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The Effect of the Geocentric Gravitational Constant on Scale

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Abstract

It is well known that the geocentric gravitational constant (GM) is a scaling factor for the reference frame realized by satellite techniques. One must be aware that its effects on the orbit and on the terrestrial reference frame (station positions) are different. The scale effect on restituted orbits is 1/3* (dGM/GM) (relative error of GM) for all kinds of satellites. But the effect on the terrestrial frame depends on the height of the satellites, on tracking techniques and on the solved for parameters. For ranging techniques such as SLR, the scale variation of the terrestrial frame is $1/3*(dGM/GM)*(r_{Sat}) / (r_{Earth})$, if the range biases are not solved for. For GPS the GM error is mostly absorbed by the clock estimates (or eliminated by the double differences), only the remaining few percents go into the scale of terrestrial reference frame. For instance if one is using a GM value of $3.986004418\ 10^{14}\ m^3/s^2$ instead of $3.986004415\ 10^{14}\ m^3/s^2$ (relative variation is $7.5\ 10^{-10}$) the scale variation of the terrestrial frame is only about $6\ 10^{-11}$.

Physically, the error in the z-direction of the antenna phase center offsets on board GPS has nothing to do with GM. But its effect on the terrestrial reference frame is practically equivalent to an error in GM. For instance, if all GPS satellites have a 7.1 cm error in dz, the effect on the station position is equivalent to a relative error of $8\ 10^{-9}$ in GM (e.g. changing GM from 3.986004418 to $3.986004386\ 10^{14}\ m^3/s^2$).

Keywords: Gravitational Constant, GM, scale

1. Introduction

GM is an important constant for satellite geodesy. Its value will affect the scale of the terrestrial reference frame (see e.g. Boucher 1989). In order to ensure the consistency of solutions of various satellite techniques, it is required that a uniform numerical value for it is adopted. But due to various reasons this is not yet completely fulfilled. Even if all techniques would use one value, this value still has an uncertainty. Some people think that as long as we use an identical GM value, the scale of the reference frames realized by various satellite techniques would be the same. Our study shows that even this assumption is not true.

In order to ease the discussion, we introduce the 'sensitivity factor k'. The scale variation is then expressed as:

$$\Delta \text{ scale} = k * (\Delta GM / GM) \quad (1)$$

First of all we must distinguish between the sensitivity factor k for orbit determination and the one for terrestrial reference frame realization. For integrated orbits, in the along-track direction the mean value of k equals to 1. For restituted orbits, once the tracking data are used to adjust the orbit, k reduces to 1/3 (Lerch et. al. 1978), since GM is proportional to m^3. We have made a lot of test runs, no matter which tracking techniques are used, no matter what kind of satellites are considered, the k for integrated orbit is always 1; that for the restituted one is 1/3. Table 1 gives a numerical example.

Table 1. Helmert transformation parameters of two Lageos orbits with dGM/GM= 7.5 10^{-10}.

Tx	Ty	Tz	Scale	Rx	Ry	Rz
cm	cm	cm	10^{-9}	mas	mas	mas
0.00014	-.00016	0.04461	0.251	0.00021	-.00007	-.01246
0.00057	0.00057	0.00057	0.005	0.00011	0.00011	0.00013

The sigma of the related parameters are listed in the last row of the table.
In the case of the terrestrial reference frame the situation is more complicated. It depends on the observation types, on what parameters are solved for, and on the height of the satellite. Section 2 uses the Lageos satellite as an example, the next section is devoted to the GPS satellites. The results clearly show that terrestrial frames estimated from satellite laser ranging (SLR) and from GPS are affected by GM in different ways.

Section 4 has actually nothing to do with GM. It deals with the problem of the errors in the phase center offset of the GPS satellites. The interesting thing is: this type of error affects the GPS solution (stations, Earth orientation parameters and orbits) almost exactly in the same way as an error in GM. Some concluding remarks and discussions are given in the last section.

