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Subducted slabs and lateral viscosity variations: effects on the long-wavelength geoid

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SUMMARY
The characteristic broad local maxima exhibited by the long-wavelength geoid over subduction zones are investigated with a numerical model of mantle flow. In a spherical axisymmetric geometry, a synthetic model of buoyancy driven subduction is used to test the effects on the geoid caused by the depth of penetration of the lithosphere into the mantle, by the viscosity stratification and by lateral viscosity variations (LVV) in the lithosphere, upper and lower mantle. The presence of anomalous slab density in the lower mantle guarantees geoid amplitudes comparable with the observations, favouring the picture of slabs that penetrate the transition zone and sink into the deep mantle. The viscosity of the lower mantle controls the long-wavelength geoid to the first order, ensuring a clear positive signal when it is at least 30-times greater than the upper-mantle viscosity. The presence of LVV in the lithosphere, in the form of weak plate margins, helps to increase the contribution of the surface topography, causing a pronounced reduction of the geoid. Localized LVV associated with the cold slab play a secondary role if they are in the upper mantle. On the other hand, highly viscous slabs in the lower mantle exert a large influence on the geoid. They cause its amplitude to increase dramatically, way beyond the values typically observed over subduction zones. Long-wavelength flow becomes less vigorous as the slab viscosity increases. Deformation in the upper mantle becomes more localized and power is transferred to short wavelengths, causing the long-wavelength surface topography to diminish and the total geoid to increase. Slabs may be then weakened in the lower mantle or retain their high viscosity while other mechanisms act to lower the geoid. It is shown that a phase change from perovskite to post-perovskite above the core–mantle boundary can cause the geoid to reduce significantly, thereby helping to reconcile models and observations.

Key words: Numerical solutions; Gravity anomalies and Earth structure; Subduction zone processes; Dynamics: gravity and tectonics.

1 INTRODUCTION
The long-wavelength non-hydrostatic geoid is one of the surface observables associated with mantle convection (e.g. Hager 1984; Ricard et al. 1984; Richards & Hager 1984). At the global scale, it can be successfully modelled in spherical geometry in terms of a Newtonian flow induced by density perturbations inferred from seismic tomography (e.g. Hager & Clayton 1989; Forte & Peltier 1991). Using both the positive and negative velocity anomalies delivered by seismic tomography (e.g. Becker & Boschi 2002), a degree-variance reduction higher than 80 per cent can be generally reached when the geoid at spherical harmonic degree $\ell \leq 8$ is interpreted with flow models with radially dependent (1-D) viscosity distribution (e.g. Hager & Clayton 1989; Forte & Peltier 1991; Thoraval & Richards 1997; Čadek & Fleitout 1999). Nevertheless, a fit to the geoid approximately as satisfactory can be obtained using solely the negative buoyancy associated with the present-day distribution of slabs inferred from subduction history (Ricard et al. 1993; Lithgow-Bertelloni & Richards 1995; Steinberger 2000). The possibility of explaining a large portion of the geoid in terms of subducted slabs motivates then the investigation of the effects produced by characteristic slab parameters on the geoid itself.

Since long time, several authors have recognized that, at long to intermediate wavelengths, a direct correlation exists between positive geoid highs and major subduction zones (Runcorn 1967; Kaula 1972; McKenzie 1977; Davies 1981; Ricard et al. 1984; Richards & Hager 1984). Fig. 1 shows four cross-sections of the long-wavelength geoid ($2 \leq \ell \leq 8$) from the satellite mission GRACE ( Förste et al. 2006) that are roughly perpendicular to the corresponding convergent plate margins. Geoid maxima range from slightly more than 20 m over the subduction zones of South

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The distance along the cross-section is measured from east to west. Tonga-Kermadec (1), New Guinea (2), Japan (3) and South America (4).

while Čadek & Fleitout (2006) take into account L VV in the D
cross-section over four convergent plate margins of the long-

Figure 1. Cross-sections over four convergent plate margins of the long-

America and Japan up to 40 and 70 m over the Tonga and New

Most spherical models aimed at fitting the long-wavelength geoid

global scale assume a layered mantle with a laterally homoge-

in the upper mantle or reaches the mid-lower mantle or the CMB.

with a seismic tomographic model of the lower mantle to analyse

Starting from Richards & Hager (1989), LVV are commonly in-

Many studies (e.g. ˇCadek & Fleitout 1999) or the consideration of

with the exception of Čadek & Fleitout (2003), incorporating

in such models does not generally lead to an improved

In some cases (Zhong & Davies 1999; Yoshida & Nakakuki 2009),

Considering highly viscous slabs in the lower mantle causes the fit
to worsen significantly.

Starting from Richards & Hager (1989), LVV are commonly in-

in the context of geoid modelling also, for example, by Kiefer & Kellogg

With this work we aim at studying how the long-wavelength

good fit to the geoid at short wavelengths. Krien & Fleitout (2008) use

So far, the long-wavelength geoid concern is generally agreed with those

resulting from GRACE (Förste et al. 2006): Tonga-Kermadec (1), New Guinea (2), Japan (3) and South America (4). The distance along the cross-section is measured from east to west.
surrounding mantle. In view of the large effects exerted by highly viscous slabs in the lower mantle, we also consider a low-viscosity region at the bottom of the slab representing the perovskite–post-perovskite phase change and show that this structure can help to reconcile model and observations.

