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Short Note

k_0 : The role of Intrinsic and Scattering Attenuation

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Abstract Knowledge of the acceleration spectral shape is important for the prediction of ground motion. At high frequencies, the rapid decrease of the spectral amplitude, which controls the peak values, has been modeled by the spectral decay factor k , allowing an estimate of the apparent attenuation and which currently constitutes a basic input parameter for the generation of stochastic ground motion and the calibration of ground-motion prediction equations. Based on numerical simulations of ground motion, we investigate the role of intrinsic and scattering attenuation in determining the high-frequency decay of earthquake-induced ground motion. We show that the attenuation term related to scattering depends nonlinearly on the intrinsic term, meaning that the commonly used explanation for the high-frequency decay spectrum parameter might not be appropriate when analyzing signal windows of several seconds' width.

Introduction

The high-frequency parameter k introduced by [Anderson and Hough \(1984\)](#), which is derived from the high-frequency decay of the Fourier spectra of acceleration recordings, is related to the attenuation that the seismic wavefield undergoes from its generation at the seismic source to its propagation to the recording site. The k component related to attenuation below a site, k_0 ([Anderson, 1991](#)), has become a parameter of major importance in engineering seismology, in particular in seismic-hazard assessment studies (e.g., [Douglas et al., 2009](#)). However, while several approaches to estimate k have been proposed (a comprehensive review can be found in [Ktenidou et al., 2014](#)) and possible biases in its estimation identified ([Parolai and Bindi, 2004](#)), its physical significance is still not fully understood.

In general, it is assumed that k provides an estimation of the apparent attenuation ($1/Q_{\text{app}}$) below a site due to the combined effect of the intrinsic and scattering quality factors Q_i and Q_{sc} , respectively, through the relationship

$$\frac{1}{Q_{\text{app}}} = \frac{1}{Q_i} + \frac{1}{Q_{\text{sc}}}. \quad (1)$$

However, while this approximation is valid when analyzing the first impulse of a signal ([Menke and Chen, 1984](#)), as is generally done in seismic experiments, it might not be appropriate when the analysis is carried out over signal windows of several seconds' width.

In fact, as also shown in this article using numerical simulations, when analyzing a signal in the time domain, the effect of both the intrinsic attenuation and scattering is to reduce the amplitude of the first propagating impulse (Fig. 1). However, the intrinsic attenuation leads to this effect by acting as a low-pass filter on the signal, whereas the scattering leads to a diminishing of the amplitude of the first pulse due to a redistribution of the

seismic-wave energy (mainly from the low to high frequencies) from the first arrival to the later part of the signal (Fig. 1) ([Richards and Menke, 1983](#)). Moreover, the way the spectral amplitudes are affected by the scattered wavefield depends on the spectra of the impedance model ([Menke and Chen, 1984](#)).

It follows that if only a short signal window, not yet affected by the later-arriving scattered waves, is analyzed, the net attenuation effect in the frequency domain can be described by equation (1). However, when the window of the analyzed signal is wider and contains successive arrivals of waves scattered by shallow heterogeneities below the site, as in the case when k is evaluated over a several-second-length window of signal, the combined effect of intrinsic and scattering attenuation in the frequency domain might lead to more complicated patterns. In fact, the low-pass filtering intrinsic-attenuation effect is combined with the energy redistribution in the signal spectra due to scattering. At the same time, the importance of the scattered waves in the surface recording should depend in turn on the intrinsic attenuation below the site, because the scattered waves are propagating through the usually nonelastic structure below the station.

Because k_0 is used in seismic-hazard studies to account for the local modification of ground motion (e.g., [Douglas et al., 2009](#)), it is of major importance to understand its physical meaning. For example, if k_0 at one site is mainly determined by net forward scattering, using the quality factor derived by it as a proxy for the Q_i in numerical simulations of ground motion would lead to seismograms with fairly realistic peak amplitudes but strongly biased durations.

In this study, based on numerical simulations of ground motion in shallower layers, we investigate the role of intrinsic and scattering attenuation in determining the high-frequency

Figure 1. (top) Seismogram obtained from the propagation of a bandlimited pulse in a nearly elastic ($Q_S = 1000$) and homogeneous velocity medium (400 m/s). (middle) Seismogram obtained when Q_S is set to 10 in the uppermost 100 m of the model. (bottom) Seismogram obtained considering an elastic medium with random velocity variations in the vertical direction (see the [Separating Scattering and Intrinsic Attenuation](#) section and Fig. 3 for details).

decay of earthquake ground motion. In particular, simulations are carried out considering the vertical propagation of S waves in homogeneous or vertically heterogeneous models. The results are analyzed to highlight the different contributions of intrinsic attenuation and scattering to the high-frequency spectral shape and therefore to the estimation of k_0 .