2. The case of Satellite Laser Ranging

Let us take Lageos as an example, and suppose the fractional variation of GM is 7.5 10^{-10}. The scale of the restituted orbit increases by 2.5 10^{-10}, which corresponds to 3 mm at the altitude of Lageos. The least square sum of tracking data residuals ensure that the station height raises approximately the same amount. 3 mm at the earth's surface is not 2.5 10^{-10}, but 4.6 10^{-10}, or k ~ 0.61 (see table 2 for the detailed numbers). That is why in Himwich et al. (1993) it is mentioned, that 'the sensitivity of the scale of the SLR frame to the value of GM is about 2/3.'

Table 2. Helmert transformation parameters of the two station coordinate sets estimated from Lageos data with dGM/GM = 0.75 10^{-9}

Tx	Ty	Tz	Scale	Rx	Ry	Rz
cm	cm	cm	10^{-9}	mas	mas	mas
0.247	-.756	0.051	0.457	-.0025	-.0127	0.0046
0.40	0.40	0.40	0.06	0.016	0.016	0.016

Generally, for the SLR technique, if all station coordinates are free (with loose constraints), the sensitivity factor k for a terrestrial reference frame realized by a satellite with radius of r_{Sat}

is about

$$k = 1/3 * r_{Sat} / a_e \quad (2),$$

where a_e is the radius of the Earth.

In practice the situation is more complicated due to many reasons:

- Tracking data from many satellites are used for realizing a reference frame, their heights are different. So approximately the scale effect in the reference frame is a 'weighted mean' of all individual satellites.
- The above conclusion is no more true when other parameters like range biases and similar are solved for. A range bias absorbs largely the error in radial direction. For our test arc, if range biases of three stations (15% of all stations) are solved, the effect on the scale reduces from 0.46 10^{-9} to 0.40 10^{-9}.
- Some institutes may fix a few fiducial sites in their solution. Then the scale of the fiducial sites constrains the scale of the estimated stations. This also changes the effect of the GM variation on the scale of the reference frame.

3. The GPS case

If a GPS satellite is tracked by SLR (only possible for GPS-35 and GPS-36) then equation (2) could also be used. The GPS microwave technique is an 'one way' system, so two clocks are needed. If only the satellites located at the zenith are observed, then the orbit scale 'error' will be completely absorbed by clock estimates (or eliminated in the double differences). If the zenith angle is not zero, then the clock only absorbs the main part. The rest will affect the estimation of station coordinates, thus the scale of the terrestrial reference frame. The sensitivity factor in this case is

$$k = b * 1/3 * r_{Sat} / a_e \quad (3),$$

For a cut-off angle of 15 degree, the typical value for b is about 5%. Dozens of test runs give similar values for b, table 3 shows one example.

Table 3. Helmert transformation parameters of the two GPS station coordinate sets with dGM/GM = 0.75 10^{-9}

Tx	Ty	Tz	Scale	Rx	Ry	Rz
cm	cm	cm	10^{-9}	mas	mas	mas
0.001	0.017	-.208	0.058	-.0034	0.0041	-.0013
0.003	0.003	0.003	0.004	0.0010	0.0011	0.0012

4. Uncertainties in the GPS antenna phase center offset

The conventional values for phase center offsets on board the GPS Block II/IIA satellites, as given in IERS Standards 1992, are 0.279 m, 0.000 m, 1.023 m in X, Y, and Z direction, respectively. In the IERS Conventions 1996 (‘Errata from Technical Note 13') it is stated that ‘GPS dz = 0.9515 m vice 1.0229 m’. The difference is about 7 cm. Some IGS Analysis Centers now use the new value, some still keep to the old one. Viewing from a ground receiver, an increasing dz is equivalent to lower the orbit by the same amount. The latter could be the result of reducing the GM. Instinctly we would expect that these two essentially different events could lead to a similar influence on the terrestrial reference frame. In order to verify this, three test runs were made.