In the following section, we recall the mathematical model underlying the geoid computation and briefly outline the numerical technique that we have employed. In Section 3, we describe in detail the axisymmetric setting of our model and the set of parameters that we have tested. Section 4 is dedicated to the description of the results while their discussion and the conclusions follow in Sections 5 and 6.

2 GOVERNING EQUATIONS AND NUMERICAL METHOD

The geoid is traditionally modelled by solving the Poisson equation for the gravitational potential coupled with the Stokes equation for a highly viscous incompressible fluid. The Poisson equation reads

$$\nabla^2 \Phi = 4\pi G \delta \rho \text{ in } B,$$

where $$\Phi$$ is the gravitational potential induced by a density anomaly $$\delta \rho$$, $$G$$ is the gravitational constant and $$B$$ is the mantle volume. The potential must be continuous through the boundary $$\partial B$$

$$[\Phi]^+ = 0 \text{ on } \partial B,$$

where the symbol $$[\Phi]^+$$ denotes the jump of $$\Phi$$ across the interface $$\partial B$$. The contribution of the dynamic topography $$h$$ caused by the flow is taken into account as a boundary condition for the gravity acceleration. The boundary $$\partial B$$ must be considered to adjust the boundary condition (2), while $$\partial$$ is the forcing functional,

$$E(u, \rho, \lambda; \delta u, \delta \rho, \delta \lambda, \delta \Phi) = F(\delta u, \delta \Phi).$$

In eq. (9), $$E$$ is a bilinear functional,

$$\delta E = \int_B \eta \delta \Phi \, dV + \int_B p \delta u \, dV + \int_B \nabla \delta p \, dV$$

$$+ \int_{\partial B} \lambda \delta u \, dS + \int_{\partial B} \delta \lambda \delta \Phi \, dS$$

$$+ \frac{1}{2} \int_B \nabla \Phi \cdot \nabla \delta \Phi \, dV + \int_B \delta \Phi \cdot \delta \Phi \, dS,$$

where $$\lambda \equiv \tau_{rr} \mid_{\partial B}$$ and the pressure $$p$$ is used as Lagrange multipliers to adjust the boundary conditions and the incompressibility constraint, respectively, the symbol ‘:’ denotes the double scalar product of tensors, that is, $$\delta \Phi \cdot \delta \Phi = \| \delta \Phi \|^2$$.

Eq. (9) is first discretized by expanding the angular part of the pressure and gravitational potential into scalar spherical harmonics, of the flow into vector spherical harmonics and of the strain rate into tensor spherical harmonics (Martinec 2000). For the discretization of the radial part of these fields, either piecewise linear finite elements (for $$\Phi$$, $$u$$ and $$\dot{e}$$) or piecewise constant functions (for $$p$$) are adopted. Both the density $$\delta \rho$$ and viscosity $$\eta$$ are parameterized with piecewise constant functions. This allows us to set their values on a grid, thereby facilitating to prescribe the geometry of the slab. The system of linear equations obtained is then solved iteratively by pre-conditioned conjugate gradient technique (e.g. Barrett et al. 1994). Further details concerning this formulation can be found in Tosi & Martinec (2007, appendix C).

The SFE method has been thoroughly tested. For the case of radially symmetric viscosity distribution, it has been benchmarked against a matrix propagator solution (e.g. Ricard et al. 1984; Hager & Clayton 1989), while for axisymmetric viscosity, it has been compared to the semi-analytical solution derived by Tosi & Martinec (2007). In both cases, good agreement was obtained.
of two orders of magnitude with respect to the surrounding lithosphere is generally used, that is, \( \eta_{PM} = 0.01 \eta_{Lith} \) (see Section 4.2 for a discussion on the effects of this parameter). While the somewhat unrealistic shape of the trench (real subducting plates generally have a more curved bending surface) has a negligible effect on the long-wavelength geoid, it becomes important at wavelengths of the order of its size which, however, are not discussed in this work. The subducting plate has a thickness of 100 km in the upper mantle and extends from the surface to a depth \( d_{SI} \). The thickening of subducted slabs in the lower mantle due to their compression and folding is a well-recognized phenomenon confirmed by seismic observations (e.g. Ribe et al. 2007) and numerical models (e.g. Běhounková & Čížková 2008; Goes et al. 2008). Therefore, in those models where the slab penetrates the 670 km boundary, its thickness in the lower mantle is increased according to the viscosity contrast between upper and lower mantle by a factor \( t = \ln(\eta_{LM}/\eta_{UM}) \) (Ricard et al. 1993). When LVV in the mantle are considered, the values of the viscosity of the slab in the upper and lower mantles are \( \eta_{Sl}^{UM} \) and \( \eta_{Sl}^{LM} \), respectively. These two values are chosen in such a way that the viscosity contrast between slab and surrounding mantle is constant throughout the mantle, that is, \( \eta_{Sl}^{UM}/\eta_{Sl}^{LM} = \eta_{Sl}^{UM}/\eta_{Sl}^{LM} \). From now on, we will denote this contrast simply by \( \eta_{Sl}/\eta_{PM} \), with \( \eta_{PM} \) indicating the viscosity of the mantle that surrounds the slab. In agreement with Ricard et al. (1993), we assume that the density anomaly \( \delta \rho_{Sl} \) associated with the slab is 60 kg m\(^{-3}\). This density anomaly is imposed from the base of the lithosphere down to the depth \( d_{SI} \) reached by the slab. To compute the dynamic topography (eq. 8), the values that we use for the jump in the reference density are \([\rho_0] = 3000 \) and 4500 kg m\(^{-3}\) for the surface and the CMB, respectively.

