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Ensemble Kalman-Filtering of Earth rotation observations with a global ocean model

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Abstract

We study the changes in ocean angular momentum which lead to changes in the Earth’s rotation. The focus lies on the consistency of model, model-forcing and observations. To check this consistency an ensemble-based Kalman-Filter approach was applied to ocean angular momentum time series which we derived from Earth rotation observations. The filter propagates a reduced rank error-covariance matrix with an optimal ensemble. This way, a complex system like the utilized global ocean circulation model can be assimilated with 32 ensemble members only. The Kalman-Filter improved the ocean-model’s trajectory with respect to the observations, i.e., the observed ocean angular momentum was better reproduced by our model. A subsequent analysis of the changes which were induced by the filter revealed that the utilized atmospheric forcing is insufficient. When the filter is not entitled to change the forcing fields no improvement in the model trajectory was possible.

Keywords: Earth rotation, data assimilation, ocean modelling, freshwater-flux

1. Introduction

Data assimilation is a powerful and well used tool in the fields of ocean and atmosphere sciences. Assimilation combines model dynamics with informations obtained by observation. In the case of Earth rotation observations the studies using data assimilation are still rare. Ponte et al. (2001) and Chen (2005) show that the assimilation of oceanic and hydrographic data results in better Earth Rotation Parameter (ERP) estimates of the respective models. Saynisch et al. (2011a,b) assimilated observations of ERP with an adjoint 4D-VAR method and a very basic and coarse resolution ocean model. This way, the distinction of current and mass change induced angular momentum anomalies became possible and the following processes could be linked to oceanic Earth rotation excitations. Oceanic length of day excitation could be attributed to total ocean mass change

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and oceanic polar motion excitation could be linked to geostrophic currents dominated by atmospheric momentum forcing (Saynisch et al., 2011a). Furthermore, this approach can help to solve the problem that estimations of ocean angular momentum based on ocean circulation models are often much smaller than estimations based on observations (Chen, 2005; Ponte and Stammer, 2000). Since oceanic angular momentum is closely connected to the atmospheric forcing the assimilation of ERP can reveal problems in the latter (see e.g., Masaki, 2008; Trenberth et al., 2011).

The purpose of our current study is to continue the work of Saynisch et al. (2011a,b). We want to reproduce and refine the findings therein with a more realistic, complex and higher-resolution global ocean model. Furthermore, we want to extend the mostly seasonal results to shorter, i.e., daily, time-scales. The adjoint method needs a huge amount of preliminary work, i.e., the derivation of the adjoint code, when applied to a new model. For this reason we use an ensemble based Kalman-Filter approach in the recent study. In contrast to 4d-VAR a Kalman-Filter can be flexibly adapted to new and advanced models.

The structure of the paper is as follows. The next section introduces basic Earth rotation theory. The methodology section bundles the descriptions of the utilized ocean model (3.1), the used data and its processing (3.2) and gives a description of the used Kalman-Filter approach (3.3). The study’s results are presented and discussed in Section 4 and we close with a conclusion in the last section.

2. Earth rotation theory

The rotation of the Earth is not uniform. Deviations in the angular velocity vector arise from external torques as well as from mass movements in the Earth system. In this paper we consider only the internal changes. On time-scales of a few years and longer the Earth’s mantle and core contribute the most to these internally-caused deviations (Gross et al., 2005; Pais and Hulot, 2000). On annual and seasonal time scales the atmosphere and the oceans play the major role. While anomalies of Earth’s revolution speed are dominated by the atmosphere (Rosen and Salstein, 1991), the polar motion is excited to equal extent by the atmosphere and the oceans (Gross, 2007). Changes in the ocean-mass distribution as well as mass movements relative to the terrestrial reference frame can lead, among other polar motion (PM) signals, to the excitation of the damped free wobble of the Earth. This phenomenon is called the Chandler Wobble (Lambeck, 1980) and has a period of 433.1 sidereal days (Vicente and Wilson, 1997). In addition, changes in the revolution speed of the Earth can arise. These are equivalent to the so-called excess of length of day (LOD). All these excitations can be stated in the form of the following inhomogeneous
partial differential equation (Munk and MacDonald, 1960):

\begin{align}
\chi_1 &= x_p + \frac{\dot{y}_p}{\sigma_{ch}} \\
\chi_2 &= y_p - \frac{\dot{x}_p}{\sigma_{ch}} \\
\chi_3 &= \frac{\Delta \Lambda}{\Lambda_0}
\end{align}