Test 1: A GM value of 3.986004418 10^{14} m^3/s^2 is used, dz is the new one;

Test 2: same GM as above, but 7.1 cm added to dz of all satellites (including non Block II satellites);

Test 3: dz is the same as in test 1, but GM intentionally is reduced to 3.986004386 10^{14} m^3/s^2.

In all three tests the solved for parameters are station coordinates with loose constrains, orbits, clocks, parameters of solar radiation model and tropospheric corrections.

Table 4 to 6 show the comparison of test 1 with test 2, 1 with 3, and 2 with 3, respectively. Test 2 and 3 are astonishingly similar. For the estimates of station coordinates, earth orientation parameters, etc. an error in dz is practically identical with an error in GM. 7.1 cm in dz corresponds to a dGM/GM of 8.02 10^{-9}.

Table 4. Helmert transformation parameters of the two GPS station solutions with dz difference = 7.1 cm

Tx	Ty	Tz	Scale	Rx	Ry	Rz
cm	cm	cm	10^{-9}	mas	mas	mas
0.014	0.000	0.036	0.576	0.0008	-.0006	-.0008
0.002	0.002	0.002	0.003	0.0007	0.0008	0.0008

Table 5. Helmert transformation parameters of the two GPS station solutions with dGM/GM difference = 8.02 10^{-9}

Tx	Ty	Tz	Scale	Rx	Ry	Rz
cm	cm	cm	10^{-9}	mas	mas	mas
0.015	0.000	0.038	0.576	0.0008	-.0007	-.0007
0.002	0.002	0.002	0.003	0.0007	0.0007	0.0008

Table 6. Helmert transformation parameters of the two GPS station sets, one with dGM/GM = 8.02 10^{-9}, the other with delta dz = 7.1 cm

Tx	Ty	Tz	Scale	Rx	Ry	Rz
cm	cm	cm	10^{-9}	mas	mas	mas
-.0001	-.0001	0.0018	0.0004	-.00002	-.00008	0.0004
0.0001	0.0001	0.0001	0.0002	0.00004	0.00004	0.00004

The reason for this is simple: adding 7.1 cm to dz for all satellites is equivalent to reducing the height of the satellites (which refers to the mass-center) by 7.1 cm, while keeping dz unchanged. Reducing GM by 8.02 10^{-9} also reduces the orbital height by 7.1 cm. Therefore the two physically different things show the same effect on the scale of the estimated terrestrial reference frames.

Generally, the error in dz (same for all GPS satellites) corresponds to an error in GM, which amounts to:

$$dGM/GM = - 3 * \Delta (dz) / r_{Sat} \qquad (4),$$

where r_{Sat} is the geocentric distance of a GPS satellite.

The actual situation may be more complicated. For instance, different types of GPS space vehicles could have different errors in dz (Block II/IIA is different from Block IIR). In this case, the scale for the station coordinates is affected by some 'weighted mean' of the different dz's and the effect will also not be limited only to the radial direction and to scale.

5. Concluding remarks.

1) The sensitivity factor k of GM to scale can be expressed as

Application	**Sensitivity Factor k**
Restituted satellite orbit	1/3
Terr. ref. frame from SLR	$1/3* r_{Sat} /a_e$
Terr. ref. frame from GPS	$0.05*1/3* r_{Sat} /a_e$

where r_{Sat} and a_e are the radius of the satellite and the earth, respectively.

2) The phase center offsets on board GPS are not well determined, especially in the z direction. If all GPS satellites have the same error Δdz, then their effects on station coordinates, EOP and clock estimates are practically identical to a GM error which is shown in equation (4).

Remarks:

(1) The terrestrial reference frames realized by various techniques have different scales, and they change with time (see Altamimi et. al. 2000). One of the important reasons is the uncertainty in GM. Even if all IGS Analysis Centers use the same GM as required by the IERS Conventions, the influence on various techniques is still different. Even for the same technique like SLR, in different years the composition of tracked satellites changed. For instance, if in one year more low altitude satellites (like Starlette, GFZ-1, Stella) are being tracked, and in another year more high altitude satellites (like ETALON, Lageos), then the scale of the reference frames realized in these two years will be different, because the scale effect depends on the height (or radius) of the satellites. Therefore, we need not only to use the same GM for all Analysis Centers, but wish to have the same composition of analyzed satellites.

