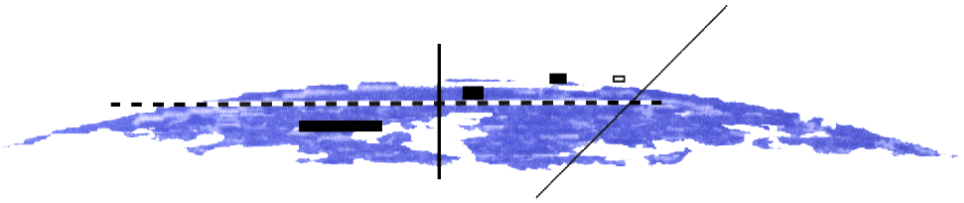

GeoForschungsZentrum Potsdam
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ABOUT THE INFLUENCE OF A POSSIBLE RELATIVE ROTATION OF THE EARTH'S INNER CORE ON THE POLAR MOTION, THE GEOMAGNETIC FIELD AND THE GRAVITY FIELD

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ABSTRACT

Provided that the figure axis of the inner core coincides with the dipole axis of the geomagnetic field, the relative rotation of the oblate inner core with respect to the outer core and the mantle can be determined. Because of the density difference between the inner and outer core, this motion is accompanied by a mass redistribution causing long-term variations of polar motion. Assuming standard density models, it is found that variations of polar motion caused by the relative inner-core rotation are similar to the decadal variations derived from pole coordinates. To examine the assumption on the orientation of the figure axis, its angle with respect to the dipole axis is investigated. The dynamo process is simulated by a prescribed electric current system within the outer core. Coincidence of both axes can be reached, if the magnitude of the angular velocity of the inner core is sufficiently high and if the current system is concentrated in a thin sheet near the outer core-inner core boundary. Calculations of the gravity potential show that the rotation of the inner core causes gravity changes which may be detectable by modern satellite methods during the next decade.

1 INTRODUCTION

Precessional motions of an oblate inner core caused by mutual gravitational torques between the inner core and mantle were investigated by SMYLIE et al. (1984) and SZETO and SMYLIE (1984, 1989). Assuming that the dipole field is frozen within the inner core, they suggested that the relative retrograde precession of the inner core resulting from their model can possibly be detected by a corresponding relative precession of the geomagnetic dipole axis. With regard to decade fluctuations in the Earth's rotation and the geomagnetic dipole field, JOCHMANN (1989) investigated the consequences of a relative precession of an oblate inner core on the inertia tensor of the whole Earth and on polar motion. To have a measure of associated variations of the moments of inertia, he identified the observed variations of the geomagnetic dipole axis with that of the figure axis of the inner core. He obtained variations of polar motion, which particularly agree with the observations. For example, he found that the 30-year periods in the theoretical and observed variations have similar amplitudes, while the amplitudes of the nearly 60-year periods disagree.

Concerning the hypothesis about the dipole axis, SCHMUTZER (1977, 1978) investigated the magnetic dipole moment $\mathbf{M}^1$ of the field $\mathbf{B}^1$ produced by the rotation of an electrically conducting sphere within a homogeneous magnetic field $\mathbf{B}^0$. In this model, the inner core is surrounded by free space. SCHMUTZER found that the direction of the total dipole moment, $\mathbf{M} = \mathbf{M}^0 + \mathbf{M}^1$, of the resulting field $\mathbf{B} = \mathbf{B}^0 + \mathbf{B}^1$ and the direction of the rotational vector $\mathbf{w}$ of the inner core coincide, if the magnitude of $\mathbf{w}$ is approximately 10^{-7} s^{-1}. SCHMUTZERS'S (1978) result was confirmed by GREINER-MAI (1997) for an improved physical model in which a prescribed electrical current system within the outer core is responsible for the generation of a homogeneous magnetic field inside and an axial dipole field outside the inner core if $\mathbf{w} = 0$.

The assumed relative precession of an oblate inner core also influences the gravity field. Associated variations can be computed by conventional methods. Consequently, by the assumed relative precession of the oblate inner core the physical relation between the variations of three observable quantities is described: the polar motion, the geomagnetic field and the gravity field. After a description of the theoretical models used, we will discuss their consistency with data for the three quantities.

2 THEORETICAL MODELS

2.1 Alignment of the geomagnetic dipole axis

The investigation of GREINER-MAI (1997) is concerned with solutions of the induction equation (e.g. KRAUSE and RÄDLER, 1980) for an electrically conducting sphere rotating relative to a surrounding shell with the angular velocity $\mathbf{w}$ ($w = ({w_x}^2 + {w_y}^2 + {w_z}^2)^{1/2}$). All vectors are defined in a mantle-fixed geocentric coordinate system, x, y, z. In the shell, an axial magnetic dipole field is produced by electric currents maintained by a prescribed electric field, $\mathbf{E^e}$, that replaces the dynamo process. In order to relate this model to SCHMUTZER's (1978) model, the structure of $\mathbf{E^e}$ must ensure that the field within the inner sphere is homogeneous if there is no relative rotation. This can be achieved by a purely toroidal current density $\mathbf{j^e}$. The adequate ansatz for $\mathbf{E^e}$ is $\mathbf{E^e}{=}E_0 f(r) \sin\vartheta \; \mathbf{e}_\varphi = \mathrm{curl}[\mathbf{r}\, E_0 f(r) \cos\vartheta\,]$, where $f(r)$ is a prescribed dimensionless function defining the radial distribution of the currents. The electrical conductivity of the outer core is σ_a, that of the inner core is $\sigma_0 \approx \sigma_a$; both are in the order of $10^5\ \Omega^{-1}\ \mathrm{m}^{-1}$. The radius of the sphere is taken as $r_0 = 1350$ km, that of the outer shell as $a = 3450$ km. The velocity field of the inner sphere is given by $\mathbf{u} = \mathbf{w} \times \mathbf{r}$. Let $\mathbf{E^e}$ be non-zero for $r_1 < r < r_2$ (source region), where $r_1 \geq r_0$ and $r_2 \leq a$. The induction equations are then given by