Tosi et al. (2009) have discussed the effect on the long-wavelength geoid of the phase transition from perovskite to post-perovskite in the lowermost mantle (Murakami et al. 2004; Oganov & Ono 2004). Recent first principle calculations (Oganov & Ono 2005) and experiments (Ohta et al. 2008) indicate that the ionic electrical conductivity of post-perovskite can be several order of magnitude lower than that of perovskite at the same pressure and temperature conditions. As the viscosity associated with diffusion creep is proportional to the inverse of the electrical conductivity (Yamazaki & Karato 2001), post-perovskite is then likely to be less viscous than perovskite. Therefore, in order to account for the presence of post-perovskite, in Section 4.4 we will also consider the effects produced by an area located inside the D\(^\prime\) layer (lower 200 km of the mantle) at the bottom end of the slab (depicted in light grey in Fig. 2) whose viscosity, denoted by \( \eta_{PPv} \), will be lower than the viscosity of the surrounding lower mantle. As the density increase associated with the transition from perovskite to post-perovskite amounts about 1.5 per cent (Oganov & Ono 2004), we will further assume that the post-perovskite area is associated with a density anomaly \( \delta \rho_{PPv} = 75 \) kg m\(^{-3}\).

We consider three different density models according to the depth of penetration of the slab. We denote by M1 the models in which the slab is prescribed only in the upper mantle \( (d_{SI} = 670 \) km). These can be representative of relatively shallow subductions (Fukao et al. 2001) such as those of the Pacific plate at the Japan trench or the Nazca Plate (Fig. 1, cross-sections 3 and 4, respectively). By M2, we denote the models where the slab reaches the mid lower mantle \( (d_{SI} = 1750 \) km) and by M3 the models where the slab reaches the CMB \( (d_{SI} = 2890 \) km). They can be representative of deeper subductions such as those of the Pacific Plate at the Tonga (Hall & Spakman 2002) and New-Guinea (Tregoning & Gorbatov 2004) trenches (Fig. 1, cross-sections 1 and 2, respectively).
We test the effects of the viscosity stratification by varying the viscosity contrasts between lithosphere and upper mantle ($\eta_{\text{Lith}}/\eta_{\text{UM}}$) and between lower and upper mantle ($\eta_{\text{LM}}/\eta_{\text{UM}}$). The tested values of $\eta_{\text{Lith}}/\eta_{\text{UM}}$ are 1, 30, 100 and 1000. Even though the effective viscosity of the lithosphere is expected to be even more than three order of magnitude greater than that of the upper mantle (Gordon 2000), we focus on smaller viscosity contrasts, for they usually result from global geoid inversion (e.g. King 1995). The tested values of $\eta_{\text{LM}}/\eta_{\text{UM}}$ are 1, 10, 30 and 100.

We analyse LVV associated with the slab by varying the viscosity of the slab with respect to the surrounding mantle. The viscosity contrast $\eta_{\text{Sl}}/\eta_{\text{M}}$ can take the values 1, 10, 50, 100 and 500. The case $\eta_{\text{Sl}}/\eta_{\text{M}} = 1$ corresponds to a radially symmetric viscosity distribution with no LVV beneath the lithosphere.

To discretize the radial coordinate, 60 elements of variable size are used to obtain a radial resolution of 60 km in the lower mantle, 30 km in the upper mantle and 15 km in the lithosphere. In order to resolve properly the viscosity structure of the thin subducted lithosphere and plate margins, we consider angular expansions of the field variables up to harmonic degree $\ell_{\text{max}} = 600$, corresponding to a wavelength of approximately 66 km.

## 4 RESULTS

### 4.1 Radially symmetric viscosity models

We first discuss the long-wavelength geoid calculated for a mantle where no LVV are present. For each density model, we show in Fig. 3 the results for all the viscosity stratifications tested. Black lines refer to model M1, red lines to M2 and blue lines to M3. For an isoviscous mantle ($\eta_{\text{LM}}/\eta_{\text{UM}} = 1$), the three models share a common feature: they result in a negative geoid that is little influenced by the viscosity of the lithosphere. The amplitude of the minimum increases with the depth of penetration of the slab although the differences between models M2 and M3 are small. For model M1, as long as $\eta_{\text{Lith}}/\eta_{\text{UM}} < 1000$, an increase in the lower mantle viscosity of a factor of 10 is sufficient to revert the sign of the geoid, although the amplitudes obtained are very small (lower than 10 m). This tendency is also characteristic of models M2 and M3. They show a weakly positive geoid when $\eta_{\text{Lith}}/\eta_{\text{UM}} = 1$ which becomes negative as this parameter increases. As soon as $\eta_{\text{LM}}/\eta_{\text{UM}} = 30$, all three models agree, giving rise to positive geoid anomalies comparable to the observed ones for $\eta_{\text{Lith}}/\eta_{\text{UM}} < 1000$ and a weakly negative anomaly for $\eta_{\text{Lith}}/\eta_{\text{UM}} = 1000$. For $\eta_{\text{LM}}/\eta_{\text{UM}} = 30$, the effect of the viscosity of the lithosphere is significant for model M1 only when $\eta_{\text{Lith}}/\eta_{\text{UM}} = 100$. In this case a nearly vanishing geoid signal is induced, while low positive amplitudes (around 15 m) are obtained for $\eta_{\text{Lith}}/\eta_{\text{UM}} < 1000$. The viscosity of the lithosphere affects more significantly models M2 and M3 causing a systematic reduction of the geoid as $\eta_{\text{Lith}}/\eta_{\text{UM}}$ is raised from 1 to 1000.

