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Polar motion at seasonal frequencies

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Abstract. Studying the planetary-dynamic behaviour of the Earth connected with geophysical processes as sources of excitation is of particular importance. In order to make a contribution concerning seasonal influences of near-surface geophysical processes on polar motion and Earth rotation, numerical investigations based on a time series of Earth Rotation Parameters and Atmospheric Angular Momentum parameters from 1976 to 1987 were performed. Atmospheric Angular Momentum changes are the dominant cause for polar motion and length-of-day variation at seasonal frequencies. Since there is another excitation source, the problem is to be considered in this way that the total excitation portion is composed of an atmospheric and a non-atmospheric portions. Particularly in polar motion, the paper deals with the separation of these portions for the seasonal constituents and presents interesting information about their character.

Key words: Polar motion, excitation, seasonal constituents, atmospheric portions, non-atmospheric portions

1 Introduction

The planetary-dynamic behaviour of the Earth is extremly complicated. There are irregularities in both the rate of rotation and the direction of the rotation axis in space and in the Earth. The variations with respect to the terrestrial reference frame are known as polar motion and length-of-day variation. The study of the Earth's rotation is a problem of significant value as a unique global measure of changes within the atmosphere, hydrosphere, cryosphere, and interior of the Earth. Therefore, the number of papers on this topic is very large. For a thorough discussion of this problem see e. g. Lambeck (1980).

The state of the art in investigations of polar motion is that the excitation sources are only partially understood. At seasonal frequencies, air and water mass redistribution are the certain causes, but the details remain uncertain. The water storage contribution at the annual frequency is much smaller than that of air mass redistribution. A discrepancy in prograde annual excitation determined from meteorological and polar motion data suggests the existence of another source. See e. g. Van Hylckama (1956); Jochmann (1976); Wilson and Haubrich (1976); Kikuchi (1977); Jochmann (1981); Chao and O'Connor (1988); Wilson and Kuehne (1990); Chao (1993); Kuehne et al. (1993); Wilson (1993). The purpose of this paper is to review the previous papers of the author concerning the polar motion excitation at seasonal frequencies, which was published in German.

2 Time series and periodicities

The data sets used in this study are time series of Earth Rotation Parameters (ERP) computed by the International Earth Rotation Service (IERS) and Atmospheric Angular Momentum (AAM) parameters computed by the U. S. National Meteorological Center (NMC) at daily intervals from 1976 to 1987 and MJD from 42960.0 to 47160.0, respectively. In Figures 1 and 2, the input data are shown to give a visual impression. For a better clearness in space and time, the 2-D motions are represented by spatial curves in perspective space-time-views instead of plane form (cf. Höpfner 1994b). Figure 1 shows the polar motion m = m_1 + i m_2 from the usual mathematical perspective in space-time-view. The corresponding Atmospheric Angular Momentum function estimates χ(IB) = χ_1(IB) + i χ_2(IB) are perspectivly represented in Figure 2, i. e., the pressure terms including the inverted barometer hypothesis (assuming isostatic response

