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The evolution of the core-surface flow over the last seven thousands years

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Abstract. We analyze the core-surface flow over the last seven thousands years derived from CALS7k.2, a continuous model of the geomagnetic field and its secular variation for the period 5000 BC to 1950 AD. This model relies on indirect observations of the geomagnetic field through the sampling of natural archives (lake sediments, lavas) and archaeological artifacts. An extensive comparison between gufm1, a historical field model spanning the interval 1590 to 1990, and CALS7k.2 is carried out. While both are similar in their descriptions of the large scale core field, their agreement compares poorly in terms of secular variation. The differences are primarily caused by the different data quality used to build these models. Such a comparison is essential for assessing the reliability of the results of flow inversion. We seek instantaneous core flow solutions that are purely toroidal and tangentially geostrophic for which we can reasonably neglect the effect of magnetic diffusion. Our flow solutions suggest that the core-surface flow undergoes different regimes of zonal flow direction, with epochs of mainly westward flow alternating with epochs of mainly eastward flow. The episodes of eastward and westward flow are robust as they are obtained for purely toroidal and tangentially geostrophic flow. The changes in the large scale zonal flow direction seem to recur with a typical period of between 540 and 1050 years. Variations in thermal and magnetic winds occurring on time scales comparable with the convective overturn of the liquid outer core could account for the observed zonal flow variations.
1. Introduction

The variation of the geomagnetic field (known as secular variation) and other observables of core dynamics indicate a variability on a broad range of timescales. However, our knowledge of the detailed dynamics is rather incomplete, because only observations of the radial magnetic field can unambiguously be used in order to garner information about core dynamics. Other observations, like the variation of the Earth’s rotation (apart from tidal braking and glacial rebound) can be associated only indirectly to variations in the angular momentum of the core and core dynamics. Those changes in angular momentum are imposed on Earth’s rotation due to coupling of the core and mantle, where the coupling can be viscous, electromagnetic, gravitational or topographical [c.f. Bloxham, 1998, and references therein].

Several attempts have been made to study core dynamics on the decadal to centennial timescale using historical geomagnetic field models describing the core field and its temporal variation, like gufm1 [Jackson et al., 2000]. These models have been successfully inverted for core surface motions [Bloxham and Jackson, 1991; Jackson et al., 1993; Amit and Olson, 2006] to provide insights to core dynamics on these timescales.

Archeomagnetic field models of the last few millennia, like the CALS7k.2 model [Korte et al., 2005; Korte and Constable, 2005], which covers the period from 5000 BC to 1950 AD, allow the study of the geomagnetic field and hence core dynamics on longer timescales. These models are built from indirect observations of the Earth’s magnetic field taken from natural archives (lake sediments, lavas) and archaeological artifacts [Hongre et al., 1998; Korte and Constable, 2003]. A detailed analysis of the temporal variability of the
magnetic field of the CALS7k.2 model revealed no particular periodicity, however quasi-periodic fluctuations with increased power at periods between 1000 and 2000 years [Korte and Constable, 2006] were found.

Recently, Dumberry and Bloxham [2006] sought time–dependent azimuthal flows from an archeomagnetic field model and compared the variations in the core angular momentum computed from these flow solutions to the observed variations deduced from records of historical solar and lunar eclipses [Stephenson and Morrison, 1995]. Their study shows that assumptions made to derive core angular momentum changes on a decadal timescale prove to be invalid for millennia. In contrast to Dumberry and Bloxham [2006] we seek to invert the CALS7k.2 geomagnetic field model for a series of snapshots of core surface flows (instantaneous solutions).

We begin in section 2 by examining the azimuthal drift of magnetic field features and comparing CALS7k.2 with gufm1. Our methodology of constructing instantaneous flow solutions is outlined in section 3, where we also justify our neglect of magnetic diffusion and describe the two dynamical constraints we employ (tangential geostrophy and toroidal flow). In section 4 the results of the core flow inversions, their robustness and their geophysical implications are discussed.

2. Pre-examinations of secular variation in CALS7k.2

2.1. Westward and eastward drift

First, we investigate the persistence and spatial distribution of westward (and eastward) drift of the magnetic field. The results may provide insights into the temporal variability of CALS7k.2, which can be related to the fluid flow. We computed the drift rate by using a method of Bullard et al. [1950]; Whitham [1958]; Yukutake [1968].
Let $C(\theta, \phi, t)$ be any main field or secular variation component or the potential $V$ at any fixed latitude $\theta = \theta_0$. If $C$ is known as a function of longitude $\phi$ at times $t_1, t_2$, then the drift $\Delta \phi$ can be estimated by minimizing

$$\chi = \sum_i [C(\theta_0, \phi_i, t_2) - C(\theta_0, \phi_i + \Delta \phi, t_1)]^2$$

with respect to $\Delta \phi$. Assuming that the temporal variation of the field is due to longitudinal drift, the results indicate how much of the temporal variation can be explained by pure drift. In Fig. 1 the overall analysis of the CALS7k.2 radial component of the field is shown. It shows that the highest drift are found in latitudinal bands from $-40^\circ$ to $-90^\circ$ and $20^\circ$ to $90^\circ$, whereas hardly any drift can be diagnosed in the equatorial regions using this method. Occasionally, the drift rates in the northern and southern hemisphere tend to be opposed.