Numerical Simulations

Separating Scattering and Intrinsic Attenuation

To understand the different roles of scattering and intrinsic attenuation on the high-frequency slope of the Fourier spectra of ground acceleration, we first carried out simulations using three extreme models. The synthetic seismograms were calculated using a semianalytical method that consists of an improved Thompson–Haskell propagator matrix algorithm (Wang, 1999). Although the formulation of the propagator algorithm was very clear and could be applied to an unlimited number of layers, its numerical results showed the same instability known from dynamic solutions. Wang (1999) approached the problem from a physical point of view and proposed an algorithm that avoids numerical instabilities between incident waves from the source at each layer interface using an orthonormalization approach.

First, a half-space model with an S -wave velocity of 400 m/s and a constant quality factor for S waves Q_S (corresponding to the Q_i in all the analyses from here on) equal to 1000 is used. This value is chosen arbitrarily and is large enough to allow the simulation of the propagation of a plane wave in a nearly elastic medium. The transverse source with an arbitrary amplitude is located at 500 m depth (the same position is used for all the following simulations). The corresponding simulated surface recording (Fig. 1) approximates the bandlimited (0–100 Hz) S -wave Green’s func-

Figure 2. The spectrum of the reference seismogram (thin black line here, and the top record in Fig. 1), the spectrum of the recording obtained by considering the effect of intrinsic attenuation only (gray line here, and the middle record in Fig. 1), and the spectrum of the seismogram obtained in a scattering medium (black thick line here, and the bottom record in Fig. 1). The absolute amplitude values are arbitrary.

tion of the medium. The Fourier spectrum of this signal is the reference spectra for the following analysis (Fig. 2). Figure 2 shows that the Fourier spectrum is nearly flat with a very slight slope due to the high Q_S value used.

Second, the role of the intrinsic attenuation alone is studied by setting Q_S equal to 10 in the uppermost 100 m. A frequency-independent quality factor is adopted, consistent with standard engineering practice (Parolai *et al.*, 2010). The frequency dependence of the quality factor is still an open issue (Morozov, 2008, 2010; Cantore *et al.*, 2012); and, in any case, its introduction would not modify the main outcomes of this study regarding the respective roles of intrinsic and scattering attenuation. Figure 1 (middle) shows that the main effect of considering the intrinsic attenuation in the calculated seismogram is a decrease in amplitude and a broadening of the pulse when analyzed in the time domain. Figure 2 (gray) depicts the corresponding Fourier spectrum of the signal, showing a clear decay with increasing frequency. When a spectral fitting (here, we adopted a nonlinear least squares Marquardt–Levenberg algorithm) is carried out to model the high-frequency decay of the spectrum (considering the 10–50 Hz frequency band), the correct value of $Q_S = 10$ is retrieved.

Third, scattering in the uppermost 100 m is simulated by subdividing this section into 100 layers of 1 m thickness. The travel time within each layer is perturbed randomly, starting from the average value (0.0025 s) derived from the S -wave velocity of the homogeneous model (400 m/s) and considering a Gaussian distribution of the perturbations with a standard deviation equal to 0.3 of the average value. The only constraint used is that the sum of the travel times in each layer

Figure 3. (left) S -wave velocity model used to simulate net forward scattering. (right) Q_S velocity models to simulate the Q_S variability when the net forward scattering is considered (see the [Scattering and Intrinsic Attenuation Combined Effect](#) section for details). Three different reference values are considered: Q_S equal to 10 (black), 30 (dark gray), and 50 (light gray).

over the whole 100 m equals the original average travel time (0.25 s). The S -wave velocity model derived in this way is shown in Figure 3 (left). Figure 1 (bottom) shows the resulting simulated surface recording. The amplitude of the first arrival is strongly reduced, and several later arrivals can be identified. Considering the frequency range computed in this simulation and the considered velocity model and thicknesses of the layers, these later phases are mainly due to Rayleigh and Mie scattering regimes (Wu and Aki, 1988). Figure 2 (thick black line) in turn shows that attempts to estimate k from the associated spectrum (gray line) would strongly depend on the frequency band chosen for the spectral fitting. For example, using the 10–50 Hz frequency band, as suggested by Ktenidou *et al.* (2014), and considering the relationship

$$k = \sum_{i=1}^n \frac{t_i}{Q_{S_i}}, \quad (2)$$

in which n is the number of layers in the model and t_i the travel time in each layer (Hough and Anderson, 1988), would lead to an apparent Q_S of around 17. As is clear from Figure 2, a frequency band between 15 and 25 Hz would see a much smaller attenuation being estimated, corresponding to an apparent Q_S of 91, whereas considering a frequency band between 15 and 20 Hz would even lead to a negative value of $Q_S = -2$.