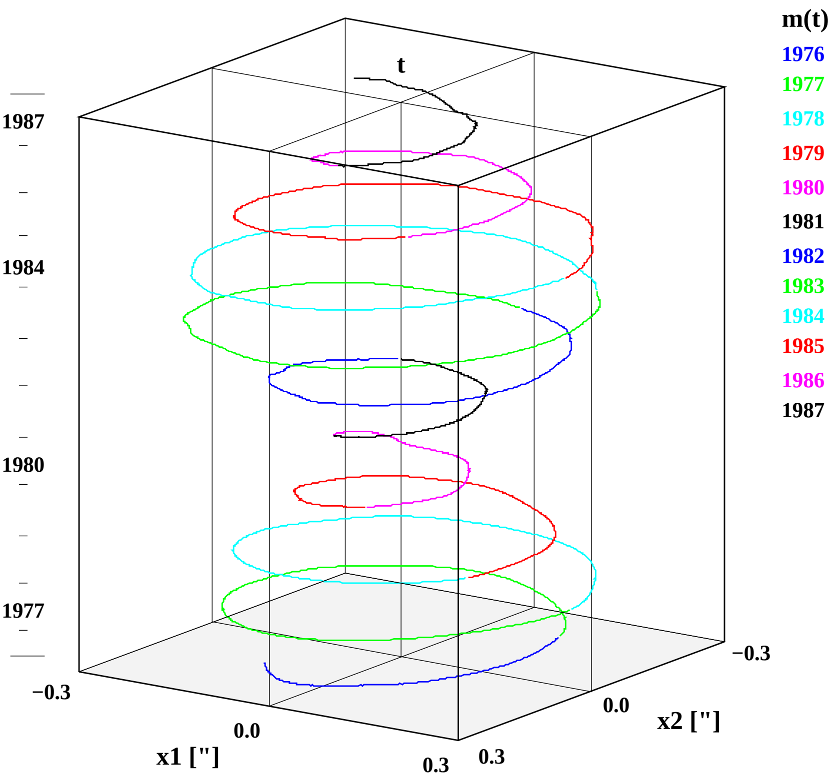

Figure 1. Polar motion m = m_1 + i m_2

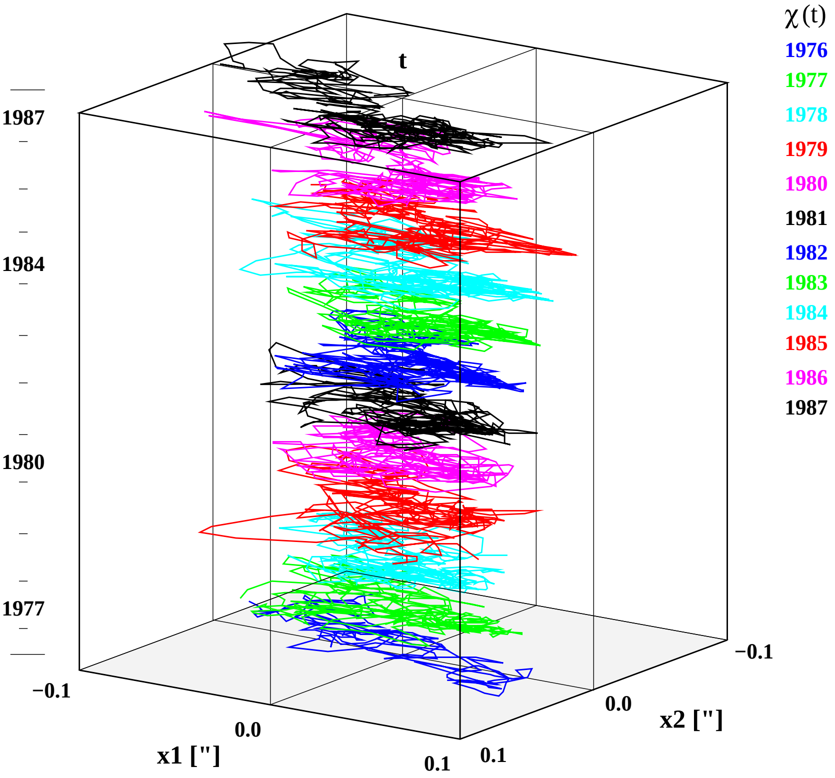

Figure 2. Atmospheric Angular Momentum function χ(IB) = χ_1(IB) + i χ_2(IB)

of oceans to pressure fluctuations) are used. The x_1-axis points towards the Greenwich meridian, the x_2-axis towards 90^o East longitude.

For interpretation purposes, the dominant periodic portions were derived by the least squares adjustment from the time series quoted, where the parameters and their accuracies for the different equivalent representation forms (trigonometric, exponential and geometric forms) and portion types (oscillation of the real and imaginary parts, circular and elliptical types), respectively, were computed (Höpfner 1993, 1994a, c, 1995a). The periodic components are displayed by ellipses in Höpfner (1995a). For the perspective representation of the 2-D periodicities by elliptic spirals see Höpfner (1994b). The portions with the same periods derived from the time series m = m_1 + i m_2 and χ(IB) = χ_1(IB) + i χ_2(IB) are of annual and semi-annual nature. Therefore, the investigations cited here on excitation of polar motion refer to the seasonal constituents.

3 Excitation portions and excited portions

For the expediency, the considerations are performed in the complex number plane (x_1, x_2). Consequently, circular-periodic portions x_f (t) in the time domain are given by exponential representations in the form

$$x_f(t) = X_f \ \exp(i \, 2\pi \ f \ t), \tag{1}$$

where t = time and f = frequency.