In the mid-to-high latitude bands the drift alternates between epochs of westward and eastward motion. Similar episodes of westward and eastward motion have previously been reported by Dumberry and Finlay [2007], who extracted the azimuthal part of secular variation by applying suitable filtering techniques to the CALS7k.2 model. The transitions between these two directions of azimuthal field evolution appear to be abrupt. This is more obvious in Fig. 2, where profiles of Fig. 1 along $-67^\circ, 0^\circ, 67^\circ$ latitude are shown. Dumberry and Finlay [2007] suggested that abrupt changes in the drift direction could be linked to the occurrence of archeomagnetic jerks [Gallet et al., 2003].

A detailed comparison between the known archeomagnetic jerks [Gallet et al., 2003] and the drift rate along a similar latitude as the time series of that study (48°N) latitude is given in Fig. 3. Two of the archeomagnetic jerks show a close correlation to changes in the drift rate around 200 AD and 800 AD. The event around 1400 AD is loosely linked...
to changes in drift rate and falls in the middle of two changes. However, the drift rate exhibits a very variable behavior with many narrow features, which may require further study.

2.2. Comparison with gufm1

A continuous geomagnetic field model developed by Jackson et al. [2000] describes the geomagnetic main field and its temporal variation during the period 1590 and 1990. Contrary to CALS7k.2, gufm1 was derived from direct measurements of the magnetic field and consequently is of much higher resolution and accuracy.

Both spatial and temporal resolution of CALS7k.2 are limited, due to three main reasons. First, the higher uncertainties in archeo- and paleomagnetic values compared to direct field measurements, which are very hard to assess consistently for the different data types (directional and intensity data, from archeomagnetic and lake sediment material) but play a significant role in models fit to the tolerance given by the error estimates. Second, uncertainties in dating of the material providing the data and third, the inhomogeneous and partly sparse distribution of the data over the globe. Spatial power is well resolved only up to spherical harmonic degree 4 and temporal resolution is on the order of 100 to 300 years [Korte and Constable, 2007]. Uncertainties in the model are strongest in the southern hemisphere, where the data distribution is sparse throughout the 7k years, and for the earliest and latest one to two centuries, where spline edge effects play a role. Moreover, the model quality varies with time. Similar to the increase in gufm1 resolution with increase in data, CALS7K.2 in general shows more detailed structure from around 1000 BC on, when the number of data rises strongly, see Fig. 4. However, there are two earlier periods with significant structure at about 4100 BC and 2200 BC, which go
along with rather high uncertainties in the model coefficients as determined by a bootstrap method applied to the modelling [see Korte and Constable, 2007]. In order to qualitatively assess the reliability of CALS7K.2 for core flow inversion, in this section we compare it to gufm1 over the time interval where the models overlap.

A comparison of the radial magnetic field at the CMB between gufm1 and CALS7k.2 is shown for the epochs 1600 and 1900 in Figure 5 in order to discuss the differences between the models. On large length scale both models are similar, e.g. both show a dipolar configuration. Differences are apparent on smaller scales, affecting the shape and amplitudes of some patches. In comparison to gufm1 the southern hemisphere of the radial field of CALS7k.2 appears featureless. This is clearly due to the uneven distribution of data contributing to CALS7k.2, with more data on the northern hemisphere than in the southern. The agreement slightly improves, if the gufm1 is truncated to spherical harmonic degree 5, i.e. if the spatial resolution is limited to the approximate resolution of CALS7k.2. See for instance the high latitudinal reverse patch which becomes visible in the map of 1900.

In order to provide quantitative diagnostics of the differences between gufm1 and CALS7k.2, we computed the fractional RMS deviation, $\sigma$, as a function of coefficient number

$$\sigma = \sqrt{\frac{(g_{l,m}^{m,\text{CALS7k.2}} - \langle g_{l,m}^{m,\text{gufm1}} \rangle_t)^2}{\langle g_{l,m}^{m,\text{gufm1}} \rangle_t^2}}, \quad (2)$$

where $\langle g_{l,m}^{m,\text{gufm1}} \rangle_t$ are the coefficients of degree $l$ and order $m$ of a temporally averaged gufm1 model and $g_{l,m}^{m,\text{CALS7k.2}}$ the coefficients of CALS7k.2 for a given epoch. The temporally averaged model is obtained by computing the spherical harmonic coefficients for every year between 1590 and 1990, adding and then dividing these by the number of epochs,
i.e.,
\[
\langle g_{l,\text{gufm1}}^m \rangle_t = \frac{1}{401} \sum_{i=0}^{400} g_{l}^m (t = 1590 + i),
\]
and likewise for \( h_{l}^m \), \( \dot{g}_{l}^m \) and \( \dot{h}_{l}^m \). Taking the temporally averaged \textit{gufm1} is reasonable because \textit{gufm1} is believed to be the more correct field description (otherwise a mean model of \textit{gufm1} and CALS7k.2 could be considered). If both models are of the same value, then \( \sigma \) is zero, and the larger the differences between them, the larger the fractional RMS \( \sigma \) becomes. In Figures 6 and 7 the fractional RMS deviations of the main field and secular variation coefficients at four epochs are shown, respectively. The analysis reveals that the agreement between both main field models varies with time. Early epochs, i.e. 1600 and 1700 show larger deviations from the time-averaged \textit{gufm1}. Further, \( \sigma \) vary within degrees over several orders of magnitude. However, a tendency for larger or smaller deviations in either sectorial or zonal terms is not evident. The average of \( \sigma \) is about 0.3 and about 0.9 for the main field and secular variation models, respectively.

