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# Chandler and annual wobbles based on space-geodetic measurements 

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## Polar motions with a half-Chandler period and less in their temporal variability

# Chandler and annual wobbles based on space-geodetic measurements 

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#### Abstract

In this study, we examine the major components of polar motion, focusing on quantifying their temporal variability. In particular, by using the combined Earth orientation series SPACE99 computed by the Jet Propulsion Laboratory (JPL) from 1976 to 2000 at daily intervals, the Chandler and annual wobbles are separated by recursive band-pass filtering of the $x_{1}-$ and $x_{2}$-components. Then, for the trigonometric, exponential, and elliptic forms of representation, the parameters including their uncertainties are computed at epochs using quarterly sampling. The characteristics and temporal evolution of the wobbles are presented, as well as a summary of estimates of different parameters for four epochs.


Key words: Polar motion, Chandler wobble, annual wobble, representation (trigonometric, exponential, elliptic), parameter, variability, uncertainty

## 1 Introduction

Variations in the Earth's rotation provide fundamental information about the geophysical processes that occur in all components of the Earth. Therefore, the study of the Earth's rotation is of great importance for understanding the dynamic interactions between the solid Earth, atmosphere, oceans and other geophysical fluids. In the rotating, terrestrial body-fixed reference frame, the variations of Earth rotation are measured by changes in length-of-day (LOD), and polar motion (PM).

The theoretical foundation of studying of PM was derived by Euler (1758), Lagrange (1788) and Poinsot $(1834,1851)$. Afterwards, intensive efforts at several observatories were aimed to prove its existence by resolving variations in latitude. Finally, in 1888, a real latitude variation was detected by Küstner at the Berlin Observatory. Moreover, in 1891/92, there was the discovery by Chandler that the motion of the pole consists of two superposed constituents. The first, of late referred to as Chandler wobble, has a period of about 427 days, while the second has an annual period. Within the scientific community, the problem of PM has raised considerable interest. After an effort covering 10 years, international activities in this field began in September 1899 with the establishment of the International Latitude Service (ILS) as the first permanent world-wide scientific cooperation to monitor the motion of the Earth's pole of rotation with respect to six observing sites based on continuous latitude observations. The ILS was reorganised in 1962 as the International Polar Motion Service (IPMS), and has provided valuable observations for PM over about 100 years. All ILS latitude observations have been reanalysed and combined to the PM solution within a consistent system for the period 1899.91979.0 by Yumi and Yokoyama (1980). Based on optical astrometry observations, a PM time series for the period 1899.71992.0 was obtained from a re-analysis within the Hipparcos frame by Vondrák et al. (1998). For more information on the PM time series available from mid-19th century to the present, see Höpfner (2000).

Based on optical astrometry and ILS data, a large number of polar motion studies have been made in order to derive the dominant terms of PM, including the secular drift of the Earth's pole, the Chandler and annual motions. Some pertinent examples should be noted: Wanach (1916) derived the mean parameters of the dominant motions from ILS data for the epoch 1900-1912. The radius and period variations of the Chandler motion was examined by Kimura (1917) who used the two time series of observations made at Greenwich and Pulkowa from 1825 until 1890 and the time series of the latitude variation investigations of Albrecht and Wanach. It should be noted that Kimura predicted a marked minimum in the Chandler radius to occur at around 1930. Assuming the Chandler period to be 1.2 years, Iijima (1965) separated
the secular, Chandler and seasonal components from ILS data for the period 1900.0-1963.2 with a 0.1 -year sampling. The results were then investigated with respect to annual changes. In particular, he found that the Chandler period varies from about 1.1 to 1.2 years and that the smaller period happens when the Chandler component has a smaller amplitude and vice versa. Based on the ILS data for 1900-1962, Proverbio et al. (1971) analysed the Chandler motion with time using Orlov's method, confirming a correlation between the amplitudes and periods. Using latitude variations observed at 20 stations ( 5 ILS and 15 independent stations) between 1900 to 1970 and 1900 to 1980, the properties of the Chandler wobble including amplitude, phase and ellipticity were derived by $\operatorname{Guinot}(1972,1982)$. An analysis of the homogeneous ILS time series for the period 1899-1977 made by Wilson and Vicente (1980) was concerned with annual, Chandler and long-period motions of the Earth's pole.

Dickman (1981) also studied these terms using the homogeneous ILS time series from 1899.9-1979.0, with a focus on controversial PM features apparently possessed by the older ILS data. Okubo (1982) dealt with the question of whether the Chandler period varies over time. He found that a variability may be explained for an invariant period model. Chao (1983) applied the autoregressive harmonic analysis to the homogeneous 80 -year-long ILS time series, again focusing on the dominant PM terms. Some of principal conclusions found in that work include how the Chandler wobble can be adequately modelled as a linear combination of four (coherent) harmonic components and that the annual wobble is relatively stationary both in amplitude and in phase. Based on BIH and ILS data, Lenhardt and Groten (1987) studied the character of the Chandler wobble. They concluded that the double peak structure in the ILS spectra does not reflect a two component wobble but could be attributed to a phase shift or other events. Using the longest astrometric PM time series that was available (from 1846 to 1988), Nastula et al. (1993) investigated amplitude variations in the Chandler and annual wobbles, including their prediction. For the Chandler, annual, semi-Chandler and semi-annual components of polar motion for the epoch 1976 to 1987, parameter average estimates including their uncertainties can be found in Höpfner (1995, 1996a). Vicente and Wilson (1997) estimated the Chandler frequency from a variety of PM time series derived from optical and space geodetic data spanning various intervals from 1846 through to the early 1990s. According to their results, its variation may not be significant. Earth rotation parameters obtained from the reanalysis in the Hipparcos frame for the epoch 1899.7 to 1992.0 by Vondrák et al. (1998) were studied with respect to longer-period polar motion, in particular the mean pole position, its drift and parameters of annual and Chandler wobbles, using a leastsquares fit at running intervals of 8.5 years, the Chandler frequency to be 0.845 cycles per year; see Vondrák (1999). The same PM time series and the EOP (IERS) C01 time series computed by the International Earth Rotation Service (IERS) from 1861.0 to 1997.0 were analysed by Schuh et al. (2001). Their research considered the linear drift and decadal variations of the pole and the Chandler and annual wobbles. The amplitude, phase and period variations of both wobbles were analysed, using a least-squares fit in terms of an iterative procedure with a sliding time window of 13.76 years. From this analysis, it was seen that the PM reanalysis series is more consistent than the IERS series. For an overview of polar motion studies, see Dick et al. (2000).

Since the middle of the 1970s, the Earth Orientation Parameters (EOPs) have been measured by precise space-geodetic techniques such as VLBI (Very Long Baseline radio Interferometry), LLR (Lunar Laser Ranging), SLR (Satellite Laser Ranging) and most recently, GPS (Global Positioning System). By combining independent measurements of the Earth's orientation taken by the space-geodetic techniques, more precise PM time series are now available. Therefore, compared to earlier polar motion studies, an analysis of these data should provide more precise results. In this study, we consider the major periodic components of polar motion, in particular the Chandler and annual wobbles, with a focus on quantifying their temporal variability.

## 2 Data sets used in this study

The data sets used to examine the Chandler and annual motions are the combined Earth orientation series, SPACE99, as computed by the Jet Propulsion Laboratory (JPL), from MJD 43049.0 (1976 9 28.0) to 51565.0 (2000 122.0) at daily intervals (Gross, 2000). Using a Kalman filter, this solution is based on data from space-geodetic techniques (LLR, SLR,VLBI, and, since mid-1992, GPS). Before their combination, bias-rate corrections and uncertainty scale factors are applied to the independent series to make them consistent with each other. Moreover, the combined series is referred in bias and rate to the IERS 1999 solutions, i. e., it is consistent with the IERS combined Earth orientation series EOP(IERS) C04. For more information and in particular for the uncertainties and the differences between SPACE99 and EOP(IERS) C04, see Gross (2000).