As the viscosity ratio $\eta_{\text{LM}}/\eta_{\text{UM}}$ is further increased to 100, geoid anomalies are positive in all three cases. They remain relatively low for the upper-mantle slab model M1. A peak of 30 m is obtained when $\eta_{\text{Lith}}/\eta_{\text{UM}} = 30$ and $\eta_{\text{LM}}/\eta_{\text{UM}} = 100$. When $\eta_{\text{LM}}/\eta_{\text{UM}} = 100$, the lower-mantle slab models M2 and M3 are characterized by a very significant thickening of the slab in the lower mantle, with

![Figure 3](image_url) Figure 3. Geoid profiles obtained with radially symmetric viscosity distributions. Black lines refer to model M1, red lines to model M2 and blue lines to model M3.
t being 4.6. The large density anomaly that accompanies such a thickening is responsible for the pronounced geoid highs which are far greater than those generally observed over subduction zones unless $\eta_{\text{Lith}}/\eta_{\text{UM}} = 1000.$

4.2 Lateral viscosity variations in the lithosphere

In analysing the effects of LVV we first focus on the case in which they are solely present in the lithosphere as low-viscosity plate boundaries. According to the results of the previous section, a lower mantle 30-times more viscous than the upper mantle produces geoid amplitudes that lie well within the range of observed values for all models M1, M2 and M3. The fact that this viscosity contrast is optimal to fit the data is a classical result on which most geoid models agree (e.g. King 1995). Along with this, a similar value for the ratio $\eta_{\text{Lith}}/\eta_{\text{UM}}$ generally guarantees a good fit to the long-wavelength geoid (e.g. Corrieu et al. 1995). Therefore, to discuss the effects of LVV in the lithosphere, we select as representative the models having $\eta_{\text{Lith}}/\eta_{\text{UM}} = \eta_{\text{LM}}/\eta_{\text{UM}} = 30.$ The results are illustrated in Fig. 4 for models M1 (panel a), M2 (panel b) and M3 (panel c). Black lines show the geoid obtained from the radially symmetric viscosity models (as in Fig. 3). Colour lines show the effects of different values of the viscosity of the plate margins, namely $\eta_{\text{PM}} = 0.3$ (red), 0.03 (blue) and 0.003 (green), corresponding, respectively, to a viscosity drop of a factor of 10, 100 and 1000 with respect to the viscosity of the lithosphere. Low viscosity plate margins play an important role as they cause the main geoid peak to reduce significantly: from 16 to 10 m for model M1, from 43 to 33 m for model M2 and from 60 to 48 m for model M3, when $\eta_{\text{PM}} = 0.3.$ As the value of $\eta_{\text{PM}}$ is further reduced, the geoid keeps decreasing, although the amplitudes obtained for $\eta_{\text{PM}} = 0.03$ and 0.003 are rather similar. The reason why the geoid decreases with reducing the viscosity of plate margins can be understood inspecting the changes of the dynamic topography plotted in Fig. 5. The dynamic topography of the CMB (panels d–f) is hardly influenced by the presence of lithospheric LVV. On the other hand, the amplitude of the surface topography (panels a–c) is significantly reduced, particularly when $\eta_{\text{PM}} = 0.03.$

![Figure 4](image1.png)  
**Figure 4.** Effect of LVV in the lithosphere on the geoid when $\eta_{\text{Lith}}/\eta_{\text{UM}} = \eta_{\text{LM}}/\eta_{\text{UM}} = 30$ for models M1 (a), M2 (b) and M3 (c). The effect of three different values of the viscosity of the plate margins are shown: $\eta_{\text{PM}} = 0.3$ (red), 0.03 (blue) and 0.003 (green). The outcome of the 1-D viscosity model with no LVV in the lithosphere is plotted for comparison (black).

![Figure 5](image2.png)  
**Figure 5.** Effect of LVV in the lithosphere on the surface dynamic topography (panels a–c) and CMB dynamic topography (panels d–f) when $\eta_{\text{Lith}}/\eta_{\text{UM}} = \eta_{\text{LM}}/\eta_{\text{UM}} = 30$ for models M1 (panels a and d), M2 (panels b and e) and M3 (panels c and f). Colours are as in Fig. 4.
a–c) increases dramatically already when $\eta_{\text{PM}} = 0.3$ (red lines): by 120 m for model M1, by 450 m for model M2 and by 550 m for model M3. Further reduction of $\eta_{\text{PM}}$ by one order of magnitude causes the amplitude of the surface topography to roughly double in all three cases (cf. black and blue lines). When $\eta_{\text{PM}}$ is reduced to 0.003, the surface topography is only slightly changed with respect to the case $\eta_{\text{PM}} = 0.03$. While the CMB topography remains roughly constant for all values of $\eta_{\text{PM}}$, larger depressions of the surface dynamic topography result in a significant decrease of the overall geoid signal. In the following, for all the calculations where L VV in the upper and lower mantle are considered, weak plate margins in the lithosphere with $\eta_{\text{PM}} / \eta_{\text{Lith}} = 0.01$ will be included without explicit mention.