Our conclusion is that the secular variation captured by \textit{gufm1} and CALS7K.2 are significantly different, even for low spherical harmonic degrees. Like the results of the analyses by \textit{Dumberry and Finlay} [2007], we find that the quality of the data (with larger time and measurement errors for the CALS7k.2 data) is the major factor causing the differences in the secular variation model. The differences in the data have subsequent impacts on the field model. Firstly, spatial structures beyond spherical harmonic degree 4 to 5 are not resolved reasonably in CALS7k.2. Secondly, the temporal distributions of the data are different for both models. The archeomagnetic data only allow a temporal resolution of the order 100 to 150 years [\textit{Korte and Constable}, 2007]. The temporal resolution of \textit{gufm1} is assumed to be of the order of five years for the early epochs and
of the order one or two years for the recent epochs. These limitations of CALS7k.2 place restrictions on the study of the core surface flow. In the next section we will discuss results of core flow inversions from CALS7k.2 and gufm1 to illustrate and quantify the differences produced in flow models by different input models of the main field and secular variation.

3. Fluid flow inversion

The fluid motion at the Earth’s core surface can be mapped by inverting the induction equation

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \frac{1}{\mu_0 \sigma} \nabla^2 \mathbf{B},$$

(4)

where $\mathbf{B} = \{B_r, B_\theta, B_\phi\}$ is the magnetic field and $\mathbf{u} = \{u_r, u_\theta, u_\phi\}$ the velocity field at the CMB.

In this study, it is assumed that the diffusive timescale is a factor 300 to 500 to the advective timescale. This implies, that on short timescales, i.e., of order of 100 years or less, the secular variation is well approximated by the rigidly coupled movement of the magnetic field lines with the fluid motion in the liquid outer core and therefore diffusion can to a first approximation be neglected. This is known as the frozen flux assumption [Roberts and Scott, 1965]. Under this assumption the radial component of (4) becomes

$$\partial_t B_r + \mathbf{u} \cdot \nabla_h B_r + B_r \nabla \cdot \mathbf{u} = 0,$$

(5)

with $\nabla_h = \nabla - i\nabla$ the horizontal gradient. In the next section we will further justify the neglect of diffusion in the computation of core surface flow from CALS7k.2 as a reasonable approximation.
Certain problems in core flow inversion arise, either from disregarding some of the relevant length scales of the field and its secular variation due to truncation [Hulot et al., 1992] or due to un-modelled secular variation produced by interaction of the non-resolvable small scales core flow with the core field [Rau et al., 2000; Eymin and Hulot, 2005]. The first problem is expected to have small impact on the first degrees of the flow [Hulot et al., 1992]. In fact, we compared the large scale flow for different truncation degrees, without finding any differences. Our inversion strategy is to use all spherical harmonic degrees of the regularized field model CALS7k.2 (up to degree 10, though there is little power beyond degree 5), but to truncate the flow to degree and order 5. The errors due to the second kind of problem can hardly be estimated for the inversion of CALS7k.2. But could however be simulated, by a synthetic test on geodynamo model output, where core field and secular variation, with simulated errors added, is filtered to mimic the power spectra of CALS7k.2. By inverting for similar regularized flow models, it can be proven to what extend large scale flow can reliably be retrieved (as suggested by C. Finlay, 2007, priv. comm.), however, this test is outside the scope of the study presented here. We shall proceed assuming that we can obtain useful information on the large scale flow by inverting CALS7K.2.

3.1. The role of diffusion in CALS7k.2

In general, frozen flux can be considered a useful approximation if the errors introduced by that assumption are smaller than those from other sources, e.g. limitations of the field model [Roberts and Glatzmaier, 2000]. Possible violation of the frozen flux assumption has been stated to occur in the 20\textsuperscript{th} century [Bloxham and Gubbins, 1986]. One likely mechanism causing diffusion of magnetic field from the core into the mantle
could be the radial advection of toroidal magnetic field where the toroidal field is expelled by a radially outward directed fluid motion. In the process it is converted to poloidal field and then diffuses through the boundary, as illustrated by Glatzmaier and Olson [2005]. However, several authors [Constable et al., 1993; O’Brien et al., 1997] have demonstrated that the frozen flux assumption can satisfactorily account for changes in field morphology at the CMB over the last 120 years, i.e. on centennial timescales. These results are in agreement with an earlier conclusion reached by Backus [1988], that field models are not sufficiently unique to unambiguously diagnose the presence of diffusional effects. Instantaneous core flow inversions from CALS7k.2 in fact represent averages of flow over about 100 to 300 years, due to the limited temporal resolution of the model. Moreover, the effective spatial resolution of the model is low and spherical harmonics beyond degree 6 or 7 play no significant role. Diffusive decay rates are largest for large scale structures and lie in the order of several hundred years for the smallest resolved structures of CALS7k.2 [see e.g. Table 3 in Gubbins and Roberts, 1987]. Bondi and Gold [1950] pointed out that as a consequence of the frozen flux theorem the unsigned flux \( N \) over the core surface \( \Omega \)

\[ N = \int_{\Omega} |B_r| d\Omega \tag{6} \]

must be invariant in time. Fig. 8 shows the differences of \( N \) for epochs separated by 150, 300 and 800 years. These differences have been divided by the mean unsigned flux of these intervals in order to highlight the significance of the changes in the unsigned flux. Larger values identify significant changes in the unsigned flux integral. The analysis reveals two facts. First, the relative changes of the unsigned flux integral \( dN/N \) over longer periods are larger than the changes over a shorter period, e.g. compare the curves of 150– and 800–year differences. Second, the variations seldom exceed values of 0.1 for
short term differences. Both results allow the conclusion that violation of frozen flux is becoming more important for longer timescales - timescales certainly longer than the temporal resolution of CALS7k.2.