Figure 1 shows the input data of polar motion in terms of an irregular spiral curve using the mathematical perspective in space-time view of Höpfner (1994a). The beat with a cycle of ca. six years is induced by the superposition of the two dominant oscillations, with periods of about 435 and 365 days, respectively.


Figure 1. Polar motion as computed by JPL (Gross, 2000) using the mathematical perspective in space-time view of Höpfner (1994a). The $x_{1}$-axis points towards the Greenwich meridian and the $x_{2}$-axis towards $90^{\circ} \mathrm{E}$ longitude.

## 3 Data processing and results

In studying the Chandler and annual components of polar motion, processing the data consists of the following analysis steps:
(1) Separating the low-frequency component by low-pass filtering and the Chandler and annual wobbles by recursive band-pass filtering for the $x_{1}-$ and $x_{2}-$ components with one-day sampling

Compared to other analysis methods such as least-squares fit, filtering is most appropriate for separating variable signals when studying their behaviour over time. As in our previous studies, digital filters have been applied; see Höpfner (1996b) for details dealing with constructing the zero-phase digital filters. To filter out the Chandler term from the daily values of the time series, a Chandler filter analogous to the filter developed for separating the annual term was designed. The Chandler and annual filters have a cosine shape modified over four periods as weight function. In order to best separate both periodic terms from each, the procedure applied was that of successive approximation by alternate elimination of the Chandler and annual terms. The results obtained for both wobbles are represented in Figs. 2 and 3 in a similar manner as Fig. 1. In particular, the Chandler motion is described by the elliptic spiral in Fig. 2, and, for the annual motion, the same is shown in Fig. 3. It should be noted that the filtered wobbles are truncated at the beginning and the end of the analysis intervals. The Chandler motion is referred to the time interval from MJD 44004.0 (1979 5 11.0) to 50610.0 (1997 611.0 ) and the annual motion from MJD 43843.0 (1978 12 1.0) to 50771.0 (1997 11 19.0).
(2) Calculating optimal estimates for the periods of the Chandler and annual wobbles over time

For this step, we applied a method, based on the maximum, zero crossing and minimum of a periodic function, to the filtered periodic terms separately for the $x_{1}-$ and $x_{2}-$ components (for details about this method, see the Appendix in Höpfner (2001)). This resulted in two period time series for the Chandler and annual wobbles, one for the $x_{1}-$ component and another for the $x_{2}$-component. In both cases, the differences between them were between 0 and 2 days, i. e. not significant, and we determined the final period time series as means. The standard deviations of the period means are $\pm 0.48$ and $\pm 0.54$ days. Figure 4 shows the period variability of the Chandler and annual wobbles over time.
(3) Deriving the trigonometric Fourier coefficients $a_{1}, b_{1}$ and $a_{2}, b_{2}$ of the Chandler and annual wobbles, and their uncertainties, for the $x_{1}-$ and $x_{2}-$ components

The expressions for the periodic terms have the trigonometric form $a_{1} \cos (2 \pi \mathrm{t} / \mathrm{T})+b_{1} \sin (2 \pi \mathrm{t} / \mathrm{T})$ and $a_{2} \cos (2 \pi \mathrm{t} / \mathrm{T})$ $+b_{2} \sin (2 \pi \mathrm{t} / \mathrm{T})$ for the $x_{1}-$ and $x_{2}$-components respectively, where t is the time in days elapsed since 1977.0 (MJD 43144.0) and T the period of the concerned wobble in days. Using a least-squares fit, the trigonometric Fourier coefficients $a_{1}, b_{1}$ and $a_{2}, b_{2}$ and their uncertainties were derived for $x_{1}$ and $x_{2}$ from the filtered time series at running intervals of 441 days for the Chandler wobble and 375 days for the annual wobble, with quarterly sampling, taking variable periods as computed according to (2) and presented in Fig. 4.

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## CHANDLER WOBBLE



Figure 2. Chandler wobble filtered out from the series EOP (JPL) SPACE99 using the mathematical perspective in space-time view of Höpfner (1994a). The $x_{1}$-axis points towards the Greenwich meridian and the $x_{2}$-axis towards $90^{\circ} \mathrm{E}$ longitude.

## ANNUAL WOBBLE



Figure 3. Annual wobble filtered out from the series EOP (JPL) SPACE99 using the mathematical perspective in space-time view of Höpfner (1994a). The $x_{1}$-axis points towards the Greenwich meridian and the $x_{2}$-axis towards $90^{\circ} \mathrm{E}$ longitude.


Figure 4. Variations in the periods of the main periodic components of polar motion: Chandler wobble (top) and annual wobble (bottom). The dashed lines indicate the 435-day and 365-day baselines.
(4) Computing the parameters and their uncertainties of the Chandler and annual wobbles for the trigonometric, exponential and elliptic representations

The periodic terms in (3) have the equivalent expressions for the just mentioned representations as follows:

- The trigonometric form is described by $c_{1} \cos \left(2 \pi \mathrm{t} / \mathrm{T}-\alpha_{1}\right)$ and $c_{2} \cos \left(2 \pi \mathrm{t} / \mathrm{T}-\alpha_{2}\right)$ for the $x_{1}-$ and $x_{2}$-components, with the amplitudes $c_{1}, c_{2}$ and the phases $\alpha_{1}, \alpha_{2}$ of the oscillations of the real and imaginary parts. From the trigonometric Fourier coefficients $a_{1}, b_{1}$ and $a_{2}, b_{2}$, the amplitudes $c_{1}, c_{2}$ and the phases $\alpha_{1}, \alpha_{2}$ of the oscillations of the real and imaginary parts are computed according to the formulas
$c_{1}=\left(a_{1}^{2}+b_{1}^{2}\right)^{\frac{1}{2}}$ and $c_{2}=\left(a_{2}^{2}+b_{2}^{2}\right)^{\frac{1}{2}}$,
and
$\alpha_{1}=\arctan \frac{b_{1}}{a_{1}}$ and $\alpha_{2}=\arctan \frac{b_{2}}{a_{2}}$.
- The exponential form is expressed as $\left(A_{+}+\mathrm{i} B_{+}\right) \exp (\mathrm{i} 2 \pi \mathrm{t} / \mathrm{T})+\left(A_{-}+\mathrm{i} B_{-}\right) \exp (-\mathrm{i} 2 \pi \mathrm{t} / \mathrm{T})$ with the exponential Fourier coefficients $A_{+}, B_{+}$and $A_{-}, B_{-}$being circular motions of the positive and negative frequencies. Instead of ( $A_{+}$ $+\mathrm{i} B_{+}$) and ( $A_{-}+\mathrm{i} B_{-}$), another way to express this is $\left|C_{+}\right| \exp \left(\mathrm{i} \phi_{+}\right)$and $\left|C_{-}\right| \exp \left(\mathrm{i} \phi_{-}\right)$, with the circular motions having amplitudes $\left|C_{+}\right|,\left|C_{-}\right|$and phases $\phi_{+}, \phi_{-}$. In each case, an elliptical path results from the circular motions of the positive and negative frequencies. For the exponential Fourier coefficients $A_{+}, B_{+}$and $A_{-}, B_{-}$, the following relationships to the trigonometric Fourier coefficients $a_{1}, b_{1}$ and $a_{2}, b_{2}$ exist:
$A_{+}=\frac{1}{2}\left(a_{1}+b_{2}\right)$ and $B_{+}=\frac{1}{2}\left(a_{2}-b_{1}\right)$
and
$A_{-}=\frac{1}{2}\left(a_{1}-b_{2}\right)$ and $B_{-}=\frac{1}{2}\left(a_{2}+b_{1}\right)$.
The amplitudes $\left|C_{+}\right|,\left|C_{-}\right|$and the phases $\phi_{+}, \phi_{-}$of the circular motions are obtained by
$\left|C_{+}\right|=\left|A_{+}+i B_{+}\right|=\left(A_{+}^{2}+B_{+}^{2}\right)^{\frac{1}{2}}$
and
$\left|C_{-}\right|=\left|A_{-}+i B_{-}\right|=\left(A_{-}^{2}+B_{-}^{2}\right)^{\frac{1}{2}}$,