$$
\begin{aligned}
-(\mu_0\sigma_0)^{-1}\mathrm{curl\ curl}\ \mathbf{B} + \mathrm{curl}\ (\mathbf{u}\times\mathbf{B}) &= \dot{\mathbf{B}}, \quad r < r_0, && (1)\\
-(\mu_0\sigma_a)^{-1}\mathrm{curl\ curl}\ \mathbf{B} + \mathrm{curl}\ \mathbf{E^e} &= \dot{\mathbf{B}}, \quad r_1 < r < r_2, && (2)\\
-(\mu_0\,\sigma_a)^{-1}\,\mathrm{curl\,curl}\,\mathbf{B} &= \dot{\mathbf{B}}, \quad r_0 < r < r_1,\ r_2 < r < a, && (3)\\
\mathrm{curl}\ \mathbf{B} &= 0\,, \quad r > a, && (4)\\
\mathrm{div}\ \mathbf{B} &= 0\,, \quad \forall\ r\ , && (5)
\end{aligned}
$$

where μ_0 is the permeability of vacuum. The boundary conditions are the continuity of the flux density, $\mathbf{B}^+=\mathbf{B}^-$, and of the tangential component of the electric field, $\mathbf{E}^+_{\mathbf{tan}}=\mathbf{E}^-_{\mathbf{tan}}$, where plus and minus denote the inner and outer side of the boundary, respectively. The field $\mathbf{B}$ satisfying equation (5) can be split into poloidal and toroidal parts $\mathbf{B} = \mathbf{B}_p + \mathbf{B}_t$, which are defined by $\mathbf{B}_p = \mathrm{curl\ curl}\ \mathbf{r}S$ and $\mathbf{B}_t = \mathrm{curl}\ \mathbf{r}T$. The field $\mathbf{u} \times \mathbf{B}$, which is not solenoidal, can generally be represented by three scalars, $\mathbf{u} \times \mathbf{B} = \mathrm{curl}\,\mathbf{r}U + \mathbf{r}V + \mathrm{grad}\,W$, from which the toroidal part is $\mathrm{curl}\,\mathbf{r}U$. Using these definitions, equations (1)–(4) can be transformed into scalar equations. Then, the poloidal scalar is derived from

$$
\begin{aligned}
(\mu_0\sigma_0)^{-1}\Delta S + U &= \dot{S} \quad , \quad r < r_0, && (6)\\
(\mu_0\sigma_a)^{-1}\Delta S + E_0 f(r)\cos\vartheta &= \dot{S} \quad , \quad r_1 < r < r_2, && (7)\\
(\mu_0\,\sigma_a)^{-1}\,\Delta\,S &= \dot{S} \quad , \quad r_0 < r < r_1\,,\ r_2 < r < a && (8)\\
\Delta S &= 0 \quad , \quad r > a\,. && (9)
\end{aligned}
$$

As shown by Greiner-Mai (1997), the scalar U can be analytically computed as a linear function of S and w_i. S and U are usually represented by spherical harmonics given, for example, by

$$S = \sum_{n,m} \left(S^c_{nm}\cos m\varphi + S^s_{nm}\sin m\varphi\right)\ P_{nm}(\cos\vartheta). \tag{10}$$

The associated boundary conditions at any surface $r =$ const. are

$$(S^{c,s}_{nm})^+ \;=\; (S^{c,s}_{nm})^-, \quad [\frac{\partial}{\partial r}\ S^{c,s}_{nm}]^+ \;=\; [\frac{\partial}{\partial r}\ S^{c,s}_{nm}]^-, \tag{11}$$

which follow, respectively, from the continuity of the radial and tangential components of $\mathbf{B}$. The solution of equations (4) and (9) for $r > a$ is a potential field regular at infinity. The associated modes are given by

$$S^{c,s}_{nm} \;=\; C^{c,s}_{nm} r^{-n-1}. \tag{12}$$

The magnetic moment $\mathbf{M}$ is defined by the coefficients $C^{c,s}_{nm}$ as follows:

$$\mathbf{M} \;=\; \frac{4\pi}{\mu_0}\,(C^c_{11},\, C^s_{11},\, C_{10}). \tag{13}$$

The further derivation of the governing equations for the spherical harmonic modes of the fields S and U is outlined in Greiner-Mai (1997), who assumes $w_y = 0$ without loss of generality. Defining $\tau_0 = \mu_0\sigma_0 r_0^2$ and introducing the dimensionless quantities $\mathrm{x} = r/r_0$ (x is not the coordinate x), $t_1 = t/\tau_0$, and $\alpha_i = \tau_0 w_i$, he obtained the following system of differential equations:

$$
\left.
\begin{aligned}
DS_{10} \;+\; \alpha_x S^s_{11} &= 0\\
DS^c_{11} \;-\; \alpha_z S^s_{11} &= 0\\
DS^s_{11} \;+\; \alpha_z S^c_{11} \;-\; \alpha_x S_{10} &= 0
\end{aligned}
\right\} \mathrm{x} < 1, \tag{14}
$$

with D defined by

$$D = \frac{\partial^2}{\partial \mathrm{x}^2} + \frac{2}{\mathrm{x}}\frac{\partial}{\partial \mathrm{x}} - \frac{n(n+1)}{\mathrm{x}^2} - \frac{\partial}{\partial t_1} \tag{15}$$