Another necessary condition for the frozen flux hypothesis to hold is that the magnetic flux $F$ through individual patches remains constant with time

$$F(t) = \int_S B r dS = \text{constant} \quad (7)$$

where $S$ is the area encircled by a null-flux curve. The analysis of flux patches of reverse polarity with respect to their vicinity (reverse flux patches) reveals temporal variations of the magnetic flux through these patches in CALS7k.2. The most prominent of such reverse flux patches appears around the year 4500 BC and stays until the year 3750 BC, with an averaged area of $8 \times 10^{12} \text{m}^2$. The mean radial field strength is 180000 nT. The flux though this patch varies from 610 MWb to 2200 MWb. (The currently observed magnetic flux through reverse flux patches is not larger than 600 MWb.) However, the presence of this particular patch is only confirmed by a single data set. Apart from that patch the magnetic flux through other reverse flux patches reaches 500 MWb at maximum and shows rather low values of about 10 MWb. These patches usually do not live longer than some hundred years.

Several studies have dealt with testing the frozen flux assumption on numerical geodynamo models. Roberts and Glatzmaier [2000] studied a particular high resolution geodynamo model and found that the unsigned flux integrated over the core surface, which must be conserved under the frozen flux assumption, was indeed approximately unchanged over timescales of a few centuries. Frozen flux also turned out to be a reasonable first approximation to account for the majority of secular variation in several different dynamo
models studied by Rau et al. [2000]. Results by Amit et al. [2007] suggest that the neglect of diffusion does not influence the major, large scale flow pattern significantly, but that limitations in field model resolution can cause displacements and artefacts in the flow structure.

Based on all these arguments from both observational and theoretical perspectives we propose that the frozen flux assumption can be taken as a reasonable approximation on timescales less than 300 years when considering the evolution of the large scale field. We feel we are therefore justified in attempting instantaneous core flow inversions of CALS7k.2 model. The consequences of the limited resolution of CALS7k.2 will be discussed further in section 4.1.

3.2. Constructing solution

The formalism of the flow calculation is based on a poloidal–toroidal decomposition of the velocity field as formulated by Roberts and Scott [1965]. Assuming $\nabla \cdot \mathbf{u} = 0$, the velocity field can be expressed in terms of two scalar functions of positions on the sphere, $S$ and $T$. These functions can be expanded in spherical harmonics. So the decomposition reads

$$\mathbf{u} = \mathbf{u}_p + \mathbf{u}_t,$$

where

$$\mathbf{u}_p = \nabla_H (r S) = \left(0, \frac{\partial S}{\partial \theta}, \frac{1}{\sin \theta} \frac{\partial S}{\partial \phi}\right)$$

$$\mathbf{u}_t = \nabla_H \times (r T) = \left(0, \frac{1}{\sin \theta} \frac{\partial T}{\partial \phi}, \frac{\partial T}{\partial \theta}\right),$$

(8)
are the poloidal and toroidal velocities and

\[ S(\theta, \phi) = \sum_{l=1}^{\infty} \sum_{m=0}^{l} s_l^m Y_l^m(\theta, \phi) \]

\[ T(\theta, \phi) = \sum_{l=1}^{\infty} \sum_{m=0}^{l} t_l^m Y_l^m(\theta, \phi). \]

(10)

\( Y_l^m \) is a real Schmidt quasi–normalized spherical harmonic. Substituting (8), (9), (10) into (5) and truncating the spherical harmonic expansion of \( S \) and \( T \) leads to a matrix equation

\[ \dot{b} = E t_l^m + G s_l^m, \]

(11)

where \( \dot{b} \) is a finite representation of the secular variation. Equation (11) relates secular variation coefficients with toroidal \( t_l^m \) and poloidal \( s_l^m \) coefficients making up the model vector. Here \( E, G \) are the Elsasser and Gaunt integrals [Bloxham, 1988].

Equation (11) represents a linear inverse problem, with one constraint on two unknowns. Therefore, the inversion requires further assumptions about the flow dynamics. We apply two different assumptions to construct solutions of the fluid flow: purely toroidal motion and tangential geostrophy.

The toroidal flow assumption confines the motion to a spherical surface. Therefore the divergence of the flow vanishes and (11) reduces to

\[ \dot{b} = E t_l, \]

(12)

which physically means that there is no up- or downwelling, as it might be expected in a compositionally or thermally stratified layer close to the core surface. This equation needs to be inverted to determine the components of \( \mathbf{u} \) normal to closed contours of \( \mathbf{B}_r \).

The tangentially geostrophic constraint is based on the assumption that flow motion at the core surface is controlled by the balance between the pressure gradient force and the
Coriolis force [Le Mouël et al., 1985] and that inertial, Lorentz and viscous forces can be neglected. This provides the constraint on the fluid motion

\[ \nabla_h \cdot (u \cos \theta) = 0 \]  

(13)

where \( \theta \) is the geographic colatitude. Both these constraints have no restrictions regarding the timescale over which they hold and could be certainly valid on the centuries to millennia timescales of interest in the present study. Each assumption involves rather crude approximations of the true core dynamics, and we should be aware that these may lead to erroneous flows especially at low latitudes for tangentially geostrophic constraint, and by mapping poloidal flow into toroidal flow for the purely toroidal assumption.