(2) There are laser reflectors on board of GPS-35 and GPS-36. When one computes the laser residuals by fixing the IGS orbits, systematic bias-like errors are found. When one first computes the orbit of GPS-35 by using laser data only, and then compares it with the IGS orbits, one sees a significant scale difference in the two orbits (see Zhu et. al 1997). The error in dz could play an important role here.

(3) In Section 6.4.7 of Lemoine et. al., 1998, it is mentioned that the GM values estimated from GPS and non-GPS satellites are different. The results shown in this study might be used as one possible explanation.

References

Altamimi Z, Boucher C and Sillard P (2000) Results from ITRF 2000 Analysis. Suppl. EOS Transactions, AGU, Vol. 48, 332

Boucher C (1989) Current Intercomparisons between CTS's. In: 'Reference Frames in Astronomy and Geophysics', J. Kovalevsky et al. (ed), Kluwer Academic Publishers, 327-348

Himwich W E, Watkins M M, Ma C, MacMillan D S, Clark T A, Eanes R J, Ryan J W, Schutz B E, Tapley B D (1993) The Consistency of the Scale of the Terrestrial Reference Frames Estimated from SLR and VLBI Data. In: 'Contributions of Space Geodesy to Geodynamics: Earth Dynamics' D E Smith/D L Turcotte (ed) AGU Geodynamics Series Vol. 24, 113-120

Lemoine F G, et. al. (1998) The Development of he Joint NASA GSFC and the NIMA Geopotential Model EGM96. NASA/TP-1998-206861

Lerch F J, Laubscher R E, Klosko S M, Smith D E, Kolenkiewicz R, Putney B H, March J G, J.E. Brownd J E (1988) Determination of the geocentric gravitational constant from laser ranging of near-Earth satellites. Geophys. Res. Lett., 1031-1034

McCarthy D D (ed.) IERS Standards (1992) and IERS Conventions (1996) IERS Tech. Notes 13 and 21. Observatoire de Paris

Zhu S Y, Reigber C, Kang Z (1997) Apropos laser tracking to GPS satellites. J. of Geodesy, 423-431

Satellite Antenna Phase Center Offsets and Scale Errors in GPS Solutions

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Abstract

ITRF2000 solutions (see Lareg, 2001) have shown that there are ppb level scale differences between GPS and other techniques and among various GPS Analysis Centers. The trends of the scale differences reach 0.2 ppb per year. The uncertainties of the current available Earth's gravitational constant could only cause less than 0.1 ppb scale error for GPS technique. On the other hand, the uncertainties in the satellite antenna phase center offsets could produce ppb level scale error. Various BLOCK types of GPS satellites have different phase center errors. The number of BLOCK IIR satellites increases from year to year. This could cause trend-like variations in the scale error.

Beside station positions, satellite antenna phase center errors affect also the clock, Zenith Path Delay, and other solved for parameters perceptibly.

Keywords: GPS, Phase center offset, scale

1. Introduction

ITRF2000 solution shows that the scales of reference frames realized by various techniques have considerable differences (bias and trend). Even the scales of various IGS Analysis Centers (AC) behave differently. In fig.1 (from Lareg 2001) the scales of GPS solutions are compared with ITRF2000. We try to search for possible error sources, which could affect the scale of GPS solutions.

It is well known, that GM (Earth's gravitational constant) is an important scale factor for satellite geodesy. But GPS is an exception. For instance the two GPS ACs, JPL and CODE, use the same GM value, but their scales are still significantly different, as can be seen from fig. 1. Accordingly, one must find other reasons for the difference. One of the candidates is the error in the offsets from the GPS satellite center-of-mass to the antenna phase center, especially in the z direction (denoted z_off later-on).