The complex-valued quantities X_f have the Cartesian form

$$X_f = A_f + \ i \ B_f \tag{2}$$

with A_f, B_f = exponential Fourier coefficient pair.

Interpreting geometrically, the representations of (2) are vectors. Therefore, arithmetic operations with the complex-valued amplitudes X_f can be realized as vector operations.

If a geophysical process influences the polar motion m, then it is to start from this that the transfer function I of a circular-periodic excitation portion ψ_f(t) is frequency-dependent, hence I = I_f. Thus, the mathematical relation of the excitation for circular-periodic portions is

$$m_f(t) = I_f \ \psi_f(t), \tag{3}$$

where m_f(t) = the circular-periodic excited polar motion portion and ψ_f(t) = the circular-periodic excitation portion.

Substituting the corresponding expressions given in equation (1) into equation (3), the relation in the frequency domain becomes

$$m_f = I_f \ \psi_f, \tag{4}$$

where m_f = complex-valued amplitude of m_f(t) and ψ_f = complex-valued amplitude of ψ_f(t).

In the case of investigations of seasonal changes, the Atmospheric Angular Momentum estimates χ_f instead of ψ_f can be used without making a noticeable mistake; cf. Moritz and Mueller (1987).

From the theory of the rotation of a realistic Earth model with an elastic mantle, it follows the differential equation of polar motion (Munk and MacDonald 1960; Lambeck 1980):

$$\frac{dm}{dt} + \alpha \ m = i \ \nu_{CH}(m - \psi) \tag{5}$$

where ν_{CH} (ν_{CH} = $2\pi \ f_{CH}$) is the Chandler angular frequency and α is the damping factor of the mantle.

Using the relation (3) for solving the equation (5), the transfer function of the Earth model $I_{f,M}$ is given by

$$I_{f,M} = \frac{f_{CH}}{(f_{CH} - f) + i \ \frac{\alpha}{2\pi}} \tag{6}$$

Equation (4), with the transfer function (6), is Fourier transform of equation (5). Both equations are the basis of the following considerations and estimations.

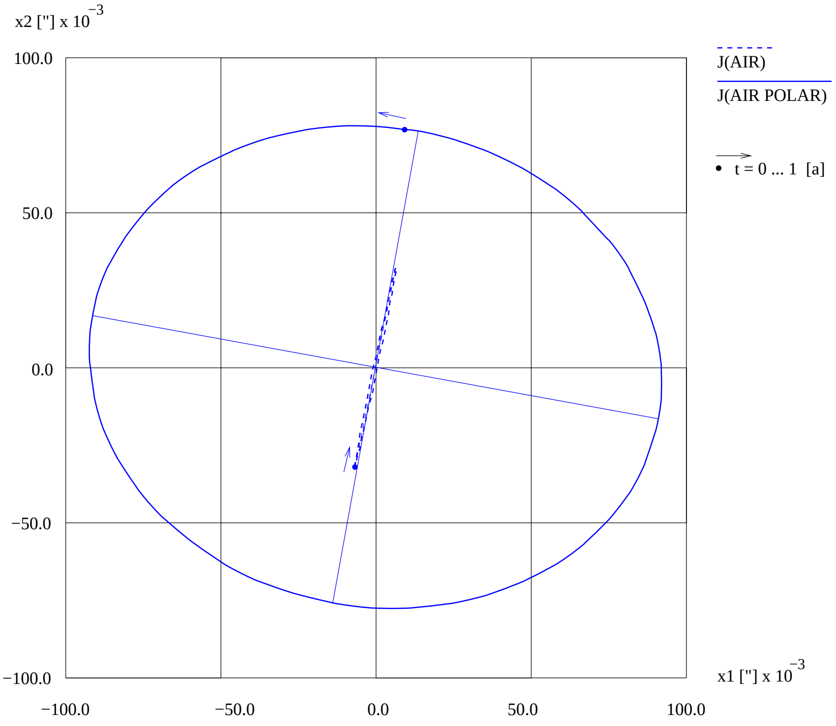

Figure 3. Annual elliptic excitation portions (dashed lines) and excited polar motion portions (solid lines): Atmospheric part