4.3 Lateral viscosity variations in the upper mantle

We now analyse the effects of L VV associated with the slab when this is located in the upper mantle (model M1). The geoid obtained from models where the slab is more viscous than the surrounding mantle is shown in Fig. 6. According to the viscosity contrast between the slab and surrounding mantle, colour lines refer to models that include L VV as follows: $\eta_{\text{Sl}} / \eta_{\text{M}} = 10$ (red), 50 (blue), 100 (green) and 500 (orange). Black lines correspond to models that do not have L VV associated with the slab but still incorporate low-viscosity plate margins (cf. blue line on Fig. 4a). The horizontal dashed line that appears on the bottom panels corresponds to the maximum geoid height observed over South America (see Fig. 1).

The presence of L VV in the upper mantle does not have a dramatic effect on the geoid. When upper and lower mantle have the same viscosity ($\eta_{\text{LM}} / \eta_{\text{UM}} = 1$) the geoid obtained for 1-D viscosity distributions is always negative, independently of the value of $\eta_{\text{Lith}} / \eta_{\text{UM}}$ and of the lateral viscosity contrast $\eta_{\text{Sl}} / \eta_{\text{M}}$. For values of $\eta_{\text{Sl}} / \eta_{\text{M}}$ smaller than 500, the differences between the geoid resulting from the 1-D viscosity model and the geoid obtained considering L VV are smaller than 4 m. The presence of L VV has the tendency to slightly decrease the geoid amplitude. This behaviour is opposite to that observed for $\eta_{\text{LM}} / \eta_{\text{UM}} \geq 30$. In this case, differently from what we will see when L VV in the lower mantle are considered (see Section 4.4), the higher is the viscosity of the slab, the lower is the (positive) geoid height generated over the subduction zone and the larger are the differences among outcomes when $\eta_{\text{Sl}} / \eta_{\text{M}}$ increases.

The amplitude of the geoid maximum over South America can be adequately fitted only if $\eta_{\text{LM}} / \eta_{\text{UM}} > 30$, $\eta_{\text{Lith}} / \eta_{\text{UM}} < 1000$ and if the slab is more viscous than the surrounding mantle.

4.4 Lateral viscosity variations in the lower mantle

In models M2 and M3, the slab penetrates the 670 km discontinuity and thickens in the lower mantle according to the viscosity

![Figure 6](https://example.com/figure6.png)

**Figure 6.** Effect of L VV in the upper mantle on the geoid for model M1. Black lines indicate the geoid obtained from models where no L VV are associated with the slab, while colour lines are plotted according to the viscosity contrast between slab and surrounding mantle: $\eta_{\text{Sl}} / \eta_{\text{M}} = 10$ (red), 50 (blue), 100 (green) and 500 (orange). In all models weak plate margins with $\eta_{\text{PM}} / \eta_{\text{Lith}} = 0.01$ are considered. The black dashed line corresponds to the geoid maximum of 24 m observed over South America (see Fig. 1).
ratio $\eta_{\text{LM}}/\eta_{\text{UM}}$ (see Section 3). Figs 7 and 8 illustrate the effects of LVV for these two models, with the black dashed line corresponding to the geoid maximum over the Tonga-Kermadec trench (see Fig. 1). As expected, the geoid amplitudes exhibited by model M3 are always larger than those of model M2. However, the two models give qualitatively very similar results if LVV are considered. When $\eta_{\text{LM}}/\eta_{\text{UM}} = 1$, the geoid predicted with LVV is rather similar in shape to that obtained without LVV, provided that the viscosity of the slab is not too high ($\eta_{\text{Sl}}/\eta_{\text{M}} \leq 100$). For $\eta_{\text{Sl}}/\eta_{\text{UM}} \leq 100$ and $\eta_{\text{Sl}}/\eta_{\text{M}} = 500$, the amplitude of the negative geoid is strongly reduced. The pronounced minimum, which was the main characteristic of the signal predicted for $\eta_{\text{LM}}/\eta_{\text{UM}} = 1$, may be even replaced by a significant elevation in some cases. If $\eta_{\text{LM}}/\eta_{\text{UM}} = 10$, the effects of LVV are more evident. The geoid obtained with 1-D viscosity structures is appreciably affected by the presence of LVV even if the slab is only weakly more viscous than the surrounding mantle. In particular, it is worth noticing that, for model M3, already when $\eta_{\text{Sl}}/\eta_{\text{M}} = 50$ (blue lines) the geoid is positive also for high values of $\eta_{\text{Sl}}/\eta_{\text{M}}$, which are associated with local geoid minima if $\eta_{\text{Sl}}/\eta_{\text{M}} = 1$. When $\eta_{\text{LM}}/\eta_{\text{UM}} \geq 30$, lower mantle LVV cause the geoid to systematically increase with $\eta_{\text{Sl}}/\eta_{\text{M}}$. The effect exerted by a highly viscous slab on the geoid is extremely significant. For example, for model M2, if $\eta_{\text{Sl}}/\eta_{\text{UM}} = \eta_{\text{LM}}/\eta_{\text{UM}} = 30$, the geoid high obtained with the radially dependent viscosity model roughly doubles with a viscosity contrast $\eta_{\text{Sl}}/\eta_{\text{M}} = 50$. For model M3, this already happens when $\eta_{\text{Sl}}/\eta_{\text{M}} = 10$. For higher values of $\eta_{\text{Sl}}/\eta_{\text{M}}$ or $\eta_{\text{LM}}/\eta_{\text{UM}}$, both models are characterized by extremely large geoid amplitudes which clearly exceed the observed maximum over Tonga region and, more generally, the values typically observed over subduction zones.