The information of z_off is not accurate. The GPS Operational Control Segment recently changed significantly the values for the Block IIR satellites, e.g. z_off of PRN14 has been changed from 0.86710 m to 1.61366 m (cf. Ray, 2001). Some IGS AC even use z_off = 0 m for all IIR satellites. For Block II & IIA, a few ACs use z_off = 0.95 m, others use 1.02 m. Table 1 gives the GM and z_off values used by all IGS ACs (IGS 2001).

In following sections we use some examples to demonstrate the effect of dz, which is the error of z_off, on GPS solutions. A similar study has already been performed by Springer (2000). In all the examples 24 GPS satellites (22 BLOCK II/IIA, two BLOCK IIR) were tracked by 42 stations. At each epoch the clock corrections were solved except one station clock was fixed as reference. The data sampling rate was 2 minutes and the cut-off angle was 15 degree. Minimal constraints were applied for the position estimates. In the reference solution z_off for the BLOCKII/IIA satellites was 0,95 m, which corresponds to the value used by JPL, that for IIR was taken as zero. Four contrast solutions were computed and compared with the reference solution. In case 1, z_off of all satellites were artificially increased by 7 cm, even for the BLOCK IIR satellites. The second case is similar to the first one, but dz was increased by 1 m instead of 7 cm for all satellites. In the third case z_off for the II/IIA satellites is 102 cm, that of BLOCK IIR is zero (these are the z_off values used at GFZ), In the last case z_off of CODE was adopted: 102 cm for BLOCK II/IIA and 120.5 cm for BLOCK IIR.

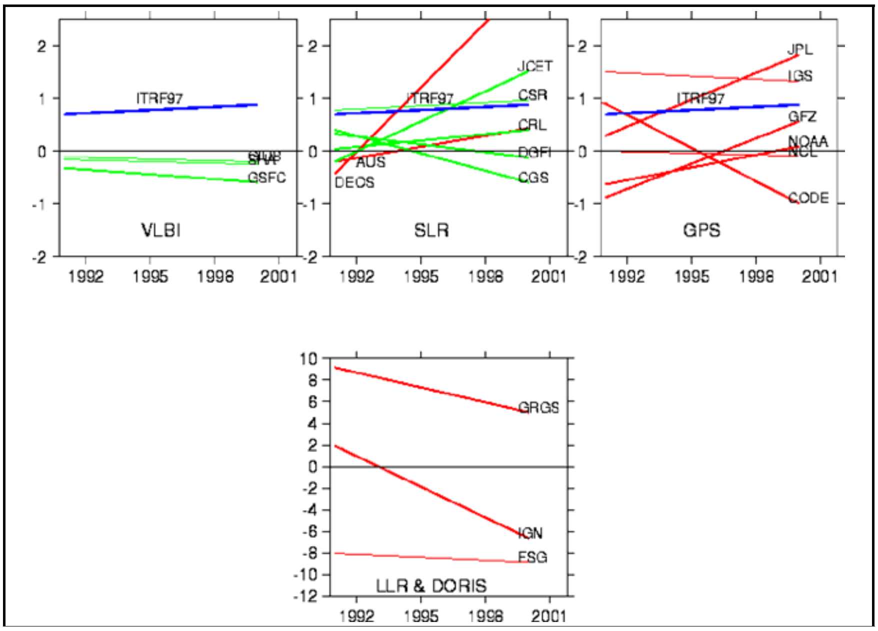

Figure 1: ITRF2000 relative scales (ppb = 10^{-9})

In all the above mentioned examples the orbits were solved together with other parameters. Test runs were repeated by fixing the IGS orbits. The effects of dz on the station, clock and ZPD estimates are same as those of orbit restitution cases.

Table 1. The GM and z_off values used by all IGS ACs.