and

$$D\,S_{11}^{c,s} = 0 \quad , \quad 1 < \mathrm{x} < \mathrm{x}_a, \tag{16}$$

$$D\,S_{10} = 0 \quad , \quad 1 < \mathrm{x} < \mathrm{x}_1\,,\ \mathrm{x}_2 < \mathrm{x} < \mathrm{x}_a, \tag{17}$$

$$D\,S_{10} = -c\,f(\mathrm{x}) \quad , \quad \mathrm{x}_1 < \mathrm{x} < \mathrm{x}_2 \tag{18}$$

for the remaining region with $c = E_0\mu_0\sigma_a r_0^2$, $\mathrm{x}_1 = r_1/r_0$, $\mathrm{x}_2 = r_2/r_0$, $\mathrm{x}_a = a/r_0$. For $f(\mathrm{x})$ a quadratic function is assumed:

$$f(\mathrm{x}) = (\mathrm{x} - \mathrm{x}_1)(\mathrm{x} - \mathrm{x}_2) \quad , \ \mathrm{x}_1 < \mathrm{x} < \mathrm{x}_2, \tag{19}$$

$$f(\mathrm{x}) = 0 \quad , \ \mathrm{x} \le \mathrm{x}_1, \mathrm{x} \ge \mathrm{x}_2\ . \tag{20}$$

Solutions for the stationary case can then be derived analytically and are explicitly given by Greiner-Mai (1997).

The orientation of the dipole moment $\mathbf{M}$ of the resulting external potential field is described by the angles φ_D and ϑ_D, where φ_D is the angle between the projection of $\mathbf{M}$ into the x-y plane and the x axis and ϑ_D is the angle between $\mathbf{M}$ and the z axis. The angles can be computed from

$$\varphi_D = \arctan\left(\frac{C_{11}^s}{C_{11}^c}\right)\ ,\ \vartheta_D = \arctan\left(\frac{\sqrt{(C_{11}^s)^2 + (C_{11}^c)^2}}{C_{10}}\right), \tag{21}$$

where C_{10}, C_{11}^s and C_{11}^c are given by the solutions of (14)–(20). The associated angels of the axis of rotation are φ_w and ϑ_w. They involve into the solutions by α_x and α_z and must be prescribed in this kinematic model. The calculations apply to the

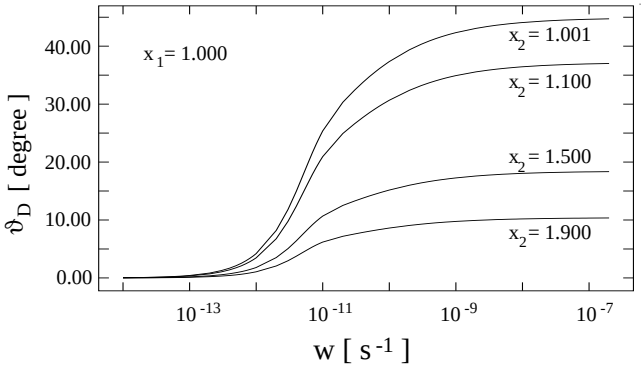

Figure 1: Angle ϑ_D as a function of w for $\mathrm{x}_1 = 1.000$ and different values of x_2 ($\vartheta_w = 45^o$)

example $\varphi_w = 0^o$, $\vartheta_w = 45^o$ and selected values of w. To obtain results for different extensions of the source region, selected values of x_1 and x_2 between the limits $\mathrm{x}_1 = 1$ and $\mathrm{x}_2 = \mathrm{x}_a$ are prescribed. The calculations show that $\varphi_D \approx \varphi_w$ for large values of w (independent on x). The main result for ϑ_D is shown in Fig. 1, where x_1 is fixed to

1.0 and x_2 is variable: $\vartheta_D \approx \vartheta_w = 45^o$ is obtained for large values of w ($\geq 10^{-9}\,s^{-1}$) and for $x_2 = 1.001$ ($x_1 = 1.000$). For the same order of w, lower values of ϑ_D are obtained for other radial distributions of $\mathbf{E}^e$. Therefore, we can produce the expected result $\mathbf{M} \parallel \mathbf{w}$ ($\vartheta_D \approx \vartheta_w$) for large values w, if we choose a very thin source region near the inner-core surface.

In the following section, we assume that the inner core is a rotational ellipsoid which rotates about its figure axis with the angular velocity $\mathbf{w}$. Further, we assume that the position of this axis is indicated by the magnetic dipole axis, which moves relative to the mantle (precessional motion of the axes). This means that $\mathbf{w}$ is time dependent. Using the result of the stationary model in the next section, we therefore imply that the angular velocity of this precessional motion is much lower than w (e.g. in the order of the dipole drift $w_D \approx 10^{-11}\,s^{-1}$). This is valid for $w = 10^{-7}\,s^{-1}$, but for $w < 10^{-9}\,s^{-1}$, the problem should be re-examined by using time-dependent solutions of the induction equations. A discussion of the model assumptions is given in section 3.

2.2 Influence on polar motion

The relative precession of the inner core with respect to the mantle is accompanied by a mass redistribution that causes variations of polar motion and the length of day. This effect is expected because the density difference at the boundary between inner and outer core is large. According to SMYLIE, SZETO and ROCHESTER (1984) and GREINER-MAI (1997), the variation of the direction of the geomagnetic dipole axis possibly indicates a similar variation of the figure axis of the earth's inner core. Using this relation the relative precessional motion of the inner core can be determined and the corresponding excitation function of polar motion and the length of day can be evaluated. In JOCHMANN (1989) it was shown that it is sufficient to consider a simple earth model with rheological properties described by the CHANDLER period and its damping coefficient, because the time scale of inner-core motion is much larger than the eigenperiods of more complicated models.