We discuss now how the dynamic topography and the flow pattern are influenced by the presence of LVV in the lower mantle. In Figs 9 and 10, for models M2 and M3, respectively, we show on the top panels the dynamic topography of the surface (solid lines) and of the CMB (dashed lines) for a mantle with radially symmetric viscosity (panels a), with LVV due to a slab/mantle viscosity contrast $\eta_{\text{Sl}}/\eta_{\text{M}} = 10$ (panels b) and $\eta_{\text{Sl}}/\eta_{\text{M}} = 100$ (panels c). In both figures, the viscosity stratification is $\eta_{\text{Sl}}/\eta_{\text{UM}} = \eta_{\text{LM}}/\eta_{\text{UM}} = 30$. Both models are characterized by a flow whose vigour is strongly reduced by the presence of LVV. For models M2 (Fig. 9), the surface and CMB dynamic topographies have similar amplitudes, which decrease as the ratio $\eta_{\text{Sl}}/\eta_{\text{M}}$ increases. For models M3, this behaviour applies only to the surface topography, while, interestingly, the CMB topography first increases when $\eta_{\text{Sl}}/\eta_{\text{M}} = 10$ and then decreases when $\eta_{\text{Sl}}/\eta_{\text{M}} = 100$. Negative surface and CMB topographies, as those observed in these cases, both contribute to the total geoid as negative density anomalies. Their reduction is thus responsible for the strongly positive geoid heights obtained for models with LVV (Figs 7 and 8). The flow patterns in Figs 9 and 10 also show that, especially when LVV associated with the slab are not considered (panel a), the plate subducts nearly vertically and is little affected by the dip angle. Being the surface free-slip, it is inevitable to
obtain a double-sided subduction with the overriding plate moving at a speed similar to that of the subducting plate (or even slightly higher). Nevertheless, as the surface dynamic topography is practically symmetric with respect to the trench, our results are little affected by the relative motion of the two plates.

4.5 Lateral viscosity variations in the D$	ext{''}$

As illustrated in Section 4.3, even moderate viscosity contrasts between slab and surrounding mantle are sufficient to generate very high geoid elevations, especially if the slab reaches the lowermost...
mantle (model M3). As shown by Tosi et al. (2009), the phase transformation from perovskite to post-perovskite at the bottom of the slab can reduce geoid amplitudes if it is accompanied by a decrease in viscosity. In Fig. 11, we show the same results as in Fig. 8 obtained for \( \eta_{\text{Lith}}/\eta_{\text{UM}} = 30 \) and \( \eta_{\text{DL}}/\eta_{\text{M}} = 10 \) (red lines) and 100 (green lines) and compare them with the geoid signals predicted for the same viscosity structure but with a 200-km-thick and 3000-km-wide low-viscosity post-perovskite area centred at the bottom of the slab (see Fig. 2). Black lines denote again the geoid obtain from the 1-D viscosity model. In Fig. 11(a), dashed lines result from calculations in which the viscosity of post-perovskite equals that of the upper mantle, that is, \( \eta_{\text{PPv}}/\eta_{\text{UM}} = 1 \), while the lower value of \( \eta_{\text{PPv}}/\eta_{\text{UM}} = 0.1 \) is used in Fig. 11(b). When \( \eta_{\text{LS}}/\eta_{\text{M}} = 10 \), the effect of the low-viscosity post-perovskite area is dramatic. Including post-perovskite helps to reduce the geoid high from 121 to 42 m when \( \eta_{\text{PPv}}/\eta_{\text{UM}} = 1 \) (panel a, dashed red line) and to 31 m when \( \eta_{\text{PPv}}/\eta_{\text{UM}} = 0.1 \) (panel b, dashed red line). These two values are even lower than the reference geoid high of 48 m obtained considering no LVV beneath the lithosphere (black lines). The effect of post-perovskite is also evident when \( \eta_{\text{LS}}/\eta_{\text{M}} = 100 \) as it causes the geoid to reduce from 205 to 171 m when \( \eta_{\text{PPv}}/\eta_{\text{UM}} = 1 \) (panel a, dashed green line) and to 169 m when \( \eta_{\text{PPv}}/\eta_{\text{UM}} = 0.1 \) (panel b, dashed green line). In this case, however, the effect is not sufficient to decrease the geoid amplitude to a value comparable with the observation over the Tonga trench (black dashed line).

5 DISCUSSION

The first set of results described in the previous sections is based on a radially symmetric viscosity distribution whose effects on the geoid were extensively studied in the past. Our results (Fig. 3) essentially confirm that a viscosity contrast larger than a factor of 10 between upper and lower mantle is necessary for reproducing the positive geoid observed over subduction zones. The optimum value for the ratio \( \eta_{\text{LM}}/\eta_{\text{UM}} \) lies between 30 and 100 (Ricard et al. 1993; Corrieu et al. 1995). Unless \( \eta_{\text{Lith}}/\eta_{\text{UM}} = 1000 \), the geoid that we obtain for models M2 and M3 and \( \eta_{\text{LM}}/\eta_{\text{UM}} = 100 \) is generally too high when compared with the observations, although there are studies suggesting that even larger values of this ratio can guarantee a good fit to the data (Kido & Čadek 1997; Steinberger 2000; Mitrovica & Forte 2004; Yoshida & Nakakuki 2009).