Stochastic inversion is applied to invert (5) core surface flows. Solutions are sought to represent large scale flows where the solution has to be spatially smooth. The objective function \( \Phi \) to minimize in order to find the solution is

\[ \Phi(m) = (y - Am)^T C_e^{-1} (y - Am) + m^T C_m^{-1} m \]  

(14)

where \( A \) is an operator which relates the model \( m \) to the data \( y \). \( y - Am \) is the misfit between data and model prediction, subject to the regularization. The entries in \( C_e \), the covariance matrix of data error estimates, are defined by the condition of minimizing the mean square secular variation misfit at the Earth’s surface which is equivalent to minimizing the mean square secular variation misfit integrated over the Earth’s surface [Le Mouël et al., 1985]. The residuals between secular variation data and secular variation predicted by the flow are weighted by this \( C_e \) matrix, which mathematically has as simple form with diagonal elements,

\[ C_e = \frac{1}{l+1} I \]  

(15)
where $I$ is the identity matrix and $l$ the spherical harmonic degree.

Following Bloxham [1988], the spatial norm is given as

$$C_s = \frac{r_c}{4\pi} \int_{CMB} \left[ (\nabla^2_h u_\theta)^2 + (\nabla^2_h u_\phi)^2 \right] dS$$

(16)

(to provide spatial smoothness and numerical convergence by the truncation degree. The geostrophic norm is

$$C_g = \int_{CMB} [\nabla_h \cdot (u \cos \theta)] dS.$$  

(17)

Together, both norms make up $C_m$, the model covariance matrix

$$C_m = \lambda_g C_g + \lambda_s C_s,$$  

(18)

where $\lambda_g$ and $\lambda_s$ are the tangentially geostrophic and spatial damping parameter, respectively. $C_m$ represents our a-priori assumption of the flow solution properties. $\lambda_g$ is set to zero for the purely toroidal flow, which leaves the solution to be constrained by the spatial norm.

Two quantities of the solution can be formalized [Pais and Hulot, 2000], the normalized misfit to the secular variation model, and the normalized flow energy. The normalized misfit is

$$\text{norm. misfit} = \sqrt{(y - Am)^T C_e^{-1} (y - Am)}$$

(19)

where $N_{sv}$ is the number of the secular variation coefficients. The normalized flow energy which quantifies the flow complexity is given by

$$\text{norm. energy} = \sqrt{\frac{C_s}{N_{sv}}}.$$  

(20)

These quantities are used in the next section to help identify the optimal solutions.
4. Results and discussion

In this section we present results of our flow inversion of the CALS7k.2 model. We inverted the geomagnetic secular variation data from CALS7k.2 for instantaneous solutions of single epoch flows, rather than for time–dependent flows [Dumberry and Bloxham, 2006] or steady solutions. The core surface flows are derived to spherical harmonic degree five, because of the limited spatial resolution of CALS7k.2, and represent averages over about two centuries rather than truly instantaneous flows due to the limited temporal resolution of the model [Korte and Constable, 2007].

The appropriate spatial regularization is chosen by considering a trade-off curve between the normalized misfit and the normalized energy of the flow, with the optimal solution chosen at the “knee” of the curve [similar to Pais and Hulot, 2000; Eymin and Hulot, 2005]. Fig. 9 shows two trade-off curves for the epoch 1900: one curve of tangentially geostrophic flow solutions and one curve of purely toroidal solutions. The curves result from varying the spatial damping parameter $\lambda_s$, and from the choice of tangentially geostrophic damping parameter $\lambda_g = 5.0 \times 10^5$ (following [Pais et al., 2004], the chosen tangentially geostrophic damping can be classified as weak) and $\lambda_g = 0.0$ (purely toroidal), respectively. Clearly, the spatial regularization controls the spatial complexity of the flow and also controls the flow velocity itself. In contrast, a relaxation of the tangentially geostrophic constraints controls the misfit to secular variation model, as it is indicated by the different levels of the curves in Fig. 9. Consequently, we find that the purely toroidal solutions always provide a better fit to the secular variation of CALS7k.2.

Throughout the period 5000 BC and 1950 AD $\lambda_g = 5.0 \times 10^5$ is fixed for all instantaneous inversions carried out. For all 139 epochs trade-off curves are derived from tangentially
geostrophic and purely toroidal solutions to find the optimal solution when the spatial damping parameter is varied. The optimal \( \lambda_s \) ranges between \( 8.0 \times 10^{-4} \) and \( 4.0 \times 10^{-3} \).

However, the usage of a constant spatial damping \( \lambda_s = 2.4 \times 10^{-4} \), which corresponds to the average optimum can be justified, because the differences to the optimal solution are marginal. Therefore, a constant spatial damping \( \lambda_s = 2.4 \times 10^{-4} \) for all epochs is applied, and in order to achieve similar flow complexity for the purely toroidal solutions the same spatial damping \( \lambda_s \) is applied as for the tangentially geostrophic solutions.