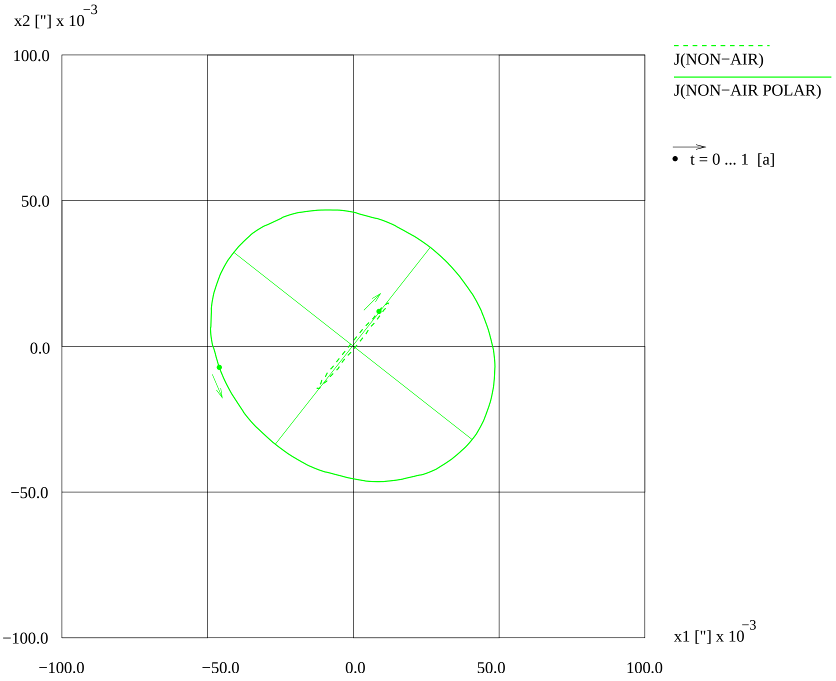

Figure 4. Annual elliptic excitation portions (dashed lines) and excited polar motion portions (solid lines): Non-atmospheric part

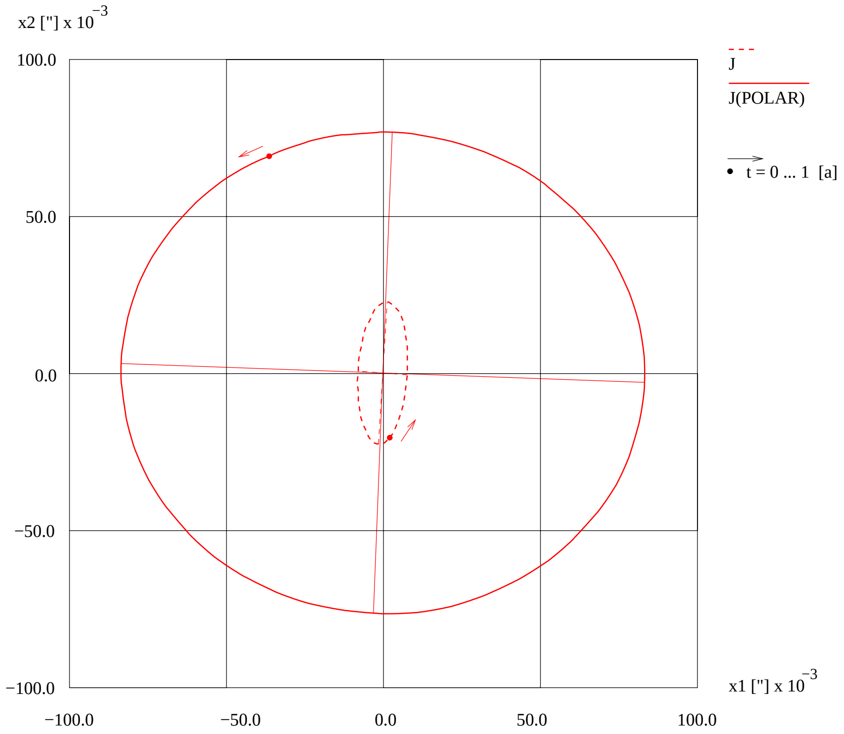

Figure 5. Annual elliptic excitation portions (dashed lines) and excited polar motion portions (solid lines): Total part