Figure 5. Variations in the semi-major and semi-minor axes (top) and direction of the major axis (bottom) of the Chandler motion. The solutions with their standard deviations are shown at quarterly sampling.
and
$\phi_{+}=\arctan \frac{B_{+}}{A_{+}}$and $\phi_{-}=\arctan \frac{B_{-}}{A_{-}}$.

- Concerning the elliptic motion, knowledge of its parameters is of particular interest in the geophysical interpretation. The major and minor semi-axes $\mathrm{a}, \mathrm{b}$ of the ellipse are given by
$a=\left|C_{+}\right|+\left|C_{-}\right|$and $b=\left|C_{+}\right|-\left|C_{-}\right|$,
and their directions $\gamma_{a}, \gamma_{b}$ by
$\gamma_{a}=\frac{1}{2}\left(\phi_{+}+\phi_{-}\right)$and $\gamma_{b}=\gamma_{a}+\frac{\pi}{2}$.
The numerical eccentricity $\epsilon$ is a dimensionless measure for the divergence of the ellipse from the circle and is obtained by
$\epsilon=\frac{\left(a^{2}-b^{2}\right)^{\frac{1}{2}}}{a}$.
If $\epsilon=0$, then there is a circle; if $0<\epsilon<1$, then there is an ellipse. For example, the orbit of the Earth has only the numerical eccentricity $\epsilon=0.0167$.

Note that, for all parameters and quantities of the periodical terms, the formulas for computing their standard deviations can be found in Höpfner (1994b). Using the results of (3), we computed the parameters and their uncertainties for the trigonometric, exponential, and elliptic forms of representation for the Chandler and annual wobbles. Figure 5 shows the variation of the semi-major and semi-minor axes (top) and the direction of the major axis (bottom) of the Chandler motion. In Fig. 6, the same is shown for the annual motion. Additional information including some results of the different parameters and the related standard deviations of the Chandler and annual wobbles at four chosen epochs (Oct./Nov. 1980, Oct./Nov. 1984, Jan./Feb. 1989, Jan./Feb. 1993) are given in Table 1.

## 4 Discussion of the results

Before discussing the results obtained from this work, it needs to be stated that, compared to the PM time series based on optical astrometric measurements, the combined Earth orientation series SPACE99 used in this study are qualitatively better. However, the separation of two signals with similar time varying periods, in particular the Chandler and annual wobbles, requires a special effort.
Table 1. Results of the different parameters of the Chandler and annual wobbles. Note: The numbers 1 and 2 are used for the $x_{1}-$ and $x_{2}-$ components respectively, and the signs + and - are for positive and negative frequencies. $s_{0}$ is the standard deviation of a single estimate. The expressions for the oscillations have the form $\mathrm{c} \cos (2 \pi \mathrm{t} / \mathrm{T}-\alpha)$, where c is the amplitude, $\alpha$ the phase, t the time in days elapsed since used are defined within the text.

