by Voose et al. (1992). Removing the atmospherically excited part of the Δlod, corrected values Δlod1 were produced, and analysed by Jochmann's (1986) Fourier method. The results are re-paint in Fig. 1, here. The figure shows that the 11 and 22 years periods vanish in Δlod1, and the amplitude of the nearly 35 years period decreases. So it can be concluded that the atmospheric excitation is responsible for the 11 and 22 years periods and for a part of the nearly 35 years period. Contrary, the higher period at about 70 years is not influenced by atmospheric excitation and probably due to other excitation mechanisms.

The investigated interval covers a period of about 80 years so that an interpretation of the higher period will be hazardous. Using the whole time series of Δlod (1657-1981), it can be proved that a dominant nearly 70 years period exists (Fig. 2). Because of defective air pressure data before 1900, it can not be compared with related Δlod1. This shows that the existence of a higher period must certainly be accepted, but its interpretation is still open. The above mentioned result (no difference between Δlod and Δlod1) and the presence of a corresponding period in geomagnetic quantities (Greiner-Mai 1993) suggested that certain processes within the earth core are responsible for these and for a part of the 35 years period. The latter will be investigated below.

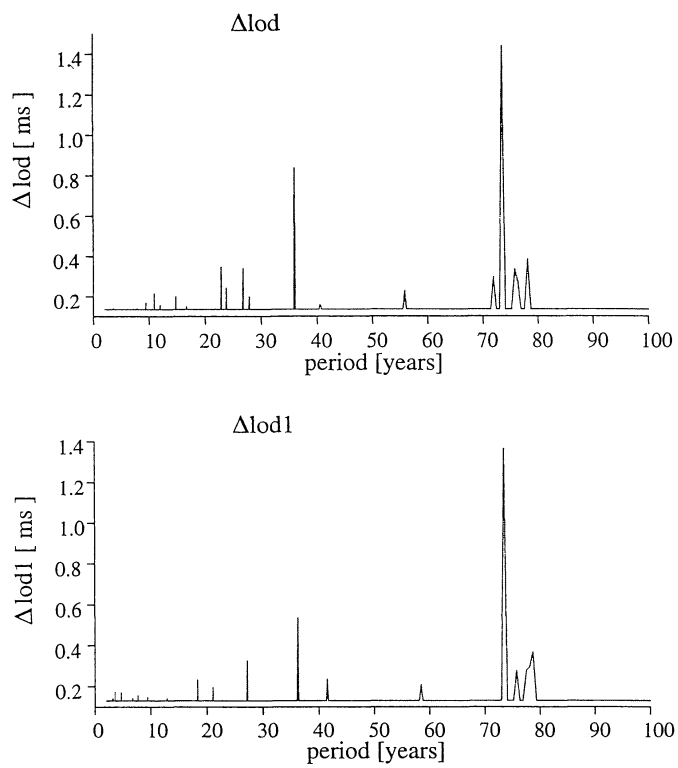

Fig. 1: The spectra of Δlod and Δlod1 for the time series beginning in 1900 (according to Jochmann and Greiner-Mai 1995)

3. The influence of relative core rotations on Δlod1

3.1. The relative core rotation

The velocity field of the relative rigid core rotation is defined by (in a mantle-fixed frame of reference)

$$\mathbf{u} = \mathbf{w} \times \mathbf{r}, \qquad \mathbf{w} = (\mathrm{w}_1, \mathrm{w}_2, \mathrm{w}_3). \tag{1}$$