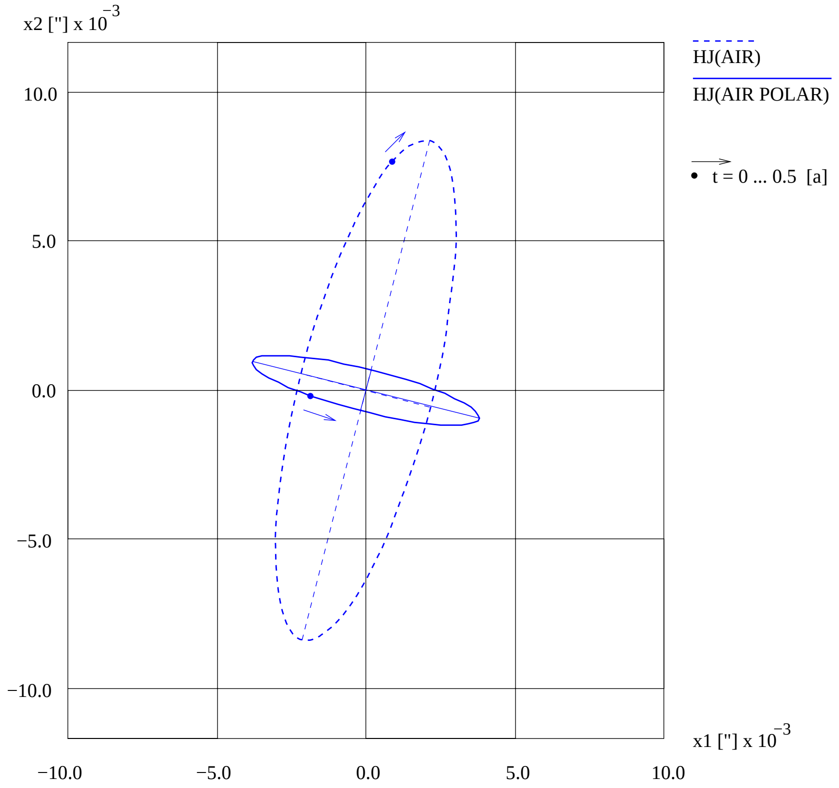

Figure 6. Semi-annual elliptic excitation portions (dashed lines) and excited polar motion portions (solid lines): Atmospheric part

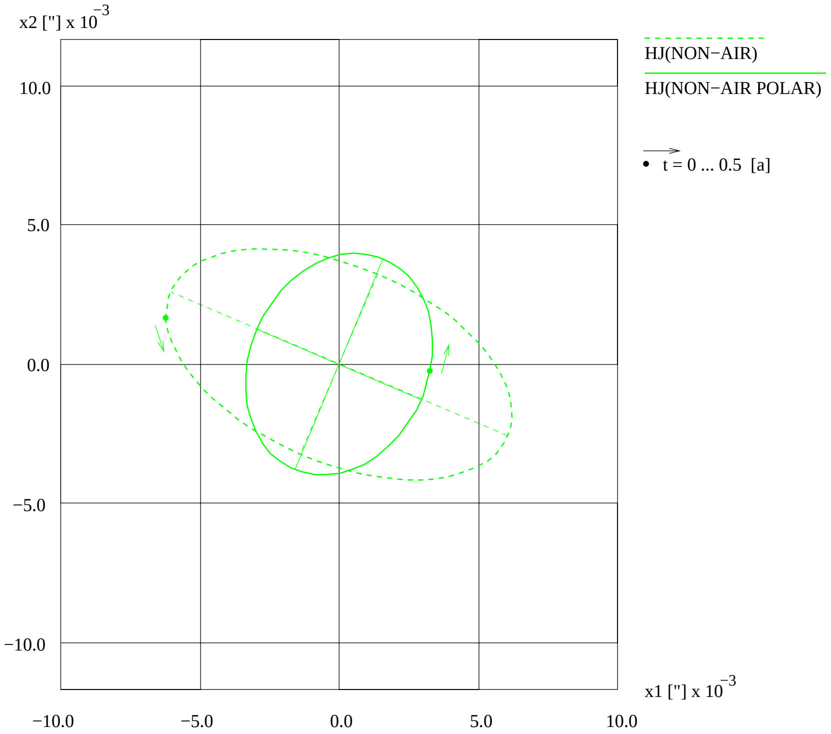

Figure 7. Semi-annual elliptic excitation portions (dashed lines) and excited polar motion portions (solid lines): Non-atmospheric part