Scattering and Intrinsic Attenuation Combined Effect

To evaluate the combined effect of scattering and intrinsic attenuation, three other case studies have been considered. In the first one, an S -wave velocity model identical to that shown in Figure 3 is used, in which the Q_S structure is considered in the uppermost 100 m either to be homogeneous (with Q_S equal to 10) or derived by randomly perturbing the ratio between the travel time and the quality factor (starting from the value of the homogeneous model) in each one-meter thick layer, under the constraint that

$$\frac{t_{\text{tot}}}{Q_S} = \sum_{i=1}^n \frac{t_i}{Q_{S_i}}, \quad (3)$$

in which t_{tot} is the total travel time in the uppermost 100 m, Q_S is the value adopted in the homogeneous model (here 10), t_i is the travel time in each randomly perturbed layer, and Q_{S_i} is the intrinsic quality factor in each layer. In the other two considered cases, the only difference is that the perturbed t_{tot}/Q_S ratio assumes Q_S values of 30 and 50, respectively. All of the resulting Q_S profiles are shown in Figure 3 (right).

In these cases, the combination of applying the spectral fitting to the 10–50 Hz frequency band and of the scattering effect leads to Q_S values equal to 7, 9, and 11 for the cases in which Q_S was set to 10, 30, and 50, respectively. The analysis of the results shows that the higher the intrinsic attenuation, the lower is the effect of the scattering. When the intrinsic attenuation is strong (e.g., when $Q_S = 10$), the decay of the high-frequency spectrum tends to the values determined when only considering the intrinsic attenuation. This effect can be appreciated when calculating the root mean square (rms) of the differences between the logarithm of the spectral amplitudes in the frequency band selected for the spectral fitting and the values derived from the linear trend, with the rms increasing from 0.53 to 1.02 when Q_S equals 10 and 50, respectively. When the intrinsic attenuation is low, only by considering a wide frequency band for spectral fitting can a robust (intrinsic attenuation-related) high-frequency decay value be obtained. In all the other cases, the scattering effect hampers any reasonable and robust high-frequency estimation (that, in any case, would remain strongly frequency-band dependent).

Conclusions

In this study, we analyzed the contribution to the high-frequency decay spectrum parameter k_0 of the intrinsic attenuation and scattering. Our results confirm the joint contributions of both effects but also that the generally used explanation of the value of this parameter, recalling the analysis of the propagation of a pulse, is not appropriate when Fourier spectra of several-second-length windows are used (equation 1). We showed that when considering net forward scattering effects, energy is redistributed at different spectral ordinates, making a robust k_0 estimation cumbersome, especially if (1) the intrinsic attenuation is weak and (2) the spectral bandwidth is narrow. Scattering effects are strongly reduced (for the same amount of impedance-contrast random variation) when the intrinsic attenuation is strong. In these cases, k_0 is mainly indicative of the quality factor below the site and could be used, following an approach similar to Hough and Anderson (1988) for estimating the Q_S structure below the site. Even more importantly, these results hint at the possibility that k_0 and the Q_S value derived from it can in these cases be used as an appropriate parameter for the numerical simulation of ground motion.

These results have been obtained only considering the effect of scattering in the uppermost 100 m. However, the results presented in Figure 4 can be easily scaled for thinner

Figure 4. (left) The spectra of the recordings obtained considering only scattering (thin black line, equivalent to the thick black line in Fig. 3), only the intrinsic attenuation (light gray, equivalent to gray line in Fig. 3), and the combined effect (thick black line). In this example, the Q_s adopted in equation (3) is fixed to 10. (center) The same as for the left panel, but for Q_s equal to 30. (right) The same as for the left panel, but for Q_s equal to 50. The absolute amplitude values are arbitrary.

or thicker sedimentary cover, because the effect of the attenuation is proportional to the travel time of the waves in the medium. It follows that, if fixing the value of the intrinsic Q , a thinner sedimentary cover would lead to a larger scattering contribution in the recordings.

Finally, we would like to remark that these results are valid when considering a nearly vertical propagation of S waves and a net forward scattering effect. Such assumptions are reasonable in the great majority of cases, because the window of horizontal signal in the high-frequency range considered is likely to be dominated by S waves, and nearly vertical arrivals for local seismicity can be guessed, considering the hypocentral depth of the events and the impedance decreasing toward the surface, but they might not always be applicable.

Data and Resources

No real data were used in this article. As stated in the text, the signals used were simulations obtained using the matrix propagation method of Wang (1999).

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