In the following, only the influence on polar motion is considered, because variations of the length of day due to mass redistributions are negligible small (JOCHMANN, 1989). The polar motion is governed by the complex differential equation

$$\dot{m} + \alpha m = i\sigma_{CH}(m - [\sigma_{EU}/\sigma_{CH}]\psi), \tag{22}$$

where $m = m_x + im_y$ are the pole coordinates, $\sigma_{CH} = 5.28\,a^{-1}$ and $\sigma_{EU} = 7.46\,a^{-1}$ are the Chandler and the Eulerian frequencies, $\alpha = 0.05\,a^{-1}$ is the damping coefficient, and

$$\psi = (c_{xz} + ic_{yz})/(C - A), \tag{23}$$

is the excitation function. In (23), c_{xz} and c_{yz} are temporally varying components of the inertia tensor of the earth. It can be decomposed into

$$\mathbf{I}(t) = \mathbf{I}_m + \mathbf{I}_c + \eta\mathbf{I}_i(t), \tag{24}$$

where $\mathbf{I}_i(t)$ describes the temporal variation of the inertia tensor due to motions of the inner core and η is the density relation $\eta = (\rho_i - \rho_e)/\rho_i$ (ρ_i is the density of the inner core and ρ_e that of the outer core at the inner core-outer core boundary). The inertia tensor of the inner core $\mathbf{I}_i(t)$ becomes a diagonal tensor $\mathbf{I}_{i0}$, if the principal axis of inertia of the inner core coincides with the z axis of the mantle-fixed coordinate system. The inertia tensor corresponding to the actual position of the inner core is obtained by a tensor transformation using a time-dependent transformation matrix $\mathbf{R}(t)$:

$$\mathbf{I}_i(t) = \mathbf{R}(t)\mathbf{I}_{i0}\mathbf{R}^{-1}(t). \tag{25}$$

The elements of the $\mathbf{R}(t)$ depend on the geographical position of the dipole axis. With the pole distance ϑ_D and the longitude φ_D, the transformation matrix reads

$$\mathbf{R}(t) = \begin{pmatrix} \cos\vartheta_D \cos\varphi_D & -\sin\varphi_D & \sin\vartheta_D \cos\varphi_D \\ \sin\varphi_D \cos\vartheta_D & \cos\varphi_D & \sin\vartheta_D \sin\varphi_D \\ -\sin\vartheta_D & 0 & \cos\vartheta_D \end{pmatrix}. \tag{26}$$

The time-dependent quantities ϑ_D and φ_D are obtained by replacing in (21) $C_{10}, C_{11}^{c,s}$ by the associated Gauss coefficients. Taking into account the relations (25) and (26), the excitation function of the inner core is obtained from equation (23):

$$\psi(t) = \frac{C_i - A_i}{C - A}\,\eta\,\frac{1}{2}\,\sin 2\vartheta_D\,\exp(i\varphi_D). \tag{27}$$

For evaluating equation (27), the flattening of the inner core and the density jump, $\Delta\rho = \rho_i - \rho_e$, between inner and outer core must be known. SMYLIE et al. (1984) determined the flattening according to Clairaut's equation. The density jump must be chosen according to theoretical earth models. In JOCHMANN (1989) the following excitation function is published:

$$\psi(t) = 4.3787 \cdot 10^{-5}\,\sin 2\vartheta_D\,\exp(i\varphi_D). \tag{28}$$

This equation was evaluated using the inner-core flattening given by SMYLIE et al. (1984) and the density jump given by BULLEN and JEFFREYS (1949) ($\Delta\rho = 2.6$ g/cm^3). GILBERT and DZIEWONSKI (1975) published an earth model with a smaller density jump ($\Delta\rho = 0.597$ g/cm^3). The corresponding excitation function is obtained, if we multiply (28) by 0.32.

In this paper, we improve the results of JOCHMANN (1989) using polar motion data corrected for the influence of atmospheric mass redistributions. According to equation (28), a time series of the excitation function of the inner-core motion was evaluated using pole distances ϑ_D and latitudes φ_D of the dipole axis of the geomagnetic field. The quantities were derived from the Gauss coefficients calculated by using secular variation coefficients according to HODDER (1981). In Fig. 2, the secular variations of the excitation function of the inner-core motion and the rotation pole of the earth (which is the same as the variation of the principle axis of inertia) are displayed. It

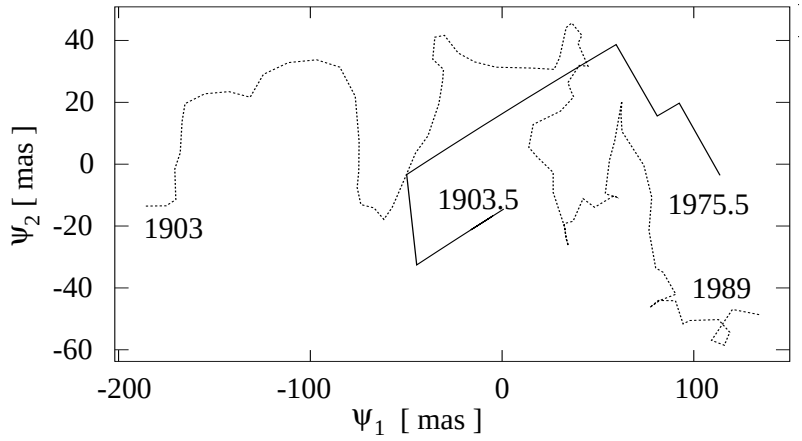

Figure 2: Secular variations of the excitation function of inner-core motion (solid line) and the secular polar motion (dotted line)

is seen that both motions are similar for a limited period of time. Since the secular motion of the pole of rotation is also influenced by other geophysical processes, a perfect agreement between both motions cannot be expected. To get better insight into relations between inner-core motion and polar motion, we calculate the amplitude spectra of both processes. The periodic terms of the excitation function, obtained by a two-dimensional Fourier analysis, are represented by elliptical motions of the principal axis of inertia. From this analysis, the following relation results for each periodic term:

$$\psi(t) = (A_+ + iB_+)\exp(i\nu t) + (A_- + iB_-)\exp(-i\nu t), \tag{29}$$

where ν is the angular frequency of the period considered. The semi-axes of the elliptical motion described by (29) are

$$\begin{aligned} a &= |A_+ + iB_+| + |A_- + iB_-|, \\ b &= |A_+ + iB_+| - |A_- + iB_-|. \end{aligned} \tag{30}$$