| (a) Chandler wobble |  |  |  |  | (b) Annual wobble |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| MJD | , | $s_{01}$ |  | $s_{02}$ | MJD | 7 | $s_{01}$ |  | $s_{02}$ |
| 44550.0 | 424.76 | $\pm 0.0007452$ |  | $\pm 0.0013794$ | 44517.0 | 357.82 | $\pm 0.0015888$ |  | $\pm 0.00057004$ |
| 46011.0 | 434.38 | $\pm 0.0008583$ |  | $\pm 0.0005646$ | 45978.0 | 370.78 | $\pm 0.0002691$ |  | $\pm 0.00017295$ |
| 47563.0 | 433.66 | $\pm 0.0010998$ |  | $\pm 0.0010907$ | 47530.0 | 366.97 | $\pm 0.0010593$ |  | $\pm 0.00162090$ |
| 49024.0 | 436.74 | $\pm 0.0004573$ |  | $\pm 0.0003590$ | 48991.0 | 363.12 | $\pm 0.0003175$ |  | $\pm 0.00047668$ |
| MJD | $a_{1}$ | $b_{1}$ | $a_{2}$ | $b_{2}$ | MJD | $a_{1}$ | $b_{1}$ | $a_{2}$ | $b_{2}$ |
| 44550.0 | $-.11534 \pm 0.00005$ | -. $10267 \pm 0.00005$ | $0.10125 \pm 0.00009$ | $-.11466 \pm 0.00009$ | 44517.0 | $0.00007 \pm 0.00012$ | $-.07988 \pm 0.00012$ | $0.07577 \pm 0.00004$ | $0.00263 \pm 0.00004$ |
| 46011.0 | $-.18036 \pm 0.00006$ | $-.02699 \pm 0.00006$ | $0.02557 \pm 0.00004$ | -. $18218 \pm 0.00004$ | 45978.0 | $-.09141 \pm 0.00002$ | -. $02622 \pm 0.00002$ | $0.02134 \pm 0.00001$ | -. $08470 \pm 0.00001$ |
| 47563.0 | $-.17876 \pm 0.00007$ | $-.03529 \pm 0.00007$ | $0.03535 \pm 0.00007$ | -. $17372 \pm 0.00007$ | 47530.0 | $-.05443 \pm 0.00008$ | $-.05760 \pm 0.00008$ | $0.05566 \pm 0.00012$ | $-.04838 \pm 0.00012$ |
| 49024.0 | $-.19167 \pm 0.00003$ | $0.06045 \pm 0.00003$ | $-.05964 \pm 0.00002$ | $-.19010 \pm 0.00002$ | 48991.0 | $0.01538 \pm 0.00002$ | -. $06070 \pm 0.00002$ | $0.05369 \pm 0.00003$ | $0.01555 \pm 0.00003$ |
| MJD | $c_{1}$ | $\alpha_{1}$ | $c_{2}$ | $\alpha_{2}$ | MJD | $c_{1}$ | $\alpha_{1}$ | $c_{2}$ | $\alpha_{2}$ |
| 44550.0 | $0.15441 \pm 0.00005$ | $221.67 \pm 0.02$ | $0.15297 \pm 0.00009$ | $311.45 \pm 0.04$ | 44517.0 | $0.07988 \pm 0.00012$ | $270.05 \pm 0.08$ | $0.07582 \pm 0.00004$ | $1.99 \pm 0.03$ |
| 46011.0 | $0.18236 \pm 0.00006$ | $188.51 \pm 0.02$ | $0.18397 \pm 0.00004$ | $277.99 \pm 0.01$ | 45978.0 | $0.09509 \pm 0.00002$ | $196.01 \pm 0.01$ | $0.08735 \pm 0.00001$ | $284.14 \pm 0.01$ |
| 47563.0 | $0.18221 \pm 0.00007$ | $191.17 \pm 0.02$ | $0.17728 \pm 0.00007$ | $281.50 \pm 0.02$ | 47530.0 | $0.07924 \pm 0.00008$ | $226.62 \pm 0.06$ | $0.07375 \pm 0.00012$ | $319.00 \pm 0.09$ |
| 49024.0 | $0.20098 \pm 0.00003$ | $162.50 \pm 0.01$ | $0.19923 \pm 0.00002$ | $252.58 \pm 0.01$ | 48991.0 | $0.06262 \pm 0.00002$ | $284.22 \pm 0.02$ | $0.05590 \pm 0.00003$ | $16.15 \pm 0.04$ |
| MJD | $A_{+}$ | $B_{+}$ | ${ }^{\text {A- }}$ | B- | MJD | ${ }^{\text {A }}$ | B+ | ${ }^{\text {A }}$ | B- |
| 44550.0 | $-.11500 \pm 0.00005$ | $0.10196 \pm 0.00005$ | $-.00034 \pm 0.00005$ | $-.00071 \pm 0.00005$ | 44517.0 | $0.00135 \pm 0.00006$ | $0.07782 \pm 0.00006$ | $-.00128 \pm 0.00006$ | $-.00205 \pm 0.00006$ |
| 46011.0 | $-.18127 \pm 0.00003$ | $0.02628 \pm 0.00003$ | $0.00091 \pm 0.00003$ | $-.00071 \pm 0.00003$ | 45978.0 | $-.08806 \pm 0.00001$ | $0.02378 \pm 0.00001$ | -. $00335 \pm 0.00001$ | -. $00244 \pm 0.00001$ |
| 47563.0 | -. $17624 \pm 0.00005$ | $0.03532 \pm 0.00005$ | $-.00252 \pm 0.00005$ | $0.00003 \pm 0.00005$ | 47530.0 | $-.05141 \pm 0.00007$ | $0.05663 \pm 0.00007$ | $-.00302 \pm 0.00007$ | $-.00097 \pm 0.00007$ |
| 49024.0 | $-.19088 \pm 0.00002$ | $-.06004 \pm 0.00002$ | $-.00079 \pm 0.00002$ | $0.00040 \pm 0.00002$ | 48991.0 | $0.01546 \pm 0.00002$ | $0.05720 \pm 0.00002$ | $-.00008 \pm 0.00002$ | $-.00351 \pm 0.00002$ |
| MJD | ${ }^{\text {C }}+$ | $\phi_{+}$ | $C_{-}$ | $\phi_{-}$ | MJD | $\mathrm{C}_{+}$ | $\phi$ | $\mathrm{C}_{-}$ | $\phi^{-}$ |
| 44550.0 | $0.15369 \pm 0.00005$ | $138.44 \pm 0.02$ | $0.00079 \pm 0.00005$ | $244.35 \pm 3.86$ | 44517.0 | $0.07784 \pm 0.00006$ | $89.01 \pm 0.05$ | $0.00242 \pm 0.00006$ | $238.07 \pm 1.45$ |
| 46011.0 | $0.18317 \pm 0.00003$ | $171.75 \pm 0.01$ | $0.00116 \pm 0.00003$ | $322.18 \pm 1.71$ | 45978.0 | $0.09121 \pm 0.00001$ | $164.89 \pm 0.01$ | $0.00414 \pm 0.00001$ | $216.04 \pm 0.16$ |
| 47563.0 | $0.17974 \pm 0.00005$ | $168.67 \pm 0.02$ | $0.00252 \pm 0.00005$ | $179.26 \pm 1.19$ | 47530.0 | $0.07648 \pm 0.00007$ | $132.23 \pm 0.05$ | $0.00317 \pm 0.00007$ | $197.74 \pm 1.28$ |
| 49024.0 | $0.20010 \pm 0.00002$ | $197.46 \pm 0.01$ | $0.00089 \pm 0.00002$ | $152.77 \pm 1.27$ | 48991.0 | $0.05925 \pm 0.00002$ | $74.87 \pm 0.02$ | $0.00351 \pm 0.00002$ | $268.63 \pm 0.34$ |
| MJD | , | b |  | + | MJD | , | b | Ya | ${ }_{0}{ }^{\epsilon}$ |
| 44550.0 | $0.15448 \pm 0.00005$ | $0.15290 \pm 0.00009$ | $191.40 \pm 1.93$ | $0.14266 \pm 0.00474$ | 44517.0 | $0.08026 \pm 0.00011$ | $0.07542 \pm 0.00005$ | $163.54 \pm 0.71$ | $0.34202 \pm 0.00448$ |
| 46011.0 | $0.18432 \pm 0.00004$ | $0.18201 \pm 0.00006$ | $246.97 \pm 0.86$ | $0.15782 \pm 0.00236$ | 45978.0 | $0.09535 \pm 0.00002$ | $0.08707 \pm 0.00001$ | $190.46 \pm 0.08$ | $0.40775 \pm 0.00059$ |
| 47563.0 | $0.18226 \pm 0.00007$ | $0.17723 \pm 0.00007$ | $173.96 \pm 0.59$ | $0.23331 \pm 0.00242$ | 47530.0 | $0.07965 \pm 0.00008$ | $0.07331 \pm 0.00012$ | $164.98 \pm 0.65$ | $0.39111 \pm 0.00432$ |
| 49024.0 | $0.20099 \pm 0.00003$ | $0.19922 \pm 0.00002$ | $175.12 \pm 0.63$ | $0.13242 \pm 0.00146$ | 48991.0 | $0.06276 \pm 0.00002$ | $0.05574 \pm 0.00003$ | $171.75 \pm 0.18$ | $0.45938 \pm 0.00135$ |



Figure 6. Variations in the semi-major and semi-minor axes (top) and direction of the major axis (bottom) of the annual motion. The solutions with their standard deviations are shown at quarterly sampling.

To derive the parameter of the Chandler and annual wobbles and the associated uncertainties in the temporal variability, we applied a 4 -step procedure as described in Sect. 3. Some of our results are plotted in Figs. 2 to 6 and summarised in Table 1. As can be seen, the parameter estimates show remarkably small uncertainties. Concerning their variability over time, this is clear without the estimated uncertainties, i.e. statistically significant, and the courses appear rather steady. Therefore, it can be said that the method used to derive these solutions for the Chandler and annual wobbles yields results of high quality. However, it should be mentioned that a few of the estimates at the beginning and end of the time series may be less accurate because of an edge effect of the recursive band-pass filtering.

The Chandler wobble is found to show a period variation between 422 and 438 days with a estimated standard deviation of only $\pm 0.48$ days (top of Fig. 4), while its amplitude mean varies from 0.15 " to 0.20 " (top of Fig. 5). Concerning a possible elliptic motion, note that the semi-major axis a only differs by 0.001 " to 0.006 " from the semi-minor axis b , resulting in the numerical eccentricity $\epsilon$ ranging between 0.10 to 0.23 . Therefore, the Chandler wobble has a quasicircular prograde (i.e. counter-clockwise) motion. Comparing the changes in amplitude with those in period, we notice that both are similar in their time dependence, i.e., there is a correlation between the amplitudes and periods over this analysis interval, as found in previous polar motion studies covering the last century over different time periods (e.g. Iijima, 1965; Proverbio et al., 1971; Vondrák, 1999). From the direction of the major axis of the Chandler wobble plotted in the bottom panel of Fig. 5, a distortion in the quasi-circular Chandler ellipse occurs between about $0^{0}$ and $200^{\circ}$, and is likely to be prograde (retrograde) if the amplitude increases (decreases).