Geoid amplitudes resulting from model M1 are always relatively low. For all the viscosity stratifications tested they reach a maximum of 30 m when \( \eta_{\text{LM}}/\eta_{\text{UM}} = 100 \). This amplitude is much smaller than those generated when the slab penetrates the lower mantle. It is then reasonable to hypothesize that the most pronounced geoid
highs correlated which subduction zones, like those in the West Pacific, indicate slabs subducted in the lower mantle. Nevertheless, it should be noted that using a larger value of the density anomaly in the upper mantle or considering additional density anomalies due, for example, to a slab stagnating in the transition zone (Fukao et al. 2001) could result in larger amplitudes of the geoid even if the slab does not sink into the lower mantle.

No matter whether LVV are present or not, there are also a few other factors that may influence our results. Our models lack a low viscosity asthenosphere (Dumoulin et al. 1999; Capitanio et al. 2007; Krien & Fleitout 2008). A weak layer underlaying the stiff lithosphere would shift the support of the slab from the CMB to the surface, increase the surface dynamic topography and hence reduce the amplitude of the geoid even in the presence of large viscosity contrasts between upper and lower mantle. It is well known that geoid predictions can be affected by the choice of the boundary conditions (Thoraval & Richards 1997). Significant changes of the dynamic topography could be obtained by considering kinematic boundary conditions at the surface or mantle layering (Wen & Anderson 1997b; Čadek & Fleitout 1999). In the latter case, lateral density anomalies due to the deformation of the phase transitions boundaries may decrease the overall slab-pull (and hence the amplitude of the dynamic topography) while simultaneously providing an additional contribution to the static geoid with relatively little effect on the total geoid. Accounting for slab-suction (Conrad & Lithgow-Bertelloni 2002; Conrad & Lithgow-Bertelloni 2004) by disconnecting the portion of the slab in the lower mantle from that in the upper mantle could also induce important changes in the computed geoid.

As shown in Fig. 4, the geoid is very sensitive to the presence of LVV in the lithosphere when these are associated with a decrease in viscosity at the plate boundaries. Weak plate margins have a large effect on the surface dynamic topography (Fig. 5) whose amplitude increases as soon as the viscosity \( \eta_{PM} \) is reduced. As a consequence, the negative dynamic component of the geoid is enhanced and the total geoid diminishes. By employing a very simple parametrization to mimic plate margins, our calculations show that, for a selected viscosity stratification, a viscosity contrast \( \eta_{PM}/\eta_{LM} \) of two order of magnitude is already sufficient to halve the geoid maximum obtained for the three slab models considered. Since the effect of LVV in the lithosphere can be very significant (Karpychev & Fleitout 1996; Wen & Anderson 1997b; Chen & King 1998), as it was recently done by Yoshida & Nakakuki (2009), low viscosity plate boundaries should also be included in 3-D spherical models used to predict the geoid at global scale.

When LVV are solely present in the upper mantle (Fig. 6), they do not affect the geoid dramatically. However, numerical models (Běhoučková & Čižková 2008) based on fairly well-constrained rheological parameters (Karato & Wu 1993) and analogue models of subduction (Funicello et al. 2008) indicate the upper mantle as the region where most likely the viscosity of slabs is higher than the ambient viscosity by several orders of magnitude. The fact that the geoid is only moderately influenced by the stiff slab in the upper mantle indicates that the mechanical coupling between the slab and the lithospheric plate at the surface, enhanced by the high viscosity of the slab, is not large enough to significantly change the surface deformation. Therefore, even though slabs are most probably very stiff in the upper mantle, neglecting their stiffness does not introduce a large error when a fit to the long-wavelength geoid is sought for.

When lower mantle slab models M2 and M3 are considered (Figs 7 and 8), the effect of LVV is much more evident. Highly viscous slabs in the lower mantle systematically increase the amplitude of the geoid. On the one hand, when the viscosity contrast between upper and lower mantle is only moderate and the 1-D viscosity models generate a negative geoid over the subduction zone, LVV in the lower mantle are capable to revert the sign of the geoid and to produce amplitudes of a few tens of metres which are comparable with the observations. On the other hand, when the geoid arising from the 1-D viscosity model is already positive, lower mantle LVV cause its amplitude to further increase. Unless the viscosity contrast between slab and surrounding mantle is relatively low (of a factor of 50 at most), the amplitudes considerably exceed those typically observed over convergent plate margins. These findings closely confirm the recent results of Yoshida & Nakakuki (2009) who also obtain very large amplitudes of the geoid over subduction zones (up to 200 m) when deep and highly viscous slabs are taken into account, although their viscosity stratification is different from ours, with higher values for the viscosity of both the lithosphere and lower mantle. The results of Zhong & Davies (1999) represent a further confirmation. They also conclude that slabs of high viscosity in the lower mantle have the tendency to generate strongly positive geoid anomalies and to deteriorate the fit to the observed geoid.

In one of the first papers devoted to the geoid induced by subducting slabs, Hager (1984) discusses the physical response of a radially stratified mantle to an internal load, showing that the sign and amplitude of the geoid strongly depend on the dynamic support of the internal density heterogeneity by surface and CMB deflections. If the viscosity of the lower mantle is higher than that of the upper mantle, the weaker upper mantle causes the weight of the slab to be transferred from the surface to the CMB. As a consequence, the surface deflection induced by the slab is shallower and the total geoid becomes larger. A similar argument may also be used in our case. The presence of a highly viscous and thick slab in the lower mantle increases the average viscosity of the lower mantle with respect to the upper mantle. The geoid amplitude is then larger than in the case where the slab has the same viscosity as the surrounding mantle. However, the reasoning based on the average viscosity is likely oversimplified and it fails when a weak plate boundary is imposed inside the lithosphere (see Section 4.2). In this case, the average viscosity of the lithosphere slightly decreases but the surface topography deepens. In fact, the low viscosity region prescribed at the trench favours the decoupling of the subducting and overriding plates and the total geoid diminishes. The behaviour of the system in this case is counterintuitive which indicates that the application of simple rules may be misleading if the viscosity structure is complex. For instance, Hager (1984) expected that prediction of the positive geoid would require even higher viscosity for the lower mantle than that determined for his layered viscosity model if the slab had a high effective viscosity. The results presented in our study suggest the opposite. To obtain reasonable amplitudes of the geoid we need a lower mantle viscosity smaller than that usually obtained from inversions of the geoid. A positive amplitude of the geoid can be obtained even for a model with \( \eta_{LM}/\eta_{UM} = 1 \) (Figs 7 and 8).