ACs	**GM** (km^3/sec^2)	**BLOCK II/IIA** z_off (meter)	**BLOCK IIR** z_off (meter)

CODE	398600.4415	1.0259	1.2053
ERM	398600.4415	1.0229	no info.
ESA	398600.4415	1.023	0.000
GFZ	398600.4418	1.0229	0.000
JPL	398600.4415	0.9529	no info.
NOAA	398600.4418	1.0230	1.0230
SIO	398600.4415	0.9519	0.0000
USNO	398600.4415	1.023	0.0000

2 The effect of dz on clock estimates

If the satellite is in zenith, dz will be completely absorbed by the clock estimates. The actual amount of dz being absorbed depends on the "observation angle", the angle between the satellite-receiver and satellite-geocenter vectors. Since the altitude of GPS is very high, the maximum of this angle is about 14 degree, see (Springer, 2000), which ensures that at any zenith angle more than 90% of dz will be absorbed. This means only a few percents of dz will affect the estimates of other parameters. It also explains why it is difficult to estimate dz from GPS data alone, see (Yoaz, 1998).

The examples of dz effects on the estimated satellite clocks are given in table 2, columns 3 and 4, with
mean dz = $\Sigma\, dz_{Sat} / n_{Sat}$, and
rms dz = $\sqrt{}\, (\Sigma(dz\_i - \text{mean } dz)^2) / n_{Sat}$.

One can easily see that a) about 95% of dzs were absorbed by the related satellite clocks, b) increasing the mean dz results in a decrease of the percentage of dz being absorbed by the clock estimates and enlarges the fluctuation of clock estimates, c) bigger rms dz corresponds to smaller percentage of dz being absorbed and larger fluctuation.

Table 2. Effect of dz (increment of z_off) on the satellite clock estimates and orbits

Case (dz)	**Mean dz rms dz**	**Average Clock Diff.**	**rms Clock Fluct.**	**Orbit Diff. (3d pos.)**
7 cm all satellites	7 cm zero	67 mm, 96%	+/- 0,1 mm	1 mm
1 m all satellites	1 m zero	943 mm, 94%	+/- 0,4 mm	6 mm
7 cm II/IIA	6,4 cm	66 mm, 95%	+/- 0,2 mm	3 mm

zero IIR	1,9 cm	< +/- .3mm		
7 cm II/IIA 1,2 m PRN11 1,2 m PRN13	16 cm 34 cm	a few mm 113 cm 94% 113 cm 94%	+/- a few mm	1 cm

The numerical results listed in table 2 depend on the elevation cut_off angle. Most GPS users adopt a cut_off angle between 10 and 20 degree. Therefore we use 15 degree in the demonstration examples. Roughly, when 95% of dz is absorbed by clock estimates using a cut_off angle of 15 degrees, then with 20 degrees it would be around 96%; with 10 degree it reduces to about 94%. The results shown in the following sections are also affected by the cut_off angle in a similar way.

3. The effect of dz on orbits

The effects of dz on the orbits are small. Larger mean dz and especially bigger rms dz indicate a degradation of the orbit quality. The examples are given in the last column of table 2. Usually the effects are at the mm level, only when the rms dz is at the few decimeter level, such as in the last case, the degradation of orbits could reach the cm range.

Even 1 m mean dz does not affect the scale of the orbit perceptibly. Table 3 lists the transformation parameters for this case.

Table 3. Orbit transformation parameters for case 2, mean dz = 1 m

Tx **cm**	**Ty** **cm**	**Tz** **cm**	**Scale** **ppb**	**Rx** **mas**	**Ry** **mas**	**Rz** **mas**
-.03	0.09	0.52	0.007	-.003	-.002	0.001
.001	.001	.001	0.002	.001	.001	.001

The sigmas of the estimated parameters are given in the last row.

4 dz effect on station coordinates

dz affects the station positions more significantly than the orbits, but mostly in the vertical component. If a 7 parameter Helmert transformation is applied, one sees clearly the scale difference (table 4). After the transformation the rms differences of station positions become significantly smaller. As expected, the rms of coordinate differences increases with mean dz, and especially increases with an increasing rms dz. Case 3 tells us that the scale of the terrestrial frame realized by GFZ should be smaller than that of JPL by 0.5 ppb. From fig. 1 the scale bias between these two institutes is about 1 ppb, that is, dz contributes half of it.