A significant change in the period of the annual wobble, from 350 to 371 days with an uncertainty of $\pm 0.54$ days, can be seen (bottom of Fig.4). Since the semi-major axis of the annual wobble is always visibly larger than the semi-minor axis (top of Fig. 6), there exists a significantly elliptic annual motion that is prograde, similar to the Chandler wobble. For the annual wobble, the semi-major axis a varies between 0.063 " and 0.096 ", and the semi-minor axis b between 0.056 " and 0.087 ", with the differences between both axes over this time interval reaching a minimum of 0.002 " and a maximum of 0.009 ". The numerical eccentricity $\epsilon$ of the annual motion ellipse ranges from 0.26 to 0.49 . Comparing the annual period curve (bottom of Fig. 4) with the semi-axis curves (top of Fig. 6) reveals the shorter (longer) periods of the elliptic annual motion are probably associated with smaller (larger) semi-axes. The direction of the major axis of the annual wobble ellipse (bottom of Fig. 6) shows considerably less change than the Chandler wobble ellipse (bottom of Fig. 5), its variability over time being around $155^{\circ}$ and $195^{\circ}$ for the analysis interval.

Recent studies are referred to following analysis intervals: From 1848 to1988 by Nastula et al. (1993), from 1976 to 1987 by Höpfner $(1995,1996$ a), from 1899.7 to 1992.0 by Vondrák (1999) and from 1861.0 to 1992.0 by Schuh et al. (2001). While it should be appropriate to use these results as a comparison with ours, there are either no or only relatively short common intervals, making such comparisons difficult. However, we can say that the average results of the Chandler and annual wobbles obtained by Höpfner $(1995,1996$ a) conform to the estimates in this work. In Vondrák (1999), the parameter estimates plotted over the time interval 1904-1988 are the semi-major and semi-minor axes and phase of the Chandler and annual wobbles, and in addition the orientation of the annual wobble ellipse. Note that the
adopted Chandler frequency is 0.845 cycles per year, i.e., a Chandler period of 432.25 days, and the time $t$ is elapsed in days since 1900.0. Considering the estimates that cover the same time interval as ours (1980-1988), there is agreement between our and their results for the semi-axes of the Chandler and annual wobbles. Also, estimates for the direction of the major axis of annual motion are similar. For the phases, we notice that they refer to different time counting and period lengths. The common interval of our parameter estimates and those by Schuh et al. (2001) is shorter (1980-1985), with the time $t$ in that work elapsed since 1945.0 compared to 1977.0 here. We find a similarity in the magnitude and time evolution of the Chandler and annual periods with our over the common interval. The same can be said for the amplitudes of the Chandler wobble, whereas the amplitudes of the annual wobble computed by Schuh et al. (2001) are similar to our in magnitude, but differ in their temporal course.

The time series EOP(IERS) C04 is a combined EOP solution available since 1962 at 1-day intervals from IERS. For the uncertainty of a daily value, see for example, IERS (2000), where it is shown that the uncertainty decreases by replacing the classical method for measuring polar motion by space-geodetic techniques. This was done over 6 periods, ranging from 30 milliarcseconds at the beginning of the series to 0.2 milliarcseconds at its end. That time series could be processed in a similar manner as described here, after which, a comparison may be made of the parameter estimates of the Chandler and annual wobbles with this work's results, revealing the significant differences between the JPL and IERS systems. Note that compared to the combined Earth orientation series SPACE99 used in this study, EOP(IERS) C04 is nearly 15 years longer. Therefore, the parameter estimates in terms of a time series in the IERS system are earlier, which would also be of special interest.

## 5 Concluding remark

Concentrating on the Chandler and annual wobbles of polar motion, the main results of our study are the characteristics and time evolution of both periodic components from 1980 to 1998 relative to the JPL system being important for geophysical interpretations, including prediction, model development and validation.

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# Polar motions with a half-Chandler period and less in their temporal variability 

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#### Abstract

Our study focuses on the observed higher-frequency polar motions that are substantially smaller than the Chandler and annual wobbles. Here, the combined Earth orientation series SPACE99 from 1976 to 2000 with one-day sampling is used as input data, after removing the low-frequency, the Chandler and annual terms. We applied a data processing procedure including four steps, each computing the amplitude spectrum by a Fast Fourier Transform in order to reveal the periodic signals in the residual motions, and then separating their components from the residual time series by band-pass filtering. In particular, the oscillations have the following periods: Semi-Chandler and semi-annual periods and those of order four, three, two, and one and a half months, as well as quasi-biennial and 300-day periods. We show to what extent the observed polar motions are irregularly occurring. A very small polar motion signal with the period of one month is still found in the remaining motions.


Key words: Polar motion, periodic components, semi-Chandler wobble, semi-annual wobble, and 4-month, 90-day, 2-month and 1.5-month components, quasi-biennial and 300-day components, magnitude, variability

## 1 Introduction

Irregularities in the Earth's rotation occur in both the direction of the rotation axis relative to a fixed reference frame in space (precession and nutation motions) and to the Earth and in the rate of rotation over many time scales. The axis orientation relative to the Earth and the temporal rotation variations are measured by polar motion (PM) and changes in length-of-day (LOD). The study and monitoring of the motion of the Earth's rotation pole began in 1899 with the establishment of the International Latitude Service (ILS) (Helmert and Albrecht, 1899). Initially, PM solutions were obtained from latitude observations made by the use of visual zenith telescopes (VZTs). Since then, new methods have allowed an improvement in the precision of such measurements by more than a factor of 100 . Specially, measurement uncertainties which were of the order of a few tenths of an arcsecond in the past are now ca. 0.1 milliarcseconds (mas).

The time resolution of these measurements has also increased with the measurement precision. Concerning the sampling of PM data from the mid-19th century to the present, some specific examples should be noted. The first ILS time series of polar motion were calculated for different periods from 1900.0 at 0.1-year intervals, for example, Albrecht and Wanach (1909), Wanach (1916) and Wanach and Mahnkopf (1932). Next, there are PM solutions at $1 / 12$-year intervals, for example Nicolini and Fichera (1970). Yumi and Yokoyama (1980) re-reduced all ILS observations in a consistent way. Also their homogeneous ILS time series computed for the period 1899.9-1979.0 is given at 1/12-year intervals.

From the International Earth Rotation Service (IERS), four combined Earth orientation parameter (EOP) solutions are available, namely EOP(IERS) C01 since 1846 at 0.05-year intervals, EOP(IERS) C02 since 1962 at 5-day intervals, EOP (IERS) C03 since 1993 at 1-day intervals and EOP(IERS) C04 since 1962 at 1-day intervals (for example, see IERS 1999). More information on the ILS and non-ILS time series including their sampling can be found in Höpfner (2000) and references therein.

Based on past latitude observations, only polar motions with relatively large amplitudes and longer periods such as the Chandler and annual wobbles are observable (see e. g. Höpfner 2002). There are polar motions at frequencies higher than the Chandler and annual wobbles, but these have a substantially smaller amplitude. Therefore, their study had to wait for improved data series, in particular these that were based on high-precision and high-resolution space-geodetic

Table 1. Parameters of the ellipses of the semi-Chandler and semi-annual wobbles obtained from the IERS data for the period 1976.5 to 1988.0

| Wobble | Period <br> (days) | Semi-major axis <br> (mas) | Direction of the <br> semi-major axis | Semi-minor axis <br> (mas) |
| :--- | :---: | :---: | :---: | :---: |
| Semi-Chandler | 206.19 | $2.82 \pm 0.66$ | $162.77^{0} \pm 12.76^{0}$ | $0.50 \pm 0.67$ |
| Semi-annual | 182.62 | $5.05 \pm 0.59$ | $32.15^{0} \pm 7.25^{0}$ | $1.10 \pm 0.59$ ) |

techniques. As a consequence of the superior measurements, other PM oscillations were revealed with periods ranging from a few hours to many years. For example, Kosek et al. (1995) discuss motions with periods ranging between 20 to 150 days and magnitudes up to 2 to 6 mas. In Kołaczek et al. (2000), the sub-seasonal oscillations are studied, in particular those with periods of 120,62 , and 49 days.