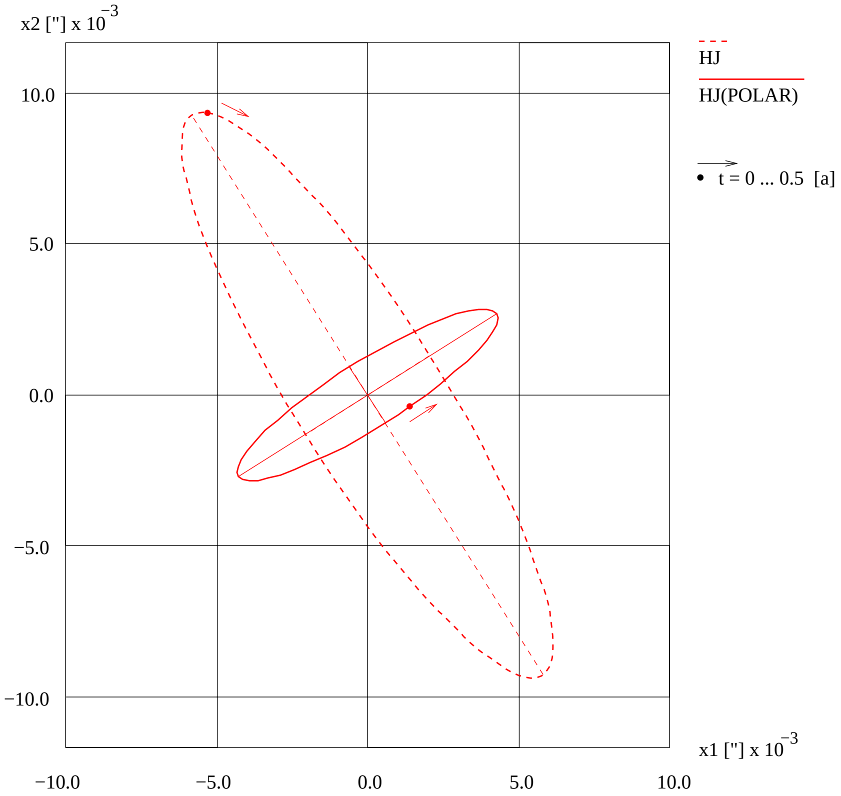

Figure 8. Semi-annual elliptic excitation portions (dashed lines) and excited polar motion portions (solid lines): Total part

4 Atmospheric and non-atmospheric portions: Separation and character

Atmospheric Angular mMmentum changes are the dominant cause for polar motion at seasonal frequencies (see e. g. Munk and MacDonald 1960; Jochmann 1976; Lambeck 1980). Since there is another excitation source, the problem is to be considered in this way that the total excitation portion is composed of an atmospheric and a non-atmospheric portions.

Specific symbols are used for the vectors which represent the complex-valued amplitudes m_f and ψ_f with the positive and negative frequencies f = $\{\pm 1, \pm 2$ cpa$\}$. In the case of the annual excitation and polar motion portions where f = ±1 cpa, names with **J** are used. Here the symbols used are:

$\mathbf{J(AIR)_+}$	vector of the complex-valued amplitude of the annual atmospheric excitation portion with positive frequency,
$\mathbf{J(NON-AIR)_+}$	the same for the non-atmospheric excitation portion,
$\mathbf{J_+}$	the same for the total excitation portion.

$\mathbf{J(AIR\ POLAR)_+}$, $\mathbf{J(NON-AIR\ POLAR)_+}$ and $\mathbf{J(POLAR)_+}$ denote the vectors for the related excited polar motion portions.

$\mathbf{J(AIR)_-}$, $\mathbf{J(NON-AIR)_-}$ and $\mathbf{J_-}$ as well as $\mathbf{J(AIR\ POLAR)_-}$, $\mathbf{J(NON-AIR\ POLAR)_-}$ and $\mathbf{J(POLAR)_-}$ are the corresponding vectors with negative frequency (f = -1 cpa).