The frozen-field equation (e.g. Backus 1968),

$$\mathrm{F} = \dot{\mathrm{B}}_\mathrm{r} + (\mathbf{u}_\mathrm{t} \cdot \nabla_\mathrm{H} \mathrm{B}_\mathrm{r}) = 0, \qquad \text{at } \mathrm{r} = \mathrm{r}_\mathrm{c}, \tag{2}$$

is an approximation of the core's induction equation for decade and global variations of the geomagnetic field near the core boundary ($r = r_c$). **u** can be estimated by inverse solutions, provided that the geomagnetic field can be re-extrapolated to $r = r_c$. The radial components B_r, $\dot{B}_r$ were approximately derived from geomagnetic potential field and its secular variation at $r = r_c$. Spherical harmonic expansions up to degree N = 5, published in geomagnetic literature (Hodder 1981; IAGA News 1985) were used. The general problem of the non-uniqueness of inverse solutions was discussed by e.g., Backus (1968), Backus and Le Mouël (1986) and Bloxham (1989), where it was shown that the solutions must be constrained by a reasonable physical assumption. The latter is arbitrarily given here by eq. (1), and the reliability can be examined by comparison of the rotational model with Δlod-data. Values of the components w_i were numerically derived by using maximum likely methods,

$$\int F \cdot F \, dS \doteq \min. \tag{3}$$

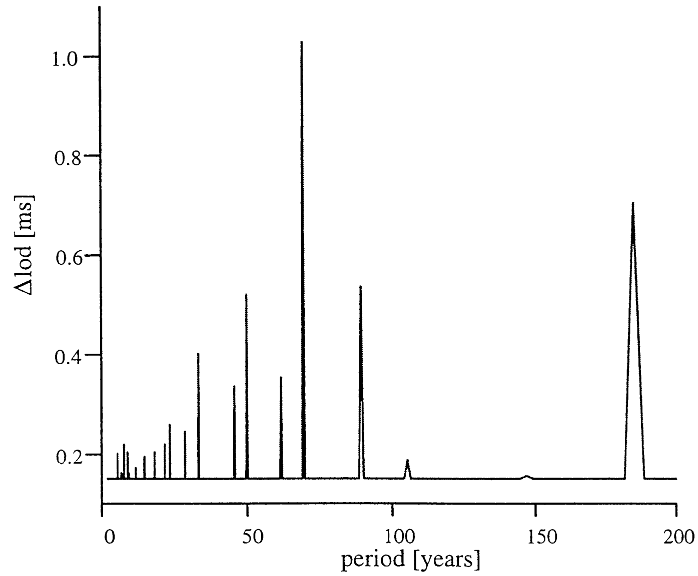

Fig. 2: The amplitude spectrum of Δlod for the time series 1657-1981 (McCarthy and Babcock 1986)

Further details of the formalism were described by Greiner-Mai (1990). w_3 can be compared with Δlod. For a force-free earth model, the balance of the angular momentum results,

$$C_m \cdot \dot{\omega}_3 = L_3 = -C\dot{w}_3, \quad C = (C_c C_m)/(C_c + C_m), \tag{4}$$

where C_c, C_m are the principle moments of inertia of the core and mantle, ω_3 is the angular velocity of the mantle, w_3 is given by eq. (1), and L_3 is the coupling torque. ω_3 can be expressed by $-(2\pi\ \Delta\text{lod})/\text{lod}^2$, so that

$$\delta w_3 = A \cdot \delta\text{lod}, \quad A = (2\pi C_m/\text{lod}^2 C), \tag{5}$$

where δ denotes the reference to the initial point of time, and lod is the reference period of 86400 s. Eq. (5) can be examined by using data of Δlod , and estimated values of w_3. It can be approximately used for Δlod1, too. Disagreements in magnitudes can be removed by taking A (or C) as free parameter of the model, and introducing a certain time shift, τ, caused by the coupling of inert bodies (e.g., Roberts 1972). The estimated values C_c' (instead of C_c) can than be interpreted as moment of inertia of a coupled core shell (Rochester 1960) of the thickness d. The result of the comparison of w_3 and Δlod, Δlod1 by applying r.m.s. methods is given by:

previously used data from Δlod	new data from Δlod	new data from Δlod1
d = 275 km	d = 470 km	d = 297 km

Fig. 3a shows that w_3 seems to precede Δlod1 by about 10 years, which was theoretically expected from the coupling process. Compared with recently derived results, the value has not changed by introducing Δlod1. The temporal behaviour of the curves after 1940 is far from being similar. So the correspondence between Δlod1 and

w_3 can not generally be claimed, and a description of the temporal behaviour by eq. (5) with constant C must be rejected or proved by using longer time series, respectively.