The \(x_p\) and \(y_p\) are the coordinates of the Celestial Intermediate Pole (CIP) in the International Terrestrial Reference Frame (ITRF, Altamimi et al., 2011). Here \(x_p\) points along the Greenwich meridian and \(y_p\) is positive towards 90°W (compare e.g., Gross, 1992). The deviations of Earth’s true rotational period compared to a sidereal day \(\Lambda_0\) (86,400 s) is given by \(\Delta \Lambda\). Together \(x_p\), \(y_p\) and \(\Delta \Lambda\) are called ERP. The \(\sigma_{ch}\) is the complex valued frequency of the Chandler Wobble which is excited, i.e., forced, by the differential equation’s inhomogeneities \(\chi_1\) and \(\chi_2\). The changes of LOD are forced by \(\chi_3\). The \(\chi_{1,2,3}\) itself are called angular momentum functions and combine angular momentum anomalies of the respective Earth system (see Eq. (4)-(6) in Section 3.1).

3. Methodology

3.1. Ocean model

In our study we use the Ocean Model for Circulation and Tides (OMCT, Dobslaw and Thomas, 2007). It is an ocean global circulation model (OGCM) with a resolution of 1.875 degrees in the horizontal and 13 layers in the vertical. The time step is 30 minutes. The model is forced with wind-stress, heat-flux, surface pressure, precipitation and evaporation. These forcings base on 6-hourly products from the European Centre for Weather Forecasts (ECMWF, Uppala et al., 2008). The OMCT is furthermore forced with river-runoff from the Land Surface Discharge Model (LSDM, Dill, 2008). Consistently, the LSDM was also forced by the ECMWF data. All freshwater-fluxes change sea surface height and therefore ocean mass at the respective input point. The correction of artificial mass change due to the Boussinesq-approximation follows Greatbatch (1994). The OMCT conserves mass and is well suited for studies of mass transport and displacement, e.g., GRACE de-aliasing (Wunsch et al., 2001).

During the ocean model simulation the effective angular momentum functions (Barnes et al., 1983) are calculated according to the conventions of the International Earth Rotation and Reference Systems Service (IERS, Petit and Luzum, 2010):

\begin{align}
\chi_1(t) &= \frac{1.10 L_{x,\text{mass}}(t) + 1.62 L_{x,\text{motion}}(t)}{\Omega(C - A)} \\
\chi_2(t) &= \frac{1.10 L_{y,\text{mass}}(t) + 1.62 L_{y,\text{motion}}(t)}{\Omega(C - A)} \\
\chi_3(t) &= \frac{0.77 L_{z,\text{mass}}(t) + 1.13 L_{z,\text{motion}}(t)}{\Omega C}
\end{align}
Where the absolute value of Earth’s mean angular velocity is one revolution per sidereal day, i.e., $\Omega$. The $L$ represent globally integrated angular momentum anomalies due to motions relative to Earth’s rotating reference frame ($L_{\text{motion}}$) and due to changes in mass distribution ($L_{\text{mass}}$). The principal moments of inertia of the solid Earth are given by $A$ and $C$. Note that core-mantle decoupling is included in the pre-factors. Calculated this way the modeled ocean angular momentum (OAM) functions are consistent with the products of angular momentum from atmosphere and hydrology which we used for the pre-processing in Section 3.2 (Dobslaw et al., 2010).

To evaluate the SEIK-induced changes we made a non-assimilation simulation with the OMCT which we call hereafter the reference-simulation. The initial state of all simulations was the result of a spin-up process described in Dobslaw and Thomas (2007).

### 3.2. Data and processing

Before the assimilation the ocean angular momentum observations had to be estimated from Earth rotation observations. For this we used the daily values of the combined ERP product from the IERS (C04 08, Bizouard and Gambis, 2009). The ERP-observations were transformed into angular momentum functions following the procedure given in Gross (1992). Afterwards, the operational angular momentum functions from land-hydrology (HAM) and atmosphere (AAM) of Dobslaw et al. (2010) were subtracted. Consistent with the forcing of our ocean model (see Section 3.1) the subtracted momentum functions also base on ECMWF’s data and ECMWF-driven LSDM-simulations. A final high-pass filter removed decadal and inter-annual signals from the residual OAM-observations. This is necessary since these time scales are supposed to be dominated by the processes in the Earth’s mantle and core (Gross et al., 2005; Pais and Hulot, 2000).