In the case of the semi-annual portions where f = ±2 cpa, analogic names with **HJ** instead of **J** are used.

The procedure for separating the atmospheric and non-atmospheric excitation and excited polar motion portions which is similar for f = $\{\pm 1, \pm 2$ cpa$\}$ may be shown by the example with f = +1 cpa.

There are the following vector equations:

(a) Excitation vector equation

$$\mathbf{J_+} = \mathbf{J(AIR)_+} + \mathbf{J(NON-AIR)_+} \tag{7}$$

(b) Polar motion vector equation

$$\mathbf{J(POLAR)_+} = \mathbf{J(AIR\ POLAR)_+} + \mathbf{J(NON-AIR\ POLAR)_+} \tag{8}$$

(c) Transfer vector equations

$$\mathbf{J(POLAR)_+} = \mathbf{I_{+1cpa,M}}\ \mathbf{J_+} = \mathbf{I_{+1cpa,M}}\ (\ \mathbf{J(AIR)_+} + \mathbf{J(NON-AIR)_+}) \tag{9}$$

$$\mathbf{J(AIR\ POLAR)_+} = \mathbf{I_{+1cpa,M}}\ \mathbf{J(AIR)_+} \tag{10}$$

$$\mathbf{J(NON-AIR\ POLAR)_+} = \mathbf{I_{+1cpa,M}}\ \mathbf{J(NON-AIR)_+} \tag{11}$$

$\mathbf{J(POLAR)_+}$ and $\mathbf{J(AIR)_+}$ with their uncertainties are obtained from the analysis of the time series $m = m_1 + i\ m_2$ and χ(IB) = χ_1(IB) + i χ_2(IB); see Höpfner (1995a).

First, according to equation (6) with values adopted for f_{CH} and α (f_{CH} = 0.8400 cpa, α = 0.0528; cf. Höpfner, 1995b, c) the complex-valued transfer function quantity $\mathbf{I_{+1cpa,M}}$ is computed.

Then, because of equation (9) the vector $\mathbf{J_+}$ is given by

$$\mathbf{J_+} = \frac{\mathbf{J(POLAR)_+}}{\mathbf{I_{+1cpa,M}}} \tag{12}$$

and, the vector $\mathbf{J(AIR\ POLAR)_+}$ is given by equation (10).

Finally, because of equations (7) and (8) it follows

$$\mathbf{J(NON-AIR)_+} = \mathbf{J_+} - \mathbf{J(AIR)_+}, \tag{13}$$

$$\mathbf{J(NON-AIR\ POLAR)_+} = \mathbf{J(POLAR)_+} - \mathbf{J(AIR\ POLAR)_+}. \tag{14}$$

For the representation of the computed complex-valued amplitudes of the circular excitation and polar motion portions as vectors see Höpfner (1995b, c). These were used for determining the parameters of the elliptic portions, i. e., the major and minor semi-axes a, b and their directions γ_a, γ_b giving a better quantitative measure of the motion for the geophysical interpretation. The expressions are as follows

$$\begin{pmatrix} a \\ b \end{pmatrix} = |A_+ +\ i\ B_+|\ \pm\ |A_- +\ i\ B_-| \tag{15}$$

Table 1. Parameters of the portions of the excitation and of the polar motion. Units: arcsec for A_+, B_+, A_-, B_- and a, b; degree for γ_a