The direction of the major semi-axis is obtained according to

$$\gamma_a = \frac{1}{2}\left(\arctan\frac{B_+}{A_+} + arctan\frac{B_-}{A_-}\right). \tag{31}$$

The quantities defined by (30) and (31) are indicative of the similarity of the periodic constituents of excitation functions derived in different ways. Fig. 3 shows that the amplitude spectra of the major semi-axis of the periodic constituents of both processes contain several common periods. In Table 1, the parameters of these periodic terms are gathered. It is seen that usually the semi-axes of the periodic terms of the excitation function of inner-core motion are smaller than those of the excitation function of polar motion (diminished by the influence of the atmosphere), although they agree within their uncertainties. On the other hand, the differences of the direction angles are too large for most periodic constituents, so that the relation between inner-core motion and polar motion is not completely proven. Only, the 16-year period can be accepted

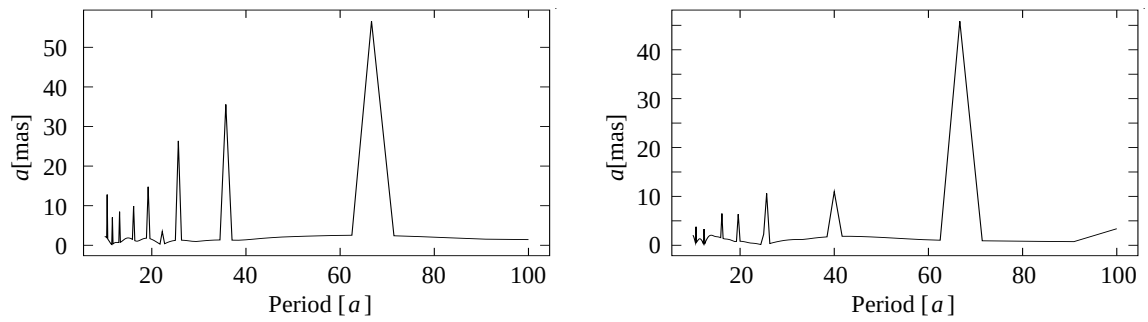

Figure 3: Amplitude spectra of the excitation function of polar motion (left) and of the excitation function of inner-core motion (right)

Period	**67**		**36(40)**		**26**		**16**	
	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)
a [mas]	56	46	36	10	26	11	15	6
b [mas]	2	9	12	4	4	0	11	4
γ_α^o	99	1	105	5	53	137	120	81

Table 1: Parameters of periodic constituents of the excitation functions of polar motion (1) and inner-core motion (2)

as being caused by inner-core motion. Improved results are expected in the future, when higher-accuracy measurements of polar motion and the geomagnetic field are available.

2.3 Influence on the gravity field

Provided that the figure axis of the oblique ellipsoidal inner core moves relative to the earth, this motion causes changes of the gravity field. Knowing the density difference between inner and outer core, the flattening of the inner core and the time variation of the orientation of its figure axis in terms of $\varphi_D(t)$ and $\vartheta_D(t)$ (Fig. 4), we can estimate these changes and compare them with the accuracy of recent gravity-field models and with the expected accuracy of planned satellite gravity missions (CHAMP, GRACE).

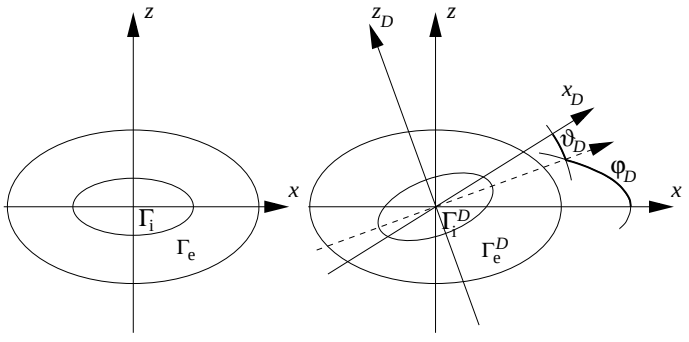

Figure 4: Aligned (left) and oblique (right) ellipsoidal inner core; Γ_i, Γ_e and $\Gamma_i^{\mathrm{D}}, \Gamma_e^{\mathrm{D}}$ are domains of inner and outer core, respectively; $\vartheta_{\mathrm{D}}, \varphi_{\mathrm{D}}$ are the co-latitude and azimuth of the figure axis z_D of the inner core with respect to the mantle-fixed coordinate system

To simplify the calculations, we introduce a second geocentrical coordinate system

which is fixed to the inner core. Its z axis is in alignment with the figure axis of the ellipsoid, respectively, the geomagnetic dipole axis. To distinguish it from the z axis of the mantle fixed coordinate system, it is hereafter referred to as z_D axis; the x_D axis is then defined by the z-z_D plane.
The gravity potential difference between an oblique and an aligned ellipsoidal inner core with the density distributions $\rho_e(\mathbf{r})$ (outer core) and $\rho_i(\mathbf{r})$ (inner core) is

$$\begin{aligned} \Delta V &= V_D - V_0 \\ \text{with} \qquad V_0 = V(\vartheta_D = 0) &= \int\limits_{\Gamma_e} \rho_e(\mathbf{r}) r^{-1}\, d\mathbf{r} + \int\limits_{\Gamma_i} \rho_i(\mathbf{r}) r^{-1}\, d\mathbf{r} \\ \text{and} \qquad V_D = V(\varphi_D, \vartheta_D) &= \int\limits_{\Gamma_e^D} \rho_e(\mathbf{r}) r^{-1}\, d\mathbf{r} + \int\limits_{\Gamma_i^D} \rho_i(\mathbf{r}) r^{-1}\, d\mathbf{r}\ . \end{aligned} \tag{32}$$