Table 4. Effect of dz on the scale of stations.

mean dz / rms dz	Variations in: Station Height and Scale	rms after Transformation
7 cm / zero	-3,5 mm / -0,58 ppb	0,1 mm
1 m / zero	-51 mm / -8,1 ppb	1,5 mm
6,4 cm / 1,9 cm	-3,3 mm / -0,52 ppb	0,2 mm
16 cm / 34 cm	-8 mm / -1,2 ppb	2.5 mm

The above results indicate that the effects of dz on the station height and scale could approximately be expressed as (rule of thumb):

d (height) ~ - 5% * mean dz (1),
d (scale) ~ - 5% * mean dz / (Earth's radius) (2).

5. Effect of dz on Zenith Path Delays (ZPD)

The dz has also an influence on the estimated ZPD. The results are summarized in table 5.

Table 5. Effect of dz on ZPD

mean dz rms dz	Average ZPD Variation	Fluctuation of estimated ZPD	Sigma / Noise
7 cm zero	0,32 mm +/- 0,005 mm	< 0,1 mm	64
1 m zero	4,4 mm +/- 0,07 mm	1,0 mm	63
6,4 cm 1,9 cm	0,28 mm +/- 0,007 mm	0,1 mm	40
16 cm 34 cm	0,7 mm +/- 0,1 mm	1,2 mm	8

Approximately, the effect of dz on the average ZPD of all stations could be expressed as:

Average delta ZPD ~ 0,4% * mean dz (3),

which is about one order of magnitude smaller than the effect on the station height.

The fluctuation is the rms difference of the individual ZPD with respect to the average one. These values are listed in the third column of table 5. The sigma of the average ZPD should be the fluctuation divided by the square root of the overall number of estimated ZPD parameters. They are given in the lower part of the second column. The values in the fourth column are the average ZPD change divided by its sigma, which is a kind of signal/noise ratio. This value decreases very significantly with increasing rms dz. The reason for that is simple, as can be seen from case 4: for each time interval, in which a ZPD is estimated for any station, the amount of BLOCK IIR tracking data is different. The more BLOCK IIR data the estimation contains, the larger the ZPD variation will be. This means, the mean dz results in a systematic bias-like error in the ZPD, the rms dz acts as an additional random error source, which causes fluctuations in the individual ZPD estimates, and at the same time degrades the other estimated parameters such as station positions. The rms dz could be more harmful for the GPS solution.

6. Similarity of dz effects and an error in GM.

Unlike dz, GM has a significant effect on the scale of the GPS orbits.

d (scale of orbit) = 1/3 * dGM / GM * a /r (4),

where a and r is the mean radius of the satellite and the Earth, respectively. dGM / GM is the fractional error of GM. Putting the effect on the orbits aside, as far as site positions (and the scale of terrestrial reference frame), clock estimates, and ZPD are concerned, the effect of the dGM is practically equivalent to an error in z_off (see Zhu et.al. 2001):

dz (same for all satellites) <--> - 1/3 * dGM/GM * a, (5)

The GM difference between JPL and GFZ is 0.0003 km^3/sec^2, or dGM/GM = 0,75 ppb. According to eq. 5, it corresponds to a dz of 6,5 mm. Table 6 gives the effects of dGM/GM on the orbit scale, the clock, station position, and the ZPD. For the sake of comparison the effect of dz (with rms dz = 0, that is, all satellites suffer from the same dz) are also listed. It is clear that the difference of GM values used by JPL and GFZ can only have a small effect on the scale of the reference frames.

Table 6. Comparison of the effects of dGM and those of dz.

