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Comparison of deterministic and stochastic earthquake simulators for fault interactions in the Lower Rhine Embayment, Germany

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SUMMARY

Time-dependent probabilistic seismic hazard assessment requires a stochastic description of earthquake occurrences. While short-term seismicity models are well-constrained by observations, the recurrences of characteristic on-fault earthquakes are only derived from theoretical considerations, uncertain palaeo-events or proxy data. Despite the involved uncertainties and complexity, simple statistical models for a quasi-period recurrence of on-fault events are implemented in seismic hazard assessments. To test the applicability of statistical models, such as the Brownian relaxation oscillator or the stress release model, we perform a systematic comparison with deterministic simulations based on rate- and state-dependent friction, high-resolution representations of fault systems and quasi-dynamic rupture propagation. For the specific fault network of the Lower Rhine Embayment, Germany, we run both stochastic and deterministic model simulations based on the same fault geometries and stress interactions. Our results indicate that the stochastic simulators are able to reproduce the first-order characteristics of the major earthquakes on isolated faults as well as for coupled faults with moderate stress interactions. However, we find that all tested statistical models fail to reproduce the characteristics of strongly coupled faults, because multisegment rupturing resulting from a spatiotemporally correlated stress field is underestimated in the stochastic simulators. Our results suggest that stochastic models have to be extended by multirupture probability distributions to provide more reliable results.

Key words: Earthquake interaction, forecasting, and prediction; Seismicity and tectonics; Statistical seismology.

1 INTRODUCTION

Seismic risk and hazard estimates are mostly based on purely stochastic models of earthquake fault systems tuned specifically to vulnerable areas of interest. Most frequently applied is a time-independent Poisson process as earthquake occurrence model. It assumes completely uncorrelated constant earthquake activity, which can typically approximate regional seismicity after removing of aftershocks. Accounting additionally for short-term earthquake clustering, the epidemic-type aftershock sequence (ETAS) model, introduced by Ogata (1988, 1998), has been shown to reproduce the regional seismicity patterns very well on short to intermediate timescales (Marzocchi & Lombardi 2009; Woessner et al. 2011; Zhuang et al. 2012). However, for specific faults, quasi-periodic recurrences of characteristic on-fault events are often assumed based on the classical elastic rebound theory (Reid 1911). Despite missing direct observational evidence from the short instrumental catalogues, quasi-periodic recurrences are indicated by paleoseismic sequences, proxy data of small repeating earthquakes (Nadeau & McEvilly 1997) as well as in numerical fault simulations (e.g. Zöller et al. 2006). Therefore, recent implementations of time-dependent hazard assessment in California and Japan (Stein et al. 2006; Field et al. 2009) assume quasi-periodic recurrence of characteristic events. The recurrences are particularly modelled by stochastic Brownian relaxation oscillators (BROs), which lead to Brownian Passage Time (BPT) distributions for single faults (Matthews et al. 2002; Zöller & Hainzl 2007). Other well-known stochastic simulators accounting for stress loading and unloading are the stress release model (Vere-Jones 1978; Bebbington & Harte 2003; Jiang et al. 2011) and the stochastic simulator for rate- and state-dependent frictional nucleation (Dieterich 1994; Stein 1999; Dieterich et al. 2000; Catalli et al. 2008).

Although these stochastic models are mostly simple to apply, they miss realistic correlations in the spatiotemporal stress evolution due to earthquake rupturing and interaction. Thus their application might be questionable. A comprehensive comparison and testing of stochastic simulators with more realistic deterministic simulations
is still missing. In order to test the applicability and limitations of stochastic simulators for seismic hazard estimates, we present in this paper a systematic comparison of stochastic simulators with deterministic physics-based earthquake simulations for a predefined fault geometry with the same stress interactions in all cases.