For the assimilation process we need to determine an error-covariance matrix of the observations (compare Section 3.3). Since the ERP observations are very precise (Gambis, 2004) the errors of the ERP-derived OAM arise mainly during the subsequent projection onto the ocean. The AAM have the largest contribution to this projection and are at the same time very poorly estimated by the atmospheric reanalyses (Hagemann et al., 2005; Trenberth et al., 2011). Therefore, we estimate errors of the ERP-derived OAM-observations through the errors of the modeled AAM we used in the reduction. These AAM-errors were derived in the following way. We calculate pairwise differences between the various, high-pass filtered (see Section 3.2), AAM-products available from the IERS, i.e., global products from ECMWF, from the National Centers for Environmental Prediction (NCEP), from the Japan Meteorological Agency (JMA) and from the British Met Office. The $3 \times 3$ covariances of these differences were calculated and a mean error-covariance matrix was derived:

$$
\mathbf{W}_{\text{AAM}} = \begin{pmatrix}
1.2 \times 10^{-15} & 3.8 \times 10^{-16} & -6.3 \times 10^{-18} \\
3.8 \times 10^{-16} & 3.1 \times 10^{-15} & -3.3 \times 10^{-17} \\
-6.3 \times 10^{-18} & -3.3 \times 10^{-17} & 5.0 \times 10^{-19}
\end{pmatrix}
$$

(7)
For comparison, this estimation gives a somewhat larger (esp. in $\chi_1$ and $\chi_2$) error-covariance than using ECMWF and NCEP alone:

$$
W_{ECMWF-NCEP} = \begin{pmatrix}
4.3 \times 10^{-16} & -1.8 \times 10^{-17} & 4.5 \times 10^{-18} \\
-1.8 \times 10^{-17} & 4.6 \times 10^{-16} & -3.0 \times 10^{-18} \\
4.5 \times 10^{-18} & -2.9 \times 10^{-18} & 4.0 \times 10^{-19}
\end{pmatrix}
$$

(8)

For computational reasons we could only assimilate seven months. We choose the time period January till July of 2003. Nonetheless we estimated OAM-observations from 1998 to 2010 to reduce the errors that the mentioned high-pass filter produces near the time series boundaries.

3.3. Kalman-Filter

The Kalman-Filter was introduced by Kalman (1960). At model time steps where observations are available the Kalman-Filter (KF) merges the observations ($y^o$) and the model-forecast ($x^f$) into a new analysis state ($x^a$):

$$
x^a = x^f + K(y^o - H[x^f])
$$

(9)

Hereby the observation operator $H$ is used to translate a model state $x$ into the observable properties $y$. The Kalman-Gain $K$ is used to control the merging with respect to error-informations:

$$
K = \frac{P^f H^T}{HP^f H^T + R}
$$

(10)

Here, $H^T$ is the transposed observation operator, $R$ is the error-covariance-matrix of the observations and $P^f$ describes the error-covariance of the model. The latter can be obtained in various ways. Evensen (1994) proposed to use an ensemble of model states to propagate an initial $P$ in time. The variance of the ensemble is then used as the error-covariance of the model $P^f$ and the ensemble-mean can be used as model state $x^f$. This famous Ensemble Kalman-Filter (EnKF) frees the original KF-technique of its limitation to sufficiently linear models. Nonetheless, since the ensemble has to represent the models entire error-covariance this leads to very impractically large ensembles. In our study we use the Singular Evolutive Interpolative Kalman-Filter (SEIK) of Pham et al. (1998). The SEIK filter is a variant of the EnKF where the ensemble is used to represent the models error-covariance in a truncated sense. Only the leading orders of the eigenvalue-decomposition of $P^f$ are considered:

$$
P^f \approx V U V^T
$$

(11)

The diagonal $r \times r$ matrix $U$ contains the leading $r$ eigen-values of $P^f$. The columns of the matrix $V$ contain the corresponding $r$ eigen-vectors. The resulting ensemble size is $r + 1$ and therefore much smaller than an EnKF ensemble of comparable performance (Nerger et al., 2007). The detailed formalism of and many useful comments on reduced rank Kalman-Filters can be found in Nerger et al. (2005).
To implement the SEIK-Filter in our ocean model we used the Parallel Data Assimilation Framework (PDAF, http://pdaf.awi.de/) by Lars Nerger. PDAF allows the coupling of a model-code to a variety of Kalman-Filters. Since PDAF is designed as a library of external routines the changes that have to be made to the model-code are few.

To derive an initial OGCM-ensemble we utilized the reference-simulation. The first time step (1st Jan. 2003) of the reference-simulation became the mean of the initial SEIK-ensemble. The temporal covariance of the whole January of the reference-simulation was reproduced in a low-rank, 2nd-order exact sense by the initial SEIK-ensemble. We follow the procedure of Pham (2001). This way we have a reasonable variation in the initial ensemble distribution. This variation becomes then propagated by the model dynamics to generate a corresponding range of model states at every time step. This range is used by the SEIK-Filter to estimate the modes error covariance-matrix at every observation step.