The reason why the geoid amplitude increases so dramatically with increasing the viscosity of the slab can be associated with the fact that the long-wavelength flow becomes less vigorous with increasing the lateral viscosity contrast between the slab and the mantle. The presence of LVV causes the density anomaly associated with the slab to have a smaller impact on the flow of the rest of the mantle than in the case without LVV. The slab cannot bend or buckle and the flow locks up in a relatively narrow region around the slab. The deformation becomes more localized so that short-wavelengths are enhanced. The presence of a highly viscous slab

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slabs are indeed everywhere much stiffer \((\eta)\) of temperature on viscosity (Ito & Sato 1991) causing the slab function of temperature but also of grain size (Riedel & Karato Chen & King 1998). Since the viscosity of olivine is not only weakened as they enter the lower mantle (Moresi & Gurnis 1996; the observations. This suggests that slabs may be progressively weakened as they enter the lower mantle (Moresi & Gurnis 1996; Chen & King 1998). Since the viscosity of olivine is not only a function of temperature but also of grain size (Riedel & Karato 1997), grain size reduction at depth could counteract the effects of temperature on viscosity (Ito & Sato 1991) causing the slab to weaken while sinking deeper than the transition zone. Another possibility is that the lower mantle is only a little more viscous than the upper mantle (approximately by a factor of 10) and that slabs are indeed everywhere much stiffer \((\eta_{LM}/\eta_{UM} > 100)\) than the surroundings. Radially symmetric viscosity models produce in this case a negative geoid whose sign can be reverted thanks to the presence of highly viscous slabs in the lower mantle. However, a low value of the ratio \(\eta_{LM}/\eta_{UM}\) is generally not supported by most geoid inversions. We have also shown that both a large value of the viscosity of the lithosphere and the presence of weak plate margins cause the geoid to decrease. As recently discussed by Yoshida & Nakakuki (2009), this is important because it may contribute to build geoid models that better constrain the viscosity of the lithosphere. This would also allow us to overcome the somewhat inconsistent outcome of most global geoid models according to which very low values of \(\eta_{L\text{inh}}\) are necessary to explain the geoid (e.g. Ricard et al. 1993).

Another option can be pictured on the basis of the results shown in Fig. 11. Very deep slabs can retain their high viscosity character also in the lower mantle while, at the same time, the phase transformation from perovskite to post-perovskite causes the viscosity of the slab to decrease in the D'' region. Such a viscosity change can increase the slab pull and hence decrease the amplitude of the geoid (Tosi et al. 2009). Nevertheless, it should be pointed out that other chemical heterogeneities which are expected to be present in the lower mantle (e.g. Trampert et al. 2004) could also have a significant impact on the geoid prediction.

On the other hand, our assumption concerning the density distribution associated with the slab might need to be revised. Lower geoid amplitudes in the presence of very stiff slabs could be obtained if the slab conserved its thickness throughout the mantle, that is, if the density anomaly in the lower mantle was significantly smaller than the one prescribed in our models. However this hypothesis seems to be unlikely in view of tomographic studies whose resolution is sufficiently high to guarantee that the blob-like character observed for subducted slabs in the lower mantle is not an artefact of the tomographic inversion (Káráson & van der Hilst 2001). A final possibility, which goes in the direction of relatively weak slabs in the lower mantle, points towards an intermediate case in which the slab thickens after penetrating the transition zone according to tomographic images but also warms up, becoming less dense as it sinks deeper into the lower mantle and preserving only a relatively thin highly viscous core which would significantly diminish the impact of LVV.

### 6 CONCLUSIONS

A flow model in spherical axisymmetric geometry has been used to investigate the long-wavelength geoid induced by a typical subduction, with the aim of predicting the characteristic broad highs observed over major subduction zones and estimating the role played by localized LVV. Several high-resolution density and viscosity structures have been systematically analysed. As far as the mantle radial viscosity structure is concerned, our case study confirms that the viscosity contrast between upper and lower mantle is responsible for a first order effect in geoid predictions. A lower mantle approximately 30-times more viscous than the upper mantle is necessary to ensure a positive geoid signal of realistic amplitude if no LVV are included. Concerning LVV, we have discussed their role according to whether they are present in the lithosphere, in the upper mantle or in the lower mantle and showed systematic trends in the way they affect the geoid. The presence of highly viscous slabs in the lower mantle may have a dramatic effect on the geoid prediction, similar to that produced by the radial changes in viscosity. If LVV are present only in the upper mantle, their impact is less significant. According to the assumption and simplifications of our models, several possibilities concerning the distribution of slab density and viscosity have been proposed which can represent a starting point for a systematic study in 3-D spherical geometry.

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