In the following, we 1) show that the calculation of the potentials according to (32) can be reduced to an integration over a rotational ellipsoid homogeneously filled with mass of density $\rho = \Delta\rho$, 2) calculate the potential for an ellipsoid in the coordinate system fixed to the inner core (Fig. 4) and 3) transform the solution to the mantle-fixed coordinate system, thus obtaining the associated perturbation of the gravitational potential in the coordinate system conventionally used. The assumption in 1) seems plausible, because an ellipsoidal inner core precessing within a surrounding with the same density has no effect on the existent gravity field of the earth.

Beginning with 1), we consider more generally the relation for the difference of the integration over two domains A and B:

$$\int\limits_A f(x)dx - \int\limits_B f(x)dx = \int\limits_{A\setminus B} f(x)dx + \int\limits_{A\frown B} f(x)dx - \int\limits_{A\frown B} f(x)dx - \int\limits_{B\setminus A} f(x)dx. \tag{33}$$

Applying (33) to (32), we must solve the following integrals over four difference regions:

$$\Delta V = \int\limits_{\Gamma_e\setminus\Gamma_e^D} \rho_e(\mathbf{r}) r^{-1}\, d\mathbf{r} - \int\limits_{\Gamma_e^D\setminus\Gamma_e} \rho_e(\mathbf{r}) r^{-1}\, d\mathbf{r} + \int\limits_{\Gamma_i\setminus\Gamma_i^D} \rho_i(\mathbf{r}) r^{-1}\, d\mathbf{r} - \int\limits_{\Gamma_i^D\setminus\Gamma_i} \rho_i(\mathbf{r}) r^{-1}\, d\mathbf{r}\ . \tag{34}$$

Because $\Gamma_e \setminus \Gamma_e^D = \Gamma_i^D \setminus \Gamma_i$ and $\Gamma_e^D \setminus \Gamma_e = \Gamma_i \setminus \Gamma_i^D$ holds, we can write

$$\Delta V = \int\limits_{\Gamma_i^D\setminus\Gamma_i} (\rho_e(\mathbf{r}) - \rho_i(\mathbf{r})) r^{-1}\, d\mathbf{r} - \int\limits_{\Gamma_i\setminus\Gamma_i^D} (\rho_e(\mathbf{r}) - \rho_i(\mathbf{r})) r^{-1}\, d\mathbf{r}\ , \tag{35}$$

and with (33) it follows that

$$\Delta V = \int\limits_{\Gamma_i^D} (\rho_i(\mathbf{r}) - \rho_e(\mathbf{r})) r^{-1}\, d\mathbf{r} - \int\limits_{\Gamma_i} (\rho_i(\mathbf{r}) - \rho_e(\mathbf{r})) r^{-1}\, d\mathbf{r}\ . \tag{36}$$

Equation (35) shows that the potential difference between an oblique and an aligned inner core depends only on the difference of the density distributions $\rho_i(\mathbf{r}) - \rho_e(\mathbf{r})$ in

the difference domains $\Gamma_i^D \setminus \Gamma_i$ and $\Gamma_i \setminus \Gamma_i^D$. Due to the small flattening of the inner core, the integrations in (35) are in fact carried out over a thin shell (thickness ≈ 3 km, Table 2). This shows that the density jump at the inner core-outer core boundary, $\Delta\rho$, is significant rather than the density distributions in the full domains, and the distribution $\rho_i(\mathbf{r}) - \rho_e(\mathbf{r})$ in (36) can be approximated by the constant density $\Delta\rho$. The calculations can then be continued according to steps 2) (using (36)) and 3).

Usually, the geopotential is given in terms of the coefficients C_{nm} and S_{nm} of a spherical harmonic expansion:

$$V(r,\varphi,\vartheta) = \sum_{n=0}^{N}\sum_{m=0}^{n} (C_{nm}\cos m\varphi + S_{nm}\sin m\varphi)\, P_{nm}(\cos\ \vartheta)\ . \tag{37}$$

These coefficients should not be mixed up with those used in section 2.1. With respect to our problem, we must compute the coefficients of the potential ΔV

$$\begin{aligned} \Delta C_{nm}(t) &= \Delta C_{nm}(\varphi_D(t),\vartheta_D(t)) \\ \Delta S_{nm}(t) &= \Delta S_{nm}(\varphi_D(t),\vartheta_D(t)) \end{aligned}$$

in the mantle-fixed coordinate system. For this, we compute the potential coefficients of the oblate inner core with respect to the above mentioned coordinate system fixed to the inner core and transform them into the coordinate system fixed to the mantle.