Fig. 3b shows that the spectrum of w_3 differs from that of Δlod1, as it was expected from Fig. 3a. Considering errors of about 15 % in the period lengthes for w_3, corresponding periods of about 27 and 35 years in w_3 and Δlod1 may exist, showing that the model (5) must be embedded into a more complicated system of excitations of Δlod. With regard to the 50 years period in w_3, a corresponding peak was found in Δlod for the longer time series (Fig. 2). So, we again suggested that a correspondence of the longer periods must be proved by using longer time series with less defective geomagnetic data.

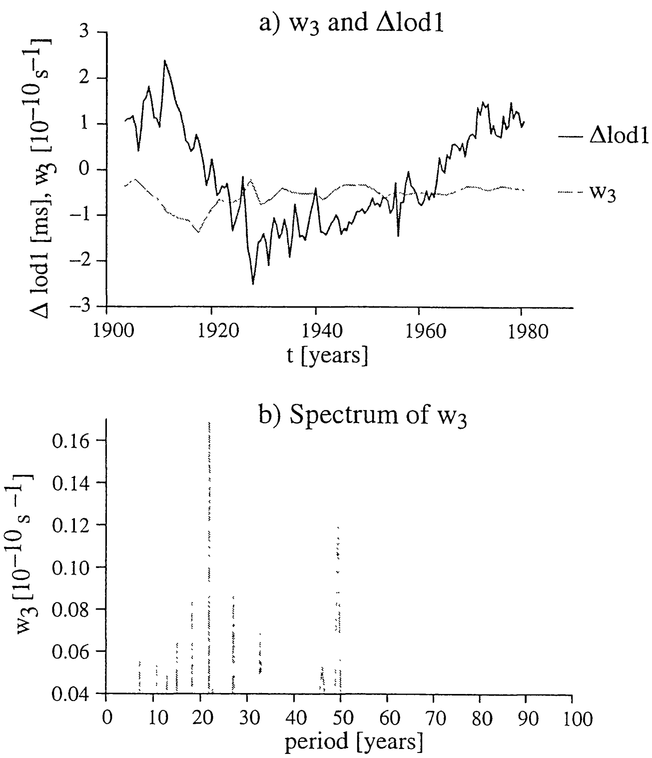

Fig. 3: The temporal variation (compared with Δlod1) and amplitude spectrum of w_3

3.2. The magnetic core-mantle coupling torques

In Section 2.1 we assumed that the torque balance between core and mantle is valid, and the relative rigid rotation is due to that derived from geomagnetic variations. If magnetic coupling was assumed, then the balance between magnetic torques on the mantle and those derived from Δlod1 (L_3^{mech})must be examined. The first part of eq. (4) shows that there is no need to restrict L_3 to the magnetic torques or the generation of them to a pure rotational motion of the fluid, respectively. Therefore, the examination of the torque balance by magnetic torques derived from w_3 can only be a first step of investigation. Differences then may be caused by geomagnetic effects of other core motions, and by other types of torques (e.g., pressure torques).

A first order approximation of the magnetic torque on the mantle was derived from

$$\mathbf{L} = \mu_o^{-1} \int_{V_m} \mathbf{r} \times [(\text{curl}\,\mathbf{B}^1) \times \mathbf{B}^\circ]\, dV, \tag{6}$$

where $\mathbf{B}^\circ$ is the potential part of the geomagnetic field (main field), and $\mathbf{B}^1$ is induced into the mantle by core surface motions. $\mathbf{B}^1$ can be derived from $\dot{\mathbf{B}}^\circ$ by a perturbation solution of the mantle's induction equation (e.g., Stix and Roberts 1984). For this, a non-zero electrical conductivity of the mantle was assumed:

$$\sigma_m(r) = \sigma_o \cdot (r_c/r)^\alpha, \qquad \sigma_o = 3000\ \mathrm{Sm}^{-1}, \qquad \alpha = 30. \tag{7}$$