Portion	Circular motions				Elliptic motions		
	Exponential Fourier coefficient pairs				Semi-axes a, b and directions γ_a		
	A_+	B_+	A_-	B_-	a	γ_a	b
(1) Annual constituent							
(a) Excitation portion							
Atmospheric	-0.00299	-0.01593	-0.00373	-0.01636	0.03298	78.27	-0.00057
	±0.00028	±0.0028	±0.0028	±0.0028	±0.0048	±0.53	±0.00031
Non-atmospheric	0.00886	0.00185	0.00009	0.00997	0.01902	50.63	-0.00092
Total	0.00587	-0.01408	-0.00364	-0.00639	0.02261	86.49	+0.00790
(b) Polar motion portion							
Atmospheric	0.01127	0.08421	-0.00174	-0.00746	0.09262	169.64	+0.07730
Non-atmospheric	-0.04589	-0.01211	0.00006	0.00455	0.05201	142.01	+0.04291
Total	-0.03461	0.07210	-0.00167	-0.00291	0.08333	177.87	+0.07662
	±0.00042	±0.00042	±0.00042	±0.00042	±0.00058	±3.61	±0.00061
(2) Semi-annual constituent							
(a) Excitation portion							
Atmospheric	0.00212	0.00239	-0.00123	0.00529	0.00863	75.77	-0.00223
	±0.00028	±0.00028	±0.00028	±0.00028	±0.00057	±2.38	±0.00044
Non-atmospheric	-0.00500	0.00073	-0.00121	0.00094	0.00658	156.96	+0.00352
Total	-0.00288	0.00312	-0.00244	0.00623	0.01093	122.03	-0.00244
(b) Polar motion portion							
Atmospheric	-0.00152	-0.00174	-0.00036	0.00156	0.00392	165.89	+0.00071
Non-atmospheric	0.00362	-0.00050	-0.00036	0.00028	0.00411	67.08	+0.00320
Total	0.00210	-0.00225	-0.00072	0.00184	0.00505	32.15	+0.00110
	±0.00042	±0.00042	±0.00042	±0.00042	±0.00059	±7.25	±0.00059

$$\gamma_a = \frac{1}{2}\left(\arctan\left(\frac{B_+}{A_+}\right) + \arctan\left(\frac{B_-}{A_-}\right)\right); \quad \gamma_b = \gamma_a + \frac{\pi}{2}. \tag{16}$$

For the seasonal constituents, the parameters of the excitation portions and of the excited polar motion portions at both circular and elliptic motions are given in Table 1. In case of exponential Fourier coefficients, the zero point of the time reckoning is fixed at the beginning of the year. In addition, it may be helpful to note that the signs of the minor semi-axes b of the ellipses indicate how the elliptic motions take place:

(a) If b $>$ 0, then the motion is prograde, i. e. counter-clockwise;
(b) if b $<$ 0, then the motion is retrograde, i. e. clockwise;
(c) if b $=$ 0, then the motion is linear.

In Figures 3 to 5, the annual elliptic-periodic excitation portions **J(AIR), J(NON-AIR)** and **J** as well as the corresponding excited polar motion portions **J(AIR POLAR), J(NON-AIR POLAR)** and **J(POLAR)** are displayed. Analogous to Figures 3 to 5, the semi-annual elliptic-periodic excitation portions **HJ(AIR), HJ(NON-AIR)** and **HJ** as well as the related excited polar motion portions **HJ(AIR POLAR), HJ(NON-AIR POLAR)** and **HJ(POLAR)** are illustrated in Figures 6 to 8. In these figures, the x_1-axis lies in the Greenwich meridian, and the x_2-axis points toward 90^o East longitude. For comparison, the characteristics of the elliptic motions are listed in Table 2.

To test a propagation of uncertainties in the Earth model parameters f_{CH} and α, three value pairs were adopted in Höpfner (1995b). The results derived for the non-atmospheric excitation are not signifantly different from each other. Compared with various water storage estimates (Van Hylckama 1956; Kikuchi 1977), there is not a full explanation of the non-atmospheric excitation at the annual frequency. This implies that another excitation source exists. With regard to the non-atmospheric excitation it is necessary to identify the sources. Other hydrospheric processes such as oceanic currents must also be contributing to excitation. In addition, cryospheric and biospheric influences are likely portions.

The non-atmospheric excited polar motion portions are much larger than to be expected. For the annual component, the effect is negative, i. e. it reduces the total result. For the semi-annual component, the effect is positive, i. e. it enlarges the total result. For a more detailed discussion of the results obtained for the polar motion at seasonal frequencies see Höpfner (1995b, c).

Table 2. Characteristics of the elliptic motions

Portion	Excitation	Polar motion
(1) Annual constituent		
Atmospheric	retrograde	prograde
Non-atmospheric	retrograde	prograde
Total	prograde	prograde
(2) Semi-annual constituent		
Atmospheric	retrograde	prograde
Non-atmospheric	prograde	prograde
Total	retrograde	prograde

5 Concluding remark

Concentrating on the annual as well as semi-annual constituents, the main achievement of the paper is the separation of the atmospheric and non-atmospheric excitations of the polar motion. The results show the character of the various portions which is important for the geophysical interpretation.

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