As mentioned above, the purely toroidal solution provides a better fit to the archeomagnetic secular variation than the tangentially geostrophic flow assumption. We find that the misfit of the tangentially geostrophic flow is almost always larger than that of the toroidal flow. If taken at face value these results imply that the flow is toroidal, rather than tangentially geostrophic, and that the uppermost core may be stably stratified. However, as pointed out by Whaler [1986], it is difficult to make definitive conclusions about the existence of stably stratified outermost core on the basis of how well purely toroidal flow fits the data. In fact, it is likely that the limited spatial resolution of the field model and the regularization applied during the inversion make it impossible to map the smaller scale poloidal flow correctly, and therefore, it is easier to fit the archeomagnetic secular variation with purely toroidal flow.

4.1. Fluid motion

In Figure 11 a series of snapshots of the tangentially geostrophic flow are shown. The tangentially geostrophic flow at the epoch 4000 BC is directed eastward under Africa, whereas the flow configuration completely changes at the epoch 2500 BC and changed again at 2000 BC. The flow map are found to be dominated by patches of westward and
eastward directed motion. Figure 12 depicts a sequence of toroidal flow maps at the same epochs in order to allow a comparison with the tangentially geostrophic flow maps. In contrast to the tangentially geostrophic flows the toroidal flows show strong contributions of motion across the equator. Like the geostrophic flows the directions of motion alternates between mainly eastward and westward. However, the detailed agreement between the two types of flow is poor.

Before we discuss further the interesting temporal variability, it is necessary to address the issue of the agreement and disagreement of the flow solutions of the time-averaged gufm1 and CALS7k.2 for the interval 1600 to 1950. In order to mimic the effect of limited field resolution, we tapered the temporally averaged gufm1 to a maximum spherical harmonic degree 5 since above this degree CALS7k.2 has little power. The top panel of Fig. 13 shows the tangentially geostrophic velocity map from this temporally averaged and spatially low-pass filtered version of the gufm1 field model. The flows obtained in this case are generally similar to flows of previously published studies by Bloxham [1992] and Amit and Olson [2006]. In detail: like [Amit and Olson, 2006], who sought temporally averaged helical core surface flow for the period 1840 to 1990, we find a large anti-cyclonic pattern located south of Africa and fairly weak flow in the northern hemisphere. The flow structure under the eastern Pacific is not seen by [Amit and Olson, 2006], but it is evident in the tangentially geostrophic flows of [Bloxham, 1992]. This feature seems to be existent in the period 1600 to 1750. However, it is less robust, maybe because of the lack of absolute intensity measurements in gufm1 for the interval 1590 to 1840 obviating the unique derivation of the flow for that period is not possible [Jackson, 2000]. More
problematic is the is very sparse data distribution and the larger error in the contributing data early in the *gufm1*, which may result in less robust flow features.

Fig. 13 shows a comparison of the flows from the time averaged and spatially low-pass filtered *gufm1* and CALS7k.2 for the epochs 1600 AD and 1900 AD. The overall impression is that the flow patterns from CALS7k.2 and *gufm1* are rather incompatible. The flow patterns from CALS7k.2 changed substantially between 1600 and 1900. This indicates that the temporal resolution is considerably less than 300 years. The reason for the differences in the flow configuration is most likely the limited spatial resolution of the archeomagnetic secular variation, i.e. due to very little data in some geographical regions (e.g. Southern Hemisphere) or regions of poor data quality, which leads to a local poor secular variation model and degrades all spherical harmonic degrees of the archeomagnetic field model. In order to quantify the discrepancies in the flow solutions, the fractional RMS deviation of the flow solutions are computed (similar to the analysis in section 2.2). The results are shown in Fig. 14 and Fig. 15. The latter shows $\sigma$ of the zonal components, the best agreement is between the $t^1_2$ term of the tangentially geostrophic solutions, the $t^2_2$ terms considerably agree from 1750 AD onward, whereas there is not much agreement between the tangentially geostrophic $t^2_3$ terms. The large scale zonal terms of the purely toroidal flow solutions has nearly no consistency, only the $t^0_2$ terms show some considerable agreement.

Although there are large differences between the flow of the time-averaged *gufm1* and CALS7k.2 due to the different quality of the model input data, we will proceed to analyze the temporal variability in the zonal flow components in more detail. Mainly because,
changes in the large scale zonal flow of CALS7k.2 seem to be robust regardless of the flow constraint.

4.2. Temporal variability of the zonal flow

The temporal variation of the large scale zonal flows are analyzed by means of time series analyzes. The multi–taper method (MTM) for estimation of the spectral density function is applied it makes use of a multiple orthogonal taper functions to describe structures in time series that are modulated in frequency and amplitude. This method provides a spectral estimate with an optimal trade–off between spectral resolution and variance. In this study, we closely follow the procedure of Thomson [1982] and Mann and Lees [1996].