The assumed value of σ_o is the critical point of the model. It is certainly due to the penetration of decadal periods of $\mathbf{B}^1$ through the mantle (Rotanova et al. 1985), but laboratory experiments, simulating the physical state of the mantle, showed that σ_o must essentially be lower (e.g., Ducruix et al. 1980). So our investigations were also motivated by the examination of possibilities to decrease σ_o in the models used for geomagnetic coupling. The rotational equation of the mantle $C_m \cdot \dot{\omega}_3 = L_3$ was used to derive $L_3 = L_3^{mech}$ from Δlod or Δlod1, respectively, by numerical differentiation. For this we applied a running linear regression over 6 years intervals. The estimated gradient was related to the central point of time of the intervals. Recent estimates, based on data by Hodder (1981) for L_3^1 and by Morrison and Stephenson (1982) for Δlod, showed nearly equal mean values of L_3^1 and L_3^{mech} on the level of about $2.5 \cdot 10^{17}$Nm. Now we will precise these values by new data of Δlod (McCarthy and Babcock 1986), Δlod1, and L_3^1 (adding data from IAGA News 1985). The mean absolute values for the torque variations are (in 10^{17} Nm):

L_3^{mech} from Δlod	L_3^{mech} from Δlod1	L_3^1
2.20	1.82	2.88 .

The higher value of L_3^1 can be fitted to the value of L_3^{mech} by decreasing σ_o in eq. (7) to 1896 Sm^{-1} for Δlod1, showing that the considaration of the atmospheric influence on Δlod may improve the assumed parameters of the geomagnetic coupling. Fig. 4 shows the periodic constituents of the torques. The main periods in the mechanical torques are the nearly 30 years periods, and they correspond with 27 years period in L_3^1. The use of Δlod1 again improves the model of σ_o for the 30 years periods by decreasing the amplitude in Δlod1, and causing a nearly 20 years period which corresponds to the 22 years period in L_3^1. The higher periods are at different places in the spectra and can probably be interpreted when longer time series will be used. The tendency will be shown in the following.

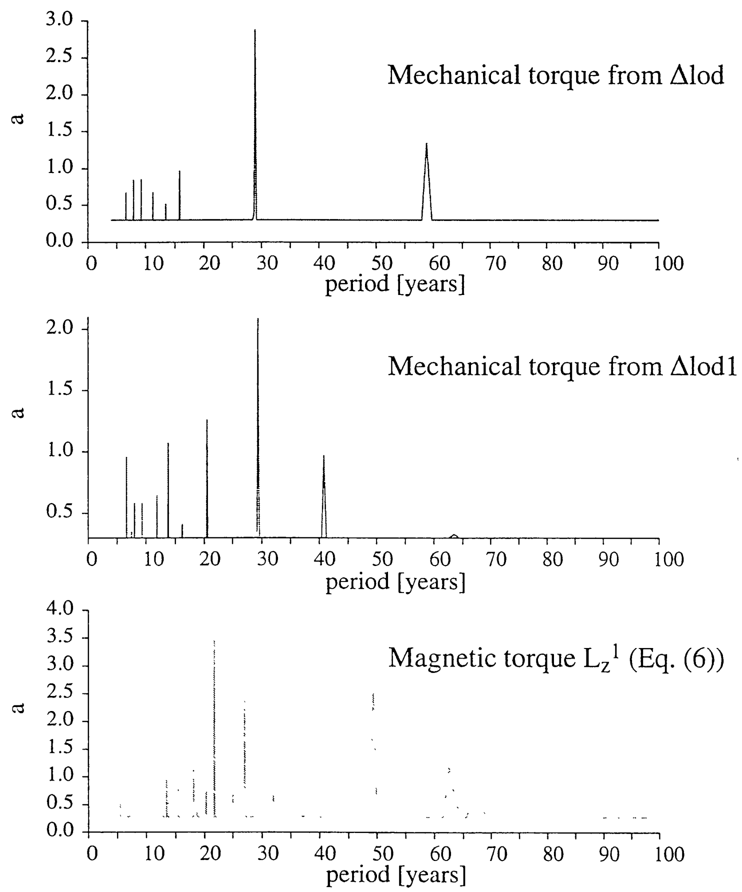

Fig. 4: The amplitude spectra of the mechanical and magnetic torques for the time series beginning in 1900 (unit of amplitudes a: 10^{17}Nm)