	Orbit Scale	**Mean Clock Variation**	**Station Scale ppb**	**ZPD Variation**
dGM/GM = -8 ppb	- 2,7 ppb	67 mm	-0.58 (-3,5 mm)	0,32 mm
dz = 7 cm	< 0,001 ppb	67 mm	-0.58 (-3,5 mm)	0,32 mm
dGM/GM = 0,75 ppb	0.25 ppb	6,2 mm	0.05 (0,3 mm)	-0,03 mm
dz = -6,5 mm	< 0,001 ppb	6,2 mm	0.05 (0,3 mm)	-0,03 mm

7. Conclusion and Summary

* GPS scales

dz is one of the major reasons which cause ppb level bias and trend error in the scale of GPS stations. The effect of GM uncertainty is an order of magnitude smaller. The numbers given in table 7 are demonstration examples. Last row explains the trend-like scale variations shown in fig. 1.

Table 7. Scale error caused by various errors

Error Sources	Effect on Scale (ppb)
gravity field model and GM	< 0,1 (bias)
all sat. are BLOCK II/IIA z_off: 1,02 m vs. 0,95 m	0,6 (bias)
one from 27 sat. is BLOCK IIR z_off: 1,2 m vs 0 m	0,3 (bias)
each year one more IIR sat. z_off: 1,2 m vs 0 m	0,3 /year (trend)

* Clock estimates

Precise GPS clock information is useful for the Precise Point Positioning (PPP), see (Zumberge et.al 1997), and for the High Precision Time and Frequency Transfer, cf. (Springer 1999).

For clock combinations or comparisons one should take into account that:
a) 95% of dz is absorbed by the related satellite clock. The actual amount can vary by 1 - 2 % due to the adopted cut_off angle and the rms dz. For BLOCK IIR satellites, this corresponds to 1 - 2 cm.
b) 0.0003 km^3/sec^2 difference (or error) in GM produces 6 mm systematic error (or difference) in clocks. The gravity field mismodeling can also cause mm level errors in clock estimates.

In PPP applications and LEO orbit determination (using the so called two step method, see Neumayer et. al. 2001) the GPS clocks are fixed to some estimated values(e.g. from IGS products). One must make sure, that the z_off values used in such solutions are identical with those used in the GPS clock estimates. Since any clock estimate is related to a certain adopted z_off value.

* For Zenith Path Delay

dz of BLOCK IIR amounts to one meter. dz values between BLOCK IIR and II/IIA, and even among various satellites of the same BLOCK type are different. This results in mm level errors in the ZPD. For precise atmospheric studies this error can not be ignored.

* Summary

In summary one can say that with the appearance of BLOCK IIR satellites, the error in z_off becomes an important error source for high accuracy GPS applications.

References

IGS (2001) http://igscb.jpl.nasa.gov/igscb/center/analysis/

Lareg (2001) http://lareg.ensg.ign.fr/ITRF/ITRF2000/.

Neumayer K.-H., Zhu S.Y., Reigber Ch., Koenig R., Meixner H. (2000) 'CHAMP Orbit Recovery - Algorithmic Approaches and Results,' EOS, Transactions, American Geophysical Union, Vol. 81, No. 48, p. F309

Ray J. (2001) E-mail message from Jim Ray (USNO 202-762-1444) Message-Id: <200102152022.PAA13699@maia.usno.navy.mil>

Springer T. (1999) 'Modeling and Validating Orbits and Clocks Using the Global Positioning System', PhD thesis, Astronomical Institute, University of Berne, Berne, Switzerland, November 1999.

Springer T. (2000) 'Common Interests of the IGS and the IVS', published in Proceedings of the IVS General Meeting, Kotzting, Germany

Yoaz E. Bar-Sever (1998) 'Estimation of the GPS Transmit Antenna Phase Center Offset', EOS Transactions, 79 (45):183

Zhu S.Y., Massmann F.-H., Yu Y., Reigber Ch. (2001), 'The effect of the geocentric gravitational constant on scale', submitted to the J. of Geodesy

Zumberge J.F., Helfin M.B., Jefferson D.C., Watkins M.M., and Webb F.H. (1997) 'Precise point positioning for the efficient and robust analysis of GPS data from large networks', J. Geophys. Res., 102 (B3), p. 5005-5018