Two other oscillations that have received considerable attention are those with a semi-annual and a half-Chandler periods. There are studies relating to these made already twenty years ago using latitude observations, for example Emetz $(1979,1980)$ and Höpfner $(1982,1985)$. Moreover, since the mid-nineties, the objective has been to analyse both oscillations using time series of polar motion. Some results are presented in Höpfner (1995, 1996a) for the analysis interval between 1976.5 to 1988.0 and Kosek and Kołaczek (1997) for the interval between 1962.0 to 1996.0. Some details on these studies should be noted.

Based on 30 time series of latitude observations made at 22 observatories between 1900 to 1969 (Fedorov et al. 1972), Emetz $(1979,1980)$ studied the spectrum of the latitude variations in the frequency range 1.4 to 2.2 cycles per year (срy). Besides the semi-annual period, oscillations with periods of $0.54,0.58,0.60$ and 0.64 years were resolved, which are equal to a half the periods found in the Chandler frequency band, in particular 1.10, 1.17, 1.20, and 1.24 years. Concerning the magnitude of the half-Chandler wobble, an amplitude of ca. 0.0037" was calculated (Höpfner 1983) from the x - and y-coordinates published in Emetz (1979).

Höpfner (1983) analysed the time series of the latitude changes obtained from observations made with a Danjon astrolabe at the Potsdam Geodetic-Astronomical Observatory between 1957.8 to 1978.0, with the temporal variations in the mean latitude and in the Chandler, annual and semi-annual wobbles discussed in Höpfner (1977, 1979). The remaining latitude time series, after removing the mean latitude and Chandler, annual and semi-annual wobbles, was studied with respect to further motions at seasonal time scales, revealing a wave with a semi-Chandler period found as a significant peak in the amplitude and power spectra as computed by Fast Fourier Transform and spectral analysis (Höpfner 1982, 1983, 1985). Using harmonic analysis, the residual latitudes at running intervals of 6 and 3 years were then used to calculate amplitude and phase values of the half-Chandler wobble. Here, referring to an observed mean Chandler period of 1.189 years (Höpfner 1980, 1983), the half-Chandler period was adopted to be 0.5945 years. For the phase variations, a linear trend was evident. Taking into consideration that this trend was caused by the small difference between the adopted and actual values of the period, an improved half-Chandler period of $(0.5839 \pm 0.0005)$ years or $(213.27 \pm 0.18)$ days was derived from the phase values affected with the trend (Höpfner 1982, 1983; Höpfner and Jochmann 1984).

Based on the combined time series EOP(IERS) C04 at daily intervals from 1976.5 to 1988.0 , the parameters of the semi-annual and semi-Chandler wobbles together with those of the annual and Chandler wobbles for the trigonometic, exponential and elliptic representations were computed by Höpfner (1995, 1996a), using a semi-Chandler period of 206.19 days or 0.5645 years. The parameters of the ellipses obtained for the semi-Chandler and semi-annual wobbles are presented in Table 1. For illustrating the four wobbles, see Höpfner (1995, 1996a). Also the atmospheric excitation portions at annual and semi-annual frequencies were derived using the time series of the Atmospheric Angular Momentum (AAM) functions, in particular $\chi(\mathrm{P}+\mathrm{IB})=\chi_{1}(\mathrm{P}+\mathrm{IB})+\mathrm{i} \chi_{2}(\mathrm{P}+\mathrm{IB})$. That is, the pressure terms which include an oceanic contribution using the Inverted-Barometer (IB) response, as computed by the U. S. National Meteorological Center (NMC).

Kosek and Kołaczek (1997) studied the semi-annual and semi-Chandler wobbles using the pole coordinate data computed by IERS for the period 1962.0 to 1996.0 and those computed by Vondrák et al. (1995) for the period 1900.0 to 1988.0. The pole paths of both separated oscillations were displayed for three intervals: 1983-1986, 1986-1989 and 1989-1992. Here the semi-Chandler period is given as 218 days.

For the semi-annual and semi-Chandler components of polar motion, it should be remarked: The semi-annual wobble has an elliptic prograde motion, where the direction of the semi-major axis is rather stable. On the contrary, the semiChandler wobble is much smaller, and the direction of the semi-major axis is extremely variable. Unfortunately, since there are no results over common intervals derived from the IERS data by Höpfner (1995) and Kosek and Kołaczek (1997), a direct comparison of results cannot be made.

Based on the precise space-geodetic measurements, we have studied the major components of polar motion including the low-frequency component, the Chandler and annual terms (Höpfner, 2002). This work should be continued with a focus on the observed higher-frequency polar motions that are substantially smaller than the Chandler and annual wobbles.

## 2 Data sets used in this study

The combined Earth orientation series, SPACE99, as computed by the Jet Propulsion Laboratory (JPL) with one-day sampling (Gross, 2000) is used as input data, after removing the low-frequency, the Chandler and annual terms; see Höpfner (2002). These data of polar motion are referred to as the residual motions. Concerning their $x_{1}-$ and $x_{2}$-components, note that the $x_{1}$-axis is in the direction of the IERS Reference Meridian (IRM) near the Greenwich meridian, and the $x_{2}$-axis is in the direction $90^{\circ} \mathrm{E}$ longitude. That is, we use the mathematical coordinate system with $x_{1}=\mathrm{x}$ and $x_{2}=-\mathrm{y}$ where x , y are the coordinates of the Celestial Ephemeris Pole (CEP) relative to the IERS Reference Pole (IRP).

In order to give a visual impression of the total motions, sums of the low-frequency, Chandler and annual terms and residual motions, henceforth referred to as residual motions (1), these series are shown in Fig. 1, with those of the $x_{1}$-component in the upper part and of the $x_{2}$-component in the lower part. Note that the total polar motions as computed by JPL are given from MJD 43049.0 (1976 9 28.0) to 51565.0 (2000 122.0) whereas the resulting sums and, therefore, the residual motions (1) are obtained from MJD 44004.0 (1979 5 11.0) to 50610.0 (1997 6 11.0).