Greiner-Mai (1993) investigated longer time series of geomagnetic data composed by values of the Gauss coefficients of different references (Barraclough 1978; Hodder 1981; IAGA News 1985) and showed time series of w_3, Δlod and the coupling tensor. We will complete our previous investigation by giving rough estimates of the torques and their spectra, here. For this, time series, covering the period from 1700 to 1989, were used. The torques can certainly be compared with mechanical torques derived from Δlod, but not from Δlod1. Then the comparison of the mean absolute variations of L_3^1 (7.06 $\cdot 10^{17}$ Nm) with those of L_3^{mech} (2.41 $\cdot 10^{17}$ Nm) shows that a further reduction of σ_o to about 1024 Sm^{-1} is possible. Additionally, Fig. 5 shows that corresponding periods at about 30 and 80 years exist, and the amplitudes may also be fitted by decreasing the σ_o-values by a similar order of magnitude (an example is given in Fig. 5b). The results derived from Δlod1 for the shorter time series suggest that a further reduction will be possible by considering atmospheric excitation, provided that more correct data of air pressure variations (and also for geomagnetic data) will be available for longer time series.

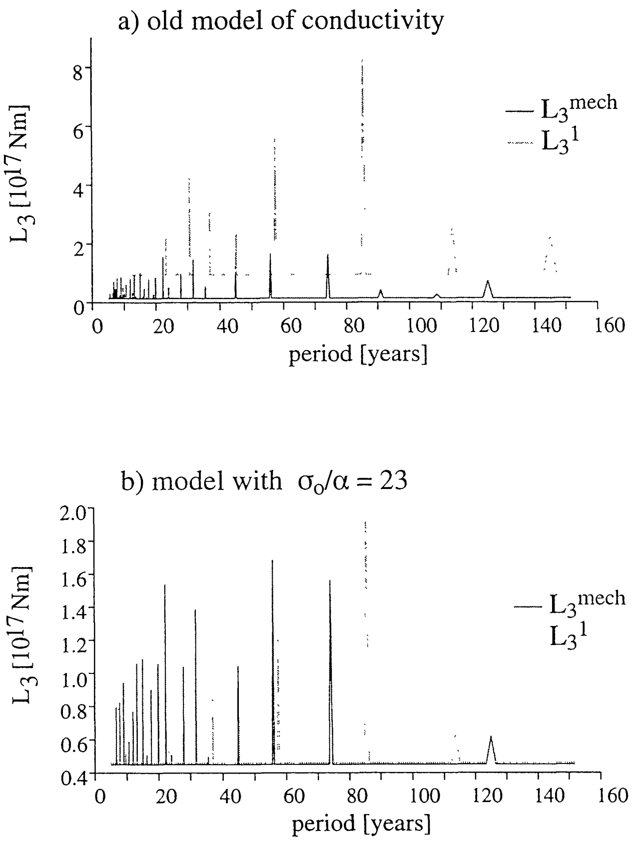

Fig. 5: The comparison of the spectra of the mechanical and magnetic torques for the time series beginning in 1700: a) for the old model of conductivity in eq. (7), b) for a model with $\sigma_o/\alpha = 23$