The spectra of the zonal flow components $t_0^0, t_0^1, t_0^2$ of the respective flows are shown in Fig 16a and b. The spectrum of the $t_0^0, t_0^2$ zonal coefficients of the purely toroidal flow shows several significant peaks at 0.00095, 0.0012, 0.00145 and 0.00185 yr$^{-1}$. These frequencies correspond to periods of approximately 1050, 840, 690 and 540 years. Shorter periods might be present, but are not clearly visible in the spectrum. The spectrum of the zonal coefficient $t_0^3$ does not show clear peaks, only broad hump around the 1000-year period. Like the spectrum of the purely toroidal $t_0^0$, the spectrum of the tangentially geostrophic $t_0^0$ zonal flow variability has significant peaks in the same spectral range. Significant frequencies are detected at 0.00095, 0.0013, 0.00162 and 0.0019 yr$^{-1}$ with corresponding periods of 1050, 770, 620 and 540 years. $t_0^2$ varies with a frequency of $1.55 \times 10^{-3}$ cycle/year, a 650 year period. The spectrum of the tangentially geostrophic $t_0^3$ shows two peaks at 0.001 and 0.0013 yr$^{-1}$, however, both peaks are little above the significance level. Higher frequency undulations are visible in the spectra of the tangentially geostrophic zonal flow, but appear to be less powerful than the frequencies above. A list of all significant
frequencies is given in Table 1. The uncertainties of these periods are not easy to estimate, but should not be larger than the temporal resolution of CALS7k.2 (100 to 300 years [Korte and Constable, 2007]).

The spectral peaks are relatively insensitive to the choice of spatial damping parameters and invariant against changes in the tangentially geostrophic damping. A strengthening of the spatial damping leads to a reduction of the power in the spectral peaks, but these peaks vanish only at unrealistically high spatial damping when the flow energy goes to zero.

As it has been noticed before by Bloxham [1990]; Jault and Le Mouël [1991] large scale zonal flows (e.g. represented by $t_{1}, t_{3}$) satisfy both the tangentially geostrophic and the purely toroidal constraint. The existence of nearly the same periods between 500 and 1000 years in both the geostrophic and purely toroidal $t_{0}$ flow indicate that the periodicities found for our flow solutions are rather robust and are required to explain the secular variation model CALS7k.2.

4.3. Interpretation of the typical flow timescales

In order to interpret the typical flow timescales some of the theoretical results of Dumberry and Bloxham [2006] need to be recalled. In their study, they decomposed the fluid motion in the outer core into two principal components: fully geostrophic rigid motions on a cylindrical surface and non–rigid motions, involving deviations from the flow parallel to the rotation axis. They demonstrate that the non–rigid motions can be caused by convective-driven changes in the thermal and magnetic winds operating on timescales of about 500 years.
Based on the grouping of the peaks found in our spectral analysis, we suggest that the first three periods may reflect a fundamental period of about 800 years. The splitting into several peaks could be either due to the superposition of the fundamental period with a shorter period or a consequence of the fundamental oscillation being quasi-periodic with period varying over the interval studied. This observation compares to theoretically predicted timescale of convection-driven variations in thermal and magnetic winds. Therefore, we interpret the 800–year period as the period of the convective fluid motion driving variation in thermal and magnetic winds. In conjunction with our finding, that the flow tends to be toroidal and therefore the outermost core to be stratified, a possible scenario of how the non–rigid flow maps onto the axis-symmetric azimuthal motion could be the penetration of columnar convection into the stratified top layer as proposed by Takehiro and Lister [2001, 2002].

4.4. Geophysical implications

What are the geophysical implications to be drawn from the time-varying core surface zonal flow that we have identified? It has been shown that variations in large scale zonal flow on decadal timescales are related to variations in the Earth’s rotation rate [Jault et al., 1988]. Moreover, Jackson et al. [1993] demonstrated that core surface flow models can be used to reconstruct the decadal variation in the length of day (LOD) in the past 150 years. In their study Dumberry and Bloxham [2006] reconstructed LOD variations of the last three thousands years (computed from records of historical solar and lunar eclipses [Stephenson and Morrison, 1995]) using a continuous core surface flow model. Their reconstruction of the LOD signal is based on simple azimuthal flow that explains part of the observed archeomagnetic secular variation. Their study shows the amplitudes
and characteristic timescales of the observed LOD changes can be explained by zonal flow variations deduced from secular variation, but, they did not obtain a detailed match. As they pointed out the reason for the mismatch is mainly that the full geostrophic assumption (c.f. Gubbins [1991] for an explanation of the difference between full geostrophic and tangentially geostrophic flows) breaks down on such a long time span. On millennial timescales, changes in the magnetic field configuration are generated by convective motions, which in turn produce observable secular variation. However, these convective motions reflect a magnetostrophic balance, the background state of geostrophic torsional oscillations, and have no requirements to be invariant on cylindrical surfaces. Therefore, attempts to extrapolate such flows into the fluid interior using the full geostrophic assumption lead to an incorrect determination of changes in the core angular momentum. Unfortunately, our core surface flows cannot therefor, at present, be used for LOD studies. Further theoretical developments concerning the structure of zonal magnetostrophic motions are first necessary.

5. Conclusion

In this study, we inverted the CALS7k.2 archeomagnetic field model for the instantaneous core-surface fluid flows every 50 years during the interval 5000 BC and 1950 AD, with spherical harmonic representation restricted to a maximum degree and order 5. A detailed comparison between the CALS7k.2 and a temporally averaged gufm1 was carried out in order to allow an evaluation of the reliability of the computed flow solutions. We find that the secular variations models differ significantly for the period 1600 AD to 1950 AD. The discrepancy is primarily caused by the larger data errors for CALS7k.2, which consequently lead to poor spatial and temporal resolution of the order of 100 to 300 years
for CALS7k.2 secular variation and impose considerable restrictions on the results of the flow inversion.

We analyzed the flow resulting from two different dynamical assumptions, namely the purely toroidal and tangentially geostrophic flow assumptions. The flow solutions are unique in the sense that they minimize the mean flow energy for the chosen damping parameters subject to fitting the data, though some ambiguous patches remain for the tangentially geostrophic flows [Backus and Le Mouël, 1986]. It is found that purely toroidal flow explains the archeomagnetic secular variation better than geostrophic flow.