In the coordinate system fixed to the inner core, the fully normalized zonal harmonic coefficients C_{n0} for the potential of a body with rotational symmetry and density ρ are given by

$$C_{n0} = \frac{1}{MR_0^n\sqrt{2n+1}} \int_{\varphi'=0}^{2\pi}\int_{\vartheta'=0}^{\pi}\int_{r'=0}^{r(\vartheta')} \rho r'^n P_n(\cos\vartheta')\ \ r'^2(\sin\vartheta')dr'\ d\vartheta'\ d\varphi' \tag{38}$$

where M is the mass of the body and R_0 is the reference radius. In our case, according to (36), the integral must be solved for the region of the ellipsoidal inner core, the density of which is $\Delta\rho$. The solution for $\rho = \Delta\rho = \text{const.}$ is

$$C_{2\ell,0} = 3\frac{m}{M}\left(\frac{a^2-b^2}{R_0^2}\right)^{\ell} \frac{-1^{\ell}}{(2\ell+1)(2\ell+3)\sqrt{4\ell+1}} \tag{39}$$

where $m = \frac{4}{3}\pi\ \Delta\rho\ a^2 b$ is the mass of the ellipsoid with axes a and b. Since we are interested to have the effect in respect to the potential of the whole earth, we must take for M the mass of the earth. Because we assume an rotational ellipsoid as shape for the inner core, only zonal coefficients C_{n0} of even degree n occur in the coordinate system fixed to the inner core and in practice it is sufficient to consider the coefficient C_{20}.

The transformation of spherical harmonic coefficients $(C_{nm}, S_{nm}) \Rightarrow (C_{np}(\alpha,\beta,\gamma), S_{np}(\alpha,\beta,\gamma))$ with respect to a coordinate system rotated by the angles α,β,γ is considered in KAUTZLEBEN (1965) and ILK (1983). We rearranged

the formulas for numerical computation of rotation by the angles φ and ϑ (Fig. 4):

$$\begin{aligned} C_{np}(\varphi,\vartheta) &= \sum_{m=0}^{n} (C_{nm}\cos m\varphi + S_{nm}\sin m\varphi)\; A_n^{pm}(\vartheta) \\ S_{np}(\varphi,\vartheta) &= \sum_{m=1}^{n} (S_{nm}\cos m\varphi - C_{nm}\sin m\varphi)\; B_n^{pm}(\vartheta) \end{aligned} \tag{40}$$

with the transformation parameters

$$\begin{aligned} A_n^{pm} &= (-1)^p \left[(2-\delta_{0,p})(2-\delta_{0,m})\frac{(n-p)!\,(n-m)!}{(n+p)!\,(n+m)!}\right]^{\frac{1}{2}} \times \\ &\sum_{j=j_1}^{j_2} \frac{m!\,p!}{(m-j)!\,j!\,(p-j)!}\; P_n^{m+p-j}(\cos\vartheta)\; \frac{(-1-\cos\vartheta)^j + (1-\cos\vartheta)^j}{2\sin^j\vartheta} \end{aligned} \tag{41}$$

$$\begin{aligned} B_n^{pm} &= (-1)^p \left[(2-\delta_{0,p})(2-\delta_{0,m})\frac{(n-p)!\,(n-m)!}{(n+p)!\,(n+m)!}\right]^{\frac{1}{2}} \times \\ &\sum_{j=j_1}^{j_2} \frac{m!\,p!}{(m-j)!\,j!\,(p-j)!}\; P_n^{m+p-j}(\cos\vartheta)\; \frac{(-1-\cos\vartheta)^j - (1-\cos\vartheta)^j}{2\sin^j\vartheta} \end{aligned} \tag{42}$$

with $\quad j_1 = \max(p+m-n, 0) \quad$ and $\quad j_2 = \min(m, p).$

Parameter	**Value**	**Source**
Mass of the Earth	$M = 5.973698995 \times 10^{27}$ kg	IERS - Standards
Reference radius of spherical harmonics	$R_0 = 6378136.49$ m	IERS - Standards
Major semi-axis of inner core	$a = 1229.5$ km	Dziewonski and Anderson (1981)
Flattening of inner core	$f = \frac{1}{415.78}$	Smylie, Szeto and Rochester (1984)
Minor semi-axis of inner core	$b = a(1-f) = 1226.54$ km	
Density jump at the inner core outer core boundary	$\Delta\rho = 0.5973$ g cm^{-3}	Dziewonski and Anderson (1981)

Table 2: Numerical values for calculating the influence of the inner core on the gravity field

The values we used to calculate the inner core influence are given in Table 2. According to equation (39), the numerical value of the coefficient C_{20} with respect to the

inner core fixed system is:

$$C_{20} = -1.240 \times 10^{-8}$$

and the time variations of the transformed coefficients $\Delta C_{nm}(\varphi_D(t), \vartheta_D(t))$ and $\Delta S_{nm}(\varphi_D(t), \vartheta_D(t))$ $(n = 2, \quad m = 0, 1, 2)$ are shown in Fig. 5. The predicted rates

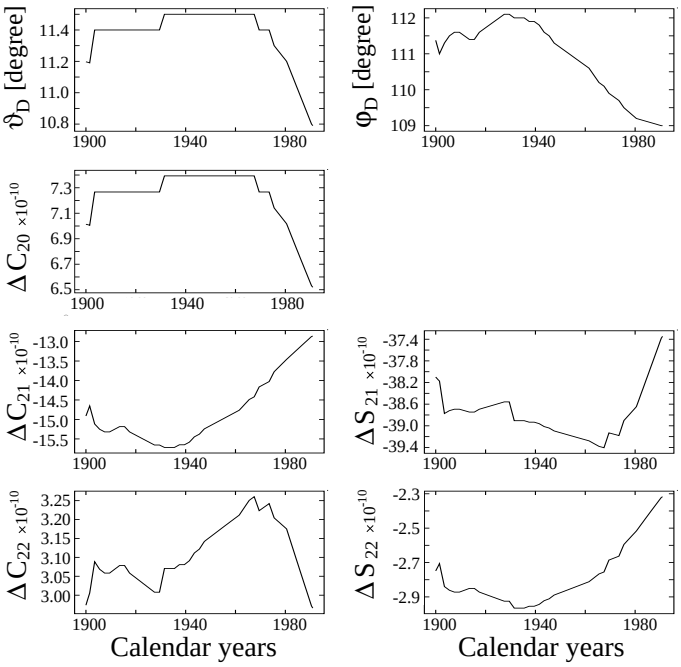

Figure 5: Predicted temporal variations of gravity due to the precession of the ellipsoidal and oblique inner core in terms of spherical harmonic coefficients ΔC_{2m} and ΔS_{2m}

of change over the last 10 years are given in Table 3. If we compare these values with the accuracy of present gravity models, e.g. GRIM4 (SCHWINTZER et al., 1997), and with the expected accuracy of the planned satellite missions CHAMP (REIGBER et al., 1997) and GRACE (TAPLEY, 1997) (Table 4), it seems to be possible to check the hypothesis of inner-core precession during the next decade. Nevertheless the problem of separating the different influences on the low degree harmonic coefficients has to be solved.