3.3. The pressure torque

Relative motions near the core surface may affect the mantle rotation by shearing forces, provided that the tangential part of the velocity field hits an effective surface. The latter will appear when the lower boundary of the mantle differs from spherical shape, e. g., by ellipticity or topography, respectively. The velocity of the fluid can be described by the equation for the local angular momentum balance in the inertial frame of reference (Stephani and Kluge 1980)

$$\partial(\varrho\, \mathbf{r} \times \mathbf{u})/\partial t \;+\; \mathrm{Div}[\mathbf{u} \circ (\varrho\, \mathbf{r} \times \mathbf{u})] \;=\; -\,\mathbf{r} \times \mathrm{grad}\, p \;+\; \mathbf{l}^1 \;+\; \mathbf{l} \tag{8}$$

(p – pressure, $\mathbf{l}^1$ – density of the Lorentz torque, $\mathbf{l}$ – density of external torques, Div – tensorial divergence, ∘ – dyadic product).

The global angular momentum balance can be derived by integration over the time dependent volume of the fluid using related rules, e.g., by Smirnow (1964):

$$\frac{d}{dt}\int (\mathbf{r} \times \varrho\, \mathbf{u})\, dV \;=\; \dot{\mathbf{H}}_c \;=\; \mathbf{L}^p \;-\; \mathbf{L}^1 \;+\; \mathbf{L}^e, \tag{9}$$

where $\mathbf{H}_c$ is angular momentum of the fluid, $\mathbf{L}^1$ is the magnetic torque, $\mathbf{L}^e$ are external torques, and $\mathbf{L}^p$ is the pressure torque defined by

$$\mathbf{L}^p \;=\; -\int_V \mathbf{r} \times \mathrm{grad}\, p\, dV \;=\; \int_V \mathrm{curl}\,(\mathbf{r}\, p)\, dV \;=\; -\int_{S_c} p\, \mathbf{r} \times \mathbf{n}\, dS. \tag{10}$$

S_c is the core mantle interface, and $\mathbf{n}$ is the normal unit vector on it. In Sect. 3.2 $\mathbf{L}^1$ was computed as torque on the mantle with positive sign. Therefore, $\mathbf{L}^1$ appears as torque on the core with a negative sign in eq. (9).

The pressure torque vanishes for radial gravitational forces (grad p $\sim \mathbf{r}$), and for spherical symmetry ($\mathbf{n} \sim \mathbf{r}$). Assuming a rigidly rotating fluid with $\mathbf{u} = \mathbf{w}_c \times \mathbf{r}$, the integral on the l. h. s. is due to $\mathbf{w}_c \cdot \mathbf{I}$ ($\mathbf{I}$ - tensor of inertia), and eq. (9) is the Euler-Liouville equation for the fluid in the inertial frame of reference (the difference to a rigid body is given by $\mathbf{L}^p$). Provided that the deviations from radial symmetry of the graviational forces are only caused by the centrifugal force, the pressure torque can be derived from $\varrho\, \mathbf{r} \times [\mathbf{w}_c \times (\mathbf{w}_c \times \mathbf{r})] = -\mathbf{w}_c \times \mathbf{r}\, (\mathbf{w}_c \cdot \mathbf{r}) = \mathbf{w}_c \times \mathbf{w}_c[\mathbf{r}^2\mathbf{E} - \mathbf{r} \circ \mathbf{r}]$ ($\mathbf{E}$-Kronecker tensor), obtaining:

$$\mathbf{L}^p = \mathbf{w}_c \times \mathbf{H}_c \tag{11}$$

Eq. (11) shows that $\mathbf{L}^p$ also vanishes for axial rotations ($\mathbf{H}_c = (0, 0, C_c w_{3c})$), so that the pressure torque will be meaningless for our investigation in Sect. 3. The Liouville equation for the core is given by

$$\dot{\mathbf{H}}_c - \mathbf{w} \times \mathbf{H}_c = \mathbf{L}^e - \mathbf{L}^1, \tag{12}$$

in a mantle-fixed frame of reference (e.g., Sasao et al. 1977; Jochmann 1988), where $\mathbf{w} = \mathbf{w}_c - \omega$ ($\mathbf{w}$ was used in Section 3.1.), and ω is the angular velocity of the mantle.