Since an ensemble forecast is a very CPU-intensive operation we had to save as much CPU-hours as possible. CPU-hours are linear in ensemble size and simulation period. Test simulations showed a saturation of the SEIK-Filters improvement for 32 ensemble members, i.e., further simulations with 64 and 128 ensemble members made only small enhancements with respect to the reproduction of OAM-observations (not shown). As already mentioned the time frame of our study had to be very small, seven months of 2003. The SEIK-Filter process shows some initial but transient oscillations. Therefore, we start the filter in January but focus our analysis on the six months from February till July.

4. Results and Discussion

Applying the described SEIK-Filter to the ocean model alone brought no satisfying results (see Fig. 1). The ocean model trajectory is only affected very little by the filter process. The rms-differences between model trajectory and observations are:

\[
\begin{pmatrix}
1.9 \times 10^{-7} \\
2.1 \times 10^{-7} \\
1.3 \times 10^{-9}
\end{pmatrix}
\]

The reason for this is simple. The angular momentum of the ocean depends to a large extent on the forcing (Saynisch et al., 2011a,b). To account for this we included parts of the atmospheric forcing in the control-vector of the SEIK-Filter. In this next experiment, every ensemble member was attributed to a unique forcing field of freshwater-flux and wind-drag. During the model simulation these additional forcing fields (AFF) are added at every time step to the forcing fields from ECMWF. In this way, every ensemble member can have a slightly different forcing. The ensemble-covariance of the AFF equals, again in the truncated sense, the temporal covariance matrix of the January reference-simulations forcing. The AFF ensemble-mean is initially zero. Since
Figure 1: Angular momentum functions of the ocean. Red: SEIK-simulation (ensemble mean); Gray: Ensemble spread; Black: reference simulation; Dots: OAM-observations.
these AFF are a part of the model state but are not changed by the models equations they remain unchanged between two observations, i.e., assimilation steps. For the same reason the AAF ensemble-covariance is not changed during the whole SEIK-simulation, i.e., the forcing error is assumed to be constant. During the SEIK update step the AFF are treated as part of the model state and are updated once a day. Therefore, the AFF ensemble-mean can change during the SEIK-simulation and influence the ocean state-estimate. In this way differences in, e.g., total ocean mass between the reference-simulation and the SEIK-simulation become possible. With the inclusion of the AFF in the control vector the rms-values reduce substantially to:

\[
\begin{pmatrix}
1.1 \times 10^{-7} \\
1.3 \times 10^{-7} \\
7.8 \times 10^{-10}
\end{pmatrix}
\]

These are approximately 60% of the values given in (12). On the other hand the correlation improves only little (e.g., from 0.3 to 0.5 for \( \chi_1 \)). Together with the rms reduction this is a hint that the changes due to the AFF improve the mean of the models time series and not their rapid temporal evolution (compare Fig.2). A close look at the AFF is necessary. There is a clear annual signal in the freshwater budget arising from the AFF. Figure 3 shows the ensemble-mean of the change in total ocean mass due to the AAF fields. The small and rapid changes of the OAM-observations are not visible here. We argue, that this is because the estimated error-budget of the observations is large. Choosing a smaller error-covariance matrix for the observation could reproduce the OAM-observations more closely (not shown). But this should be interpreted as over-fitting since that respective error-covariance matrix has to be smaller than \( W_{ECMWF\text{-}NCEP} \) (Eq. (8)). Why is the annual OMCT mass change too weak in the reference-simulation? Since the incorporated hydrology model shows a reasonable seasonal cycle (see, Saynisch et al., 2011b) and the ocean mass can only change due to external forcing we argue that the seasonal amplitude of the ECMWF forcing fields, at least over the ocean, is too small. This finding is consistent with Saynisch et al. (2011a,b), Trenberth et al. (2007) and Hagemann and Gates (2001).

5. Conclusions

To reproduce estimated ocean angular momentum observations we applied a low-rank Kalman-Filter to an OGCM-ensemble. When the atmospheric forcing was not allowed to change during the filters update step the filter had only little influence on the modeled ocean angular momentum. When the atmospheric forcing is included in the filter-process the agreement between observation and model could be enhanced. The corresponding rms-values show a 40% decrease. The filter-induced changes in the atmospheric forcing fields showed a strong annual signal. This hints to a problem with the mass budget of the utilized atmospheric reanalysis fields.
Figure 2: Angular momentum functions of the ocean. Atmospheric wind stress and freshwater-flux was allowed to be changed by the SEIK-Filter. Red: SEIK-simulation (ensemble mean); Gray: Ensemble spread; Black: reference simulation; Dots: OAM-observations.
Figure 3: Ocean mass change due to additional freshwater-flux (ensemble mean).
To study the annual cycle closer and include interannual variations in the future we want to extend the assimilation period to a few years.

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