From the flow maps it can be clearly seen that the fluid flow undergoes different regimes of zonal flow direction, that flows mainly dominated by eastward motion alternate with mainly westward flows. While the detailed flow feature structure is less certain and varies depending on the flow assumption, the episodes of eastward and westward flow are robust as they are obtained regardless of the flow assumption.

The significant periodicities of the large scale zonal flows ranges from 540 to 1050 years (with a center-period of about 800 years), and are clearly detectable in both flow solutions. These periods are consistent with the timescale predicted for variations in thermal/magnetic winds driven by convective motion in Earth’s outer core.

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References


Gallet, Y., A. Genevey, and V. Courtillot (2003), On the possible occurrence of ‘archaeomagnetic jerks’ in the geomagnetic field over the past three millennia, *Earth and


Figure 1. Drift rates, which explain the temporal variation of CALS7k.2 under the assumption that longitudinal drift is the dominating motion. Brown represents westward drift, dark blue corresponds to eastward drift whereas white, light brown and light blue indicate almost zero drift.
Figure 2. Profiles of the drift at different latitudes: the equatorial drift rate (solid), the southern drift rate at -67° Latitude (dotted) and the northern drift rate at 67° Latitude (dashed).

Figure 3. Profile of the drift rate along 48° latitudes (Paris). The gray bars mark the occurrence time of the archeomagnetic jerks. The width of the bars is 25 years.
Figure 4. Spatial (black, left ordinate) and temporal (gray, right ordinate) norms of CALS7k.2 as a measure for spatial and temporal complexity together with normalized misfit and number of data, both in 50 year bins.
Figure 5. Comparison of the radial magnetic field at the CMB. The top panels show maps derived from CALS7k.2. The middle panels show maps derived from gufm1 and in the bottom panels the maps of a gufm1, limited in resolution by truncation to spherical harmonic degree 5, are shown. All field configurations are derived for the epochs 1600 (left) and 1900 (right).
Figure 6. The fractional RMS deviation between the temporally averaged $gufm1$ and CALS7k.2 main field per coefficient numbers at four different epochs. Coefficient are numbered from 1 to 35 in the order $g_{1}^{0}, g_{1}^{1}, h_{1}^{1}, \ldots, h_{5}^{5}$.

Figure 7. The fractional RMS deviation of the secular variation models per coefficient number at four different epochs.
Figure 8. Differences of the unsigned flux integral over the core surface of 150–, 300– and 800–year intervals divided by the mean unsigned flux of these intervals.

Figure 9. Trade-off curves between the normalized energy and the normalized misfit for weak geostrophic flows and purely toroidal flows at the epoch 1900. The tetragon marks the preferred tangentially geostrophic solution and the triangle marks the preferred purely toroidal solution.
Figure 10. Trade-off curves of weak tangentially geostrophic flows at different epochs 1900 AD and 2500 BC. Again, the tetragon refers to the preferred weak tangentially geostrophic solution at 1900 AD, the pentagon marks the weak tangentially geostrophic solution for 2500 BC. The spatial damping parameter is for both solutions $8.0 \times 10^{-4}$. 
Figure 11. Snapshots of the tangentially geostrophic flow from the CALS7k.2 for three different epochs 4500 BC, 2500 BC and 2000 BC. The arrows describe the horizontal flow on the core surface, with the length indicating the magnitude and direction of the flow at certain positions. The continents are added as reference.
Figure 12. Similar to Figure 11, but for the purely toroidal flow at the core surface.
Figure 13. The tangentially geostrophic flow computed from the temporally averaged gufm1 (top). The flow configuration of the time-averaged gufm1 is the same for any epoch within 1590 and 1990. The tangentially geostrophic flow from CALS7k.2 at the epochs 1600 (middle) and 1900 (bottom). Note the different length scale of the arrows.
**Figure 14.** The fractional RMS deviation of the tangentially geostrophic (top) and the purely toroidal flows (bottom) computed from the averaged *gufm1* and CALS7k.2 at 1600 and 1900.
Figure 15. The fractional RMS deviation of the zonal flow components of the tangentially geostrophic (top) and purely toroidal (bottom) flow solutions between the averaged $gufm1$ and CALS7k.2 at 1600 and 1900.
Figure 16. Estimates of the power spectral density of the temporal variability a) large scale zonal flow, purely toroidal flow, b) large scale zonal flow, tangentially geostrophic flow. The dashed line marked as red noise indicate the significance level for the $t_1^0$ terms. The significance levels of the other $t_2^0$ and $t_3^0$ are different and below that line. The vertical gray bars mark the most dominant frequencies.
Table 1. Significant frequencies and periods of the zonal flow velocity terms.

<table>
<thead>
<tr>
<th>Tangentially Geostrophic Flow</th>
<th>Frequencies [10^{-3} cycle/year]</th>
<th>Periods [year]</th>
</tr>
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<tr>
<td>$\ell_1^0$</td>
<td>0.95, 1.3, 1.62, 1.9</td>
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<td>650</td>
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<tr>
<td>$\ell_3^0$</td>
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</tr>
<tr>
<td>$\ell_2^0$</td>
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<td>1050, 840, 690</td>
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<tr>
<td>$\ell_3^0$</td>
<td>0.95</td>
<td>1050</td>
</tr>
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