As suggested from the space-time structure of the geomagnetic variation, the core surface motions are actually more complicated, so that eq. (10) must explicitely be used within the Liouville equation for the rotating system. Therefore, we need a model which relates p to an "observable" quantity, i. e. the velocity field $\mathbf{u}'$ of the core surface region. This may be given by the geostrophic approximation, $2\, \varrho\, \omega \times \mathbf{u}' = -$ grad p, or by the magnetostrophic approximation where the Lorentz force on the r. h. s. is added. Although these equations are not complete, and represent a stationary state, the approximations can be used for the calculation of the pressure torque affecting the mantle rotation, provided that a perturbation in the velocity will be simultaneously compensated by the pressure.

Using published models of the core-mantle topography, recent estimates of this torque showed that the axial part is too large by two orders of magnitude (Jault and Le Mouël 1989), and cannot be responsible for the excitation of the Δlod. Otherwise, this value is needed for the excitation of the decade fluctuations of the polar motion.

The problem of the two refered models of core-mantle coupling is that the components of the related torques are nearly equal, but those derived from observed data differ by two orders of magnitude (10^{17} Nm for the axial components, and 10^{19} Nm for the equatorial components). Therefore, a special distribution of the topographic heights h ($\mathbf{n}$ in eq. (10) is proportional to ∇ h) will be needed to obtain an anisotropic coupling. This model must then be examined e.g., by comparison with the core-mantle topography derived from seismic tomography.

4. Discussion of other influences

We will discuss two other correlations and suggestions. Jochmann and Greiner-Mai (1995) showed that the mean temperature variations and the Δlod, Δlod1 are certainly correlated, but a physical causality could not be proved. It was suggested that the correlation is an apparent one, and may be caused by a third process which independently influences both quatities. The core-mantle coupling seems to be a possible candidate for this process. The suggestion was certainly supported by correlations between mean temperature variations and geomagnetic field variations, but the physical influence of the geomagnetic field on temperature is a matter of speculation up to now, and must still be proved. Positive results then would suggest a feedback of the magnetic field on long term variations of the lod by influencing long term atmospheric processes, too.

Using a certain model of climate change, Jochmann (1993) proved that the mass exchange between the Antarctic ice sheet and the ocean could cause an observable long term effect on Δlod so that long term temperature variation of sufficient magnitudes may influence the earth's rotation by the interaction between the hydrosphere and cryosphere. Otherwise, short period variations are not affected by this process, and the mechanism must be defined as one of some others so that the problem of the separation again appears.

Another mass exchange arises from precipitation and ground water storage. The groundwater storage must theoretically be correlated with air pressure variations, and temperature. So, we expect a certain influence on well-known periods in the excitation of the Δlod, provided that these effects can theoretically be considered in related models of mass exchange between atmosphere, ground and hydrosphere, and needed data series can be constructed.

Long term, and long period decade lod variations can theoretically be caused by changes of the density distribution within the outer core, too. Density variations are due to heterogenous temperature distributions in the D" layer at the bottom of the mantle (e.g., Zhang and Gubbins 1992), and to the decomposition of core material. The influence of Archimedean (Coriolis and magnetic) forces on the torsional oscillation of the so called H-layer (produced by decomposition) was studied by Braginsky (1993) obtaining the 65 years oscillation of the geomagnetic secular variation and the Δlod.

Additionally, the processes influence the pressure torque on the mantle by affecting the core mantle boundary or the velocity field, respectively.

These processes must be described by a complicated system of equations of the core dynamics. The determination of related parameters of these models (scaling parameters) and values of investigated quantities (temperature within the mantle, density heterogeneities) is an insufficiently solved problem up to now.

5. Conclusions

The consideration of atmospheric dynamics improves the recently used model of the geomagnetic core-mantle coupling with respect to the electric conductivity and some periods. Further refinements of the model of the excitation of the Δlod can possibly be derived from hydrospheric processes, just as well by a more realistic description of the core processes, and by using longer time series for the related quantities.

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