## 3 Data processing and results

In studying polar motions that are substantially smaller than the Chandler and annual wobbles, the data processing is given as a flow chart in Fig. 2 and includes the following four steps:
(1) To obtain the periodic signals contained within the residual motions (1), the amplitude spectrum (1) of them is computed by Fast Fourier Transform (FFT) and is presented in Fig. 3 (top-left). The semi-annual, semi-Chandler and 4 -month signals can be seen as distinct peaks with periods of 183,225 and 120 days respectively. Then, separating the semi-Chandler and semi-annual terms by recursive band-pass filtering and the 4-month term by band-pass filtering of the residual motions (1). To isolate these and the other terms considered later, we designed the band-pass filters according to Höpfner (1996b). Figure 4 presents the resulting semi-Chandler, semi-annual and 4-month oscillations, with those for the $x_{1}$-component in the upper part and those for the $x_{2}$-component in the lower part.
(2) Calculating the residual motions (2) that remain after removing the terms of (1) and amplitude spectrum (2) of them that reveals the other periodic signals shown in Fig. 3 (top-right). In particular, see the 2-month and 1.5 -month signals with periods of 60 and 45 days and the quasi-biennial signal with a period of 650 days. The 2-month and 1.5month terms are then separated by recursive band-pass filtering, and the quasi-biennial term by band-pass filtering of the residual motions (2). These terms are presented in Fig. 5, separately for the $x_{1}-$ and $x_{2}-$ components in the upper and lower parts respectively.
(3) Calculating the residual motions (3) that remain after the removal of the terms of (2) and amplitude spectrum (3) of them that reveals the other periodic signals as shown in Fig. 3 (bottom-left). In particular, the 300-day and 90-day signals can be seen which are then separated by band-pass filtering from these series. Figure 6 displays the residual motions (3) and the oscillations filtered out with periods of 300 and 90 days, again in the upper part for the $x_{1}$-component and in the lower part for the $x_{2}$-component.
(4) Computing the residual motions (4) as the remaining motions after the removal of the terms of (3) and amplitude spectrum (4) of them that reveals the other periodic signals shown in Fig. 3 (bottom-right). The residual motions (4) are also presented in Fig. 6 (bottom panels).

As noted, in Fig. 1, we can see the residual motions (1) that are computed as difference series of the total motions and the sums of the low-frequency, Chandler and annual terms. The amplitude spectra of all residual motions (1) to (4) are computed by FFT and displayed in Fig. 3. Figures 4 to 6 show to what extent the periodic terms filtered out are irregularly occurring in the $x_{1}-$ and $x_{2}$-components. Using the space-time view of Höpfner (1994), Fig. 7 presents the quasi-biennial, 300-day, semi-Chandler and semi-annual motions in terms of elliptic spiral curves, while the same is shown in Fig. 8 for the 4 -month, 90 -day, 2-month and 1.5 -month motions. Some characteristics of these oscillations, including the time interval of the resulting series, period length, type, semi-major axis and its direction, are listed in Table 2.

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No.: 02/13
Author: J. Höpfner




Figure 1. Polar motion in the $x_{1}$-component (upper part) and the $x_{2}$-component (lower part). In each part, the motions shown are: Total motion as computed by JPL (top), the sum of the low-frequency, Chandler and annual terms (centre), and the residual motion (1)(bottom).

Table 2. Characteristics of the periodic components of polar motion that are substantially smaller than the Chandler and annual wobbles

| Component | Time interval <br> (MJD) | Time interval <br> (calendar days) | Period <br> (days) | Type | Semi-major <br> axis (arcsec) | Direction of the <br> semi-major axis |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Quasi-biennial | $45861.0 \ldots 48933.0$ | $1983 / 12 / 13 \ldots 1992 / 11 / 07$ | $600 \ldots 750$ | variably elliptic | $0.003 \ldots 0.008$ | $70^{0} \ldots 110^{0}$ |
| 300-day | $47056.0 \ldots 47558.0$ | $1987 / 09 / 18 \ldots 1989 / 02 / 01$ | $290 \ldots 300$ | clearly elliptic | about 0.004 | about $170^{0}$ |
| Semi-Chandler | $44466.0 \ldots 50148.0$ | $1980 / 08 / 15 \ldots 1996 / 03 / 06$ | $200 \ldots 250$ | elliptic to circular | $0.004 \ldots 0.008$ | very variable |
| Semi-annual | $44397.0 \ldots 50217.0$ | $1980 / 06 / 07 \ldots 1996 / 05 / 14$ | $175 \ldots 190$ | elliptic | $0.004 \ldots 0.013$ | $10^{0} \ldots 35^{0}$ |
| 4-month | $44728.0 \ldots 49886.0$ | $1981 / 05 / 04 \ldots 1995 / 06 / 18$ | $110 \ldots 135$ | elliptic to circular | $0.001 \ldots 0.006$ | about $150^{0}$ |
| 90-day | $46641.0 \ldots 4937.0$ | $1986 / 07 / 30 \ldots 1990 / 03 / 23$ | $80 \ldots 95$ | elliptic | $0.001 \ldots 0.003$ | very variable |
| 2-month | $44859.0 \ldots 4955.0$ | $1981 / 09 / 12 \ldots 1995 / 02 / 07$ | $55 \ldots 70$ | clearly elliptic | $0.001 \ldots 0.005$ | very variable |
| 1.5-month | $44827.0 \ldots 49787.0$ | $1981 / 08 / 11 \ldots 1995 / 03 / 11$ | $40 \ldots 55$ | clearly elliptic | $0.001 \ldots 0.004$ | about $170^{0}$ |

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Author: J. Höpfner
(1) Residual motions (1)
Fast Fourier Transform: Amplitude spectrum (1)

Separating the semi-Chander, semi-annual and 4-month terms
(2) Calculating the residual motions (2)

Fast Fourier Transform: Amplitude spectrum (2)
Separating the 2-month, $1.5-$ month and quasi-biennial terms
(3) Calculating the residual motions (3)

Fast Fourier Transform: Amplitude spectrum (3)
Separating the 300-day and 90-day terms
(4) Calculating the residual motions (4)

Fast Fourier Transform: Amplitude spectrum (4)

Figure 2. Flow chart about the four steps of data processing.


Figure 3. Amplitude spectra of the residual motions (1), (2), (3), and (4). The arrows indicate the periodic terms that have been separated out by band-pass filtering of the residual motions (1), (2), and (3), respectively.

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No.: 02/13
Author: J. Höpfner






Figure 4. Polar motion in the $x_{1}$-component (upper part) and the $x_{2}$-component (lower part). In each part, the motions shown are: The semiChandler term (top), the semi-annual term (centre), and the 4 -month term (bottom).

Publication: Scientific Technical Report
No.: 02/13
Author: J. Höpfner


Figure 5. Polar motion in the $x_{1}$-component (upper part) and in the $x_{2}$-component (lower part). In each part, the motions shown are: Residual motion (2) (top), the 2 -month term (centre), and the 1.5 -month term (bottom). In the top panels, the quasi-biennial term is plotted by a dashed line.

Publication: Scientific Technical Report
No.: 02/13
Author: J. Höpfner







Figure 6. Polar motion in the $x_{1}$-component (upper part) and in the $x_{2}$-component (lower part). In each part, the motions shown are: Residual motion (3) (top), the 90 -day term (centre), and residual motion (4) (bottom). In the top panels, the 300 -day term is plotted by a dashed line.

Publication: Scientific Technical Report
No.: 02/13
Author: J. Höpfner

QUASI-BIENNIAL WOBBLE


SEMI-CHANDLER WOBBLE


300-DAY WOBBLE


SEMI-ANNUAL WOBBLE


Figure 7. Quasi-biennial, 300-day, semi-Chandler, and semi-annual wobbles filtered out from the series EOP (JPL) SPACE99 using the space-time view of Höpfner (1994). The $x_{1}$-axis points towards the Greenwich meridian, and the $x_{2}$-axis towards $90^{\circ} \mathrm{E}$ longitude.

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Figure 8. 4-month, 90-day, 2-month, and 1.5-month wobbles filtered out from the series EOP (JPL) SPACE99 using the space-time view of Höpfner (1994). The $x_{1}$-axis points towards the Greenwich meridian, and the $x_{2}$-axis towards $90^{\circ} \mathrm{E}$ longitude.