3 DISCUSSION OF THE MODEL ASSUMPTIONS

In more realistic studies of the influence of the inner-core rotation on the geomagnetic field, the prescribed current system in the outer core should be replaced by a system in agreement with recent dynamo models. In a next step of investigation, model extensions should be introduced, which self-consistently explain the relative rotation and its influence on the geomagnetic field. This requires the use of numerical methods. GLATZMEIER and ROBERTS (1996) show that the assumption of an angular velocity of the relative rotation of about $10^{-9}\,\mathrm{s}^{-1}$ is consistent with recent dynamo models. The relative rotation is then maintained by magnetic coupling between the inner core and

m	$\frac{d}{dt}C_{2m}$ $[a^{-1}]$	$\frac{d}{dt}S_{2m}$ $[a^{-1}]$
0	-4.74×10^{-12}	
1	$+5.69 \times 10^{-12}$	$+1.24 \times 10^{-11}$
2	-1.99×10^{-12}	$+1.89 \times 10^{-12}$

Table 3: Predicted rates of change of the normalized degree 2 spherical harmonic coefficients of the earth's gravity field caused by the hypothetic precession of the inner core averaged over the last 10 years

Model	$\sigma(C/S)$	$\sigma\left(\frac{d}{dt}(C/S)\right)$ $[a^{-1}]$
GRIM4	2×10^{-10}	4×10^{-12}
CHAMP	2×10^{-11}	1×10^{-12}
GRACE	2×10^{-12}	1×10^{-13}

Table 4: Estimated standard deviations of the low-degree ($n < 5$) harmonic coefficients and their rate of change for present and future gravity-field models; the estimates for CHAMP and GRACE are based on 1 year of data for $\sigma(C/S)$ and 5 years for $\sigma\left(\frac{d}{dt}(C/S)\right)$

the flow of the overlying liquid outer core. Recent seismological investigations (SONG and RICHARDS (1996)) have suggested that the inner core is rotating about its tilted symmetry axis relative to the mantle with an angular velocity of the same order of magnitude. The assumption of a fast relative rotation of the inner core about its tilted symmetry axis has therefore received some confirmation. However, the decade variations of the position could not yet be found by dynamo simulations and seismological investigations. This is also not considered in our model concerning the hypothesis about the dipole axis, where stationary solutions are used. Changes of the direction of the dipole axis should therefore be introduced in this kinematic model by a prescribed time dependence of $\mathbf{w}$. The expected result is a phase shift between theoretical and observed variations of polar motion. JOCHMANN (1989) estimated a mean phase shift of about 28 years for particular periods between 20 and 50 years, which is too short to be explained only by magnetic diffusion in the outer core. Alternatively, he proposed explanation of the phase shift by Alfvén wave propagation. Magnetohydrodynamic disturbances caused by inner-core motion propagate with the Alfvén wave velocity through the outer core. Using conventional parameters of the outer core, JOCHMANN (1989) estimates an associated propagation time that agrees fairly well with the phase shift mentioned above.

Particular periods of the polar motion can be explained by temporally varying gravitational torques affecting the hypothetical inner-core motion. Provided that the gravitational torques can be derived from observations, we can determine the motion of the symmetry axis of the inner core without use of the observed position of the dipole axis, and may confirm the relation between the inner-core motion and that of the geomagnetic dipole axis by comparing it with the result of the model. In a related study, JOCHMANN (1991) investigated the influence of the gravitational attraction of the sun and the moon on the motion of the inner core and found that the amplitude of the 18.6 years period in the observed polar motion is associated with the retrograde

motion of the moon's node. A corresponding period is also found in the spectrum of the theoretical polar motion calculated according to the motion of the observed dipole axis. Within the uncertainty of the Fourier-analysis method, this period may be associated with the 16-year period mentioned in section 2.2. In addition, JOCHMANN (1991) proved that long term variations of the external torques cause a secular variation of the relative motion between inner core and mantle with a period of about 320 years. The same period is discovered in the amplitude spectra of ϑ_D and φ_D by JOCHMANN and GREINER-MAI (1996), and a similar period of 360 years is found by ADAM (1983) in archaeomagnetic time series.

For the other periods, the kinematic model explains the influence on polar motion, but not the generation of these oscillations by internal core processes. A possible mechanism for the decade variations of the flow in the outer core is provided by BRAGINSKY's (1984) torsional oscillations, although the coupling with the inner-core motion is not explained up to now. In self-consistent dynamo models, the decade variations may be consequences of instabilities or transient processes constrained by additional geophysical conditions. For an excitation by gravitational variations, there is no evidence for these periods.

4 CONCLUSIONS

The results of our kinematic model show that the influence of the assumed relative precessional motion of the inner core on the magnetic field and the polar motion may cause observable changes of these quantities. Recent seismological studies and numerical dynamo simulations have shown that the relative rotation of the inner core about its figure axis with an angular velocity of about 10^{-9} s^{-1} may be real, while decade variations of its precessional motion relative to the mantle could not yet be found by these methods. The consistency of our kinematic model with the gravity observations may be proved in the near future.

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