## 4 Discussion of the results

Using the amplitude spectra of the residual motions (1) to (3) as shown in Fig. 1, we assess them with respect to periodic signals for their separating. The arrows mark in each spectrum those signals that are revealed and accordingly have been separated. As the amplitude spectrum of the residual motions (4) shows, there is still a very small 1 -month signal.

From Table 2, we can see that all of the PM signals filtered out from the remaining motions after the low-frequency, Chandler and annual terms have been removed (especially from SPACE99 computed by JPL; Höpfner, 2001), are oscillations with variable amplitudes reaching maxima of 3 to 13 mas over their analysis intervals. From Figs. 6 and 7, we note that the motions occur progradely (i.e. counter-clockwise), similar to the Chandler and annual wobbles. Note also that the persistence of the oscillations becomes less with higher frequencies. The filtered terms are now to be discussed in detail.

Over a relatively short analysis interval of ca. 9 years, there are 4.5 revolutions of the quasi-biennial wobble showing considerable variability in the amplitude (semi-major axis), in particular increasing from 3 to 8 mas at 1989 after which it decreases again to less than 3 mas, while not having the stable period as given in Table 2. The orientation of the elliptic motion, referred to the semi-major axis, lies between $70^{\circ}$ and $110^{\circ}$ with the type of quasi-biennial wobble being very variably elliptic.

The 300-day wobble found over only about 500 days has a clearly elliptic motion with a semi-major axis of ca. 4 mas in the direction of ca. $170^{\circ}$ and a semi-minor axis of about 2 mas.

In the analysis interval from 1980 to 1996, the semi-Chandler wobble shows a motion varying from elliptic to circular with an amplitude of the order of 5 mas. Especially around 1989, there is a maximum amplitude of 8 mas whose direction is $15^{\circ}$. Two other maxima are in 1985 and 1994, with an amplitude of 6 mas at $160^{\circ}$ and 7 mas at $160^{\circ}$. In 1986/1987 and 1996, distinct irregularities exist where the amplitude is much smaller, in particular about 2 mas.

Similarly, estimates for the semi-annual wobble were obtained for the period 1980-1996. From the elliptic spiral curve of this wobble, shown in Fig. 6 (bottom-right), we notice considerable variability in the amplitude (semi-major axis) with minima in 1982, 1985-1987 and 1992/1993 and maxima in 1980, 1984, 1988/1989 (where the amplitude reaches a magnitude of ca. 13 mas at a direction of ca. $10^{0}$ ) and 1995. Here, the orientation of the elliptic motion is relatively stationary with stable ellipticity.

During 1981-1995, the 4-month wobble varies considerably both in amplitude and ellipticity. Especially in 1983/1984, the amplitude reaches a maximum of about 6 mas at a direction of about $160^{\circ}$ and in 1982 and again 1985, a minimum of 1 mas. We find that the main orientation of the elliptic motion referred to the semi-major axis lies at about $150^{0}$.

Over 3.5 years between 1986 to 1990, we see the elliptic spiral curve of the 90 -day wobble in Fig. 7 (top-right) showing a similar variability in both amplitude and ellipticity as the 4 -month wobble. However, the amplitude of the 90day wobble is smaller, in particular varying between 1 and 3 mas, and its direction very variable. We can see amplitude maxima in 1988 and 1989, each with 3 mas but with directions of $220^{\circ}$ and $160^{\circ}$.

The 2-month wobble found over a 13.5-year period from 1981 to 1995 shows three obvious maxima in 1985, 1988 and 1993 of 5,4 and 3 mas, with the direction of the semi-major axis lying at $150^{\circ}, 160^{\circ}$ and $10^{\circ}$ and five minima of nearly zero mas in 1983/1984, 1985/1986, 1987, 1990 and 1994. Generally, for the orientation of the elliptic motion, there is again much variability.

Like the 2-month wobble, the 1.5 -month motion is filtered out over about 13.5 years from 1981 to 1995. Its variability is characterised by alternating smaller and larger amplitudes ranging between 1 and 4 mas. In the case of the larger amplitudes of 3 to 4 mas (in 1985, 1988, 1990 and 1995), the orientation of the elliptic motion referred to the semi-major axis lies at about $170^{\circ}$.

Especially for information on the period variability of the elliptic motions given in Table 2, we found that the period of the semi-annual wobble varies by 15 days. The other higher-frequency wobbles also show a similar variation in their periods. Compared to it, the period of the semi-Chandler wobble has a 50 -day variation interval, ranging from 200 to 250 days. Concerning the quasi-biennial and 300-day motions computed for relatively short time intervals, it should be noted that this is due to the band-pass filter method sequentially applied for this analysis according to the magnitude of a periodic signal revealed by FFT. Therefore, the estimates of these periodic components are secondary results of our paper.

Finally, some comments should be given about the geophysical explanation of the oscillations found by our analysis. Today, it is clear that all three excitation sources and in particular atmosphere, oceans, and hydrology provide contributions to the excitation of polar motion; see e.g. Wilson (2000). Note that there are a large number of papers on correlations between variations of polar motion and atmospheric excitation functions, for example Chao (1993), Kuehne et al. (1993), Nastula et al. (1997), Kołaczek et al. (2000), and further references therein. Especially at annual and semi-annual frequencies, the problem of the excitation was considered in the way that the total excitation is composed of atmospheric and non-atmospheric excitation portions (Höpfner 1996a). Using three ocean circulation models, oceanic contributions to seasonal polar motions are studied in Wünsch (2000). Schmitz-Hübsch and Schuh (1999) found that the semi-annual
prograde variations are delayed by strong El Niño events. For regional atmospheric angular momentum contributions to polar motion excitation, see e.g. Salstein and Rosen (1989), Nastula and Salstein (1999). Based on a constant-density, nearly global numerical ocean model, the results of the oceanic excitation of signals in the Earth's rotation are presented in Ponte (1997) and Ponte et al. (1998), indicating that the oceanic circulation and mass-field variability play important roles in the excitation of seasonal to fortnightly polar motion. Studies dealt with the hydrological contributions to polar motion excitation are made, for example, by Chao et al. (1987), Chao and O'Connor (1988), Wilson and Kuehne (1990), and Wilson (1993). Using the wavelet analysis, seasonal and short-period variations in the Earth's rotation measured by VLBI from 1981 to 1999 are investigated by Schmidt and Schuh (2000), and Schuh and Schmitz-Hübsch (2000). Comparing the periods of polar motion and atmospheric angular momentum time series, it was found that irregular shortperiod PM variations, in particular between two and five months, are partially caused by atmospheric excitations with a normed coherency between 0.3 and 0.6 .

Recent studies of PM including its excitation are published in the proceedings of the IAU Colloquium 178 "Polar motion: Historic and Scientific Problems" held at Cagliari, Italy in September 1999 to celebrate the 100th anniversary of the International Latitude Service (ILS), see Dick et al. (2000) and in particular the papers given by Wilson (2000), Salstein (2000), Nastula et al. (2000), and Kołaczek et al. (2000).

## 5 Concluding remarks

Estimates obtained from different methods should be similar to each other in order to resolve significant signals. For a comparison of our estimates relative to the JPL system with others, see Kosek et al. (1995), Kosek and Kołaczek (1997), and Kołaczek et al. (2000) for temporal variability, and Höpfner (1995, 1996a) for means over the study period between 1976 and 1987. Comparing the estimates, we find that they are similar in the general form but with some disagreements in the details. These differences will be elucidated in the future. In this work, since the comparison estimates are relative to the IERS system, small discrepancies are explainable by systematic errors, i.e. due to different approaches used by JPL and IERS to correct the bias and rate of the individual series before they are combined.

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