Physics-based earthquake simulations with full state-of-the-art description of all relevant physical processes related to earthquake faulting is out of sight on geological timescales. However, simplified physical models based on elastic dislocation and frictional instability, unstable slip and nucleation concepts represent reasonable compromises between physical completeness and computational efficiency. For the fault network in California, the outcome of different deterministic simulators of this model class has been recently compared (Tullis 2012; Tullis et al. 2012). One of those models contributed to the comparative study is a simulator based on rate- and state-dependent friction introduced by Dieterich (1995, 2007) and Richards-Dinger & Dieterich (2012), which is now used for our comparative study with stochastic simulators. It is based on a computationally efficient technique, where approximate solutions for earthquake nucleation and dynamics of earthquake slip are introduced such that certain characteristics of earthquake rupture are preserved which are present in fully dynamic models. The approximate solutions are inferred from experimentally derived rate- and state-dependent constitutive properties (Dieterich 1995, 2007).

In this paper, we compare such quasi-dynamic simulations with the outcome of stochastic simulators adapted to the fault network in Lower Rhine Embayment (LRE), Germany. For this area, which is characterized by low seismicity but high vulnerability, numerical models are of particular importance in order to compensate for the lack of data (Zöller et al. 2013). After brief description of the analysed models in Section 2, we focus on the comparison of the resultant recurrence time distributions and the effects of fault interactions in Section 3. In the last two sections, we finally discuss and summarize our main findings.

## 2 MODELS

In the following, we briefly reflect the models of the two simulator classes which are subject of inspection within this manuscript:

(1) the deterministic quasi-dynamic earthquake simulator based on rate- and state-dependent friction introduced by Dieterich (1995) and (2) simulators with stochastic description of the nucleation process on faults.

Both model classes have in common that they are based on the same fault geometry and thus on the same stress interactions which are calculated by the Green’s functions of the elastostatic equations for an elastic half-space (Okada 1992). Fig. 1 illustrates the general model setup in the case of a single isolated fault. Faults are discretized into rectangular fault elements (patches) for which the stresses are calculated with analytical expressions of Okada (1992). Tectonic loading rates at each patch are given by backslip loading, where \( v_{\text{back}} = -v_{\text{tec}} \) is the backslip velocity equivalent to constant tectonic creep velocity outside locked fault elements. This leads to loading rates of the shear stress \( \dot{\tau} \) and normal stress \( \dot{\sigma} \) calculated by

\[
\dot{\tau}_i = G_{ij} v_{\text{back}}^j, \tag{1}
\]

\[
\dot{\sigma}_i = G_{ij} v_{\text{back}}^j, \tag{2}
\]

with \( G_{ij} \) being the Green’s function for elastic interaction between \( i \)th and \( j \)th patch. The overall stress evolution at fault elements is simply described by

\[
\tau_i = \dot{\tau}_i + G_{ij} \delta^\text{eq} + \tau_{\text{tec}}, \tag{3}
\]

\[
\sigma_i = \dot{\sigma}_i + G_{ij} \delta^\text{eq} + \sigma_{\text{tec}}, \tag{4}
\]

where \( \delta^\text{eq} \) is the cumulative earthquake slip and and \( \tau_{\text{tec}} \) and \( \sigma_{\text{tec}} \) are pre-stress values.

### 2.1 Deterministic rate- and state-dependent model

Fully dynamic simulations on geological timescales are still out of sight and thus some simplifications are required for computational efficiency. The commonly used way to simplify is to ignore all inertia effects in the first place by dropping the time dependence in the partial differential equations and solving the so-called elastostatic equations instead. Like this, and with assuming slow (quasi-static) stress loading rates over long (geological) time periods, the
evolution of a fault system is described in a quasi-static manner. The drawback of such an approach is that the dynamic evolution of a single earthquake rupture is not accounted for; instead the solution of the static differential equation (Green’s function $G_i$) is applied instantaneously (infinite rupture propagation speed).

We adopted the quasi-dynamic approach of Dieterich (1995) and its extension by Dieterich and Richards-Dinger (2010). In addition to the quasi-static interaction, the model assumes simplified dynamics: each fault patch is either locked (state 0), nucleating (state 1) or rupturing (state 2). When a fault patch $j$ switches from state 1 into state 2 or from state 2 into state 0, the change in stressing rate is instanta- neously increased (at the onset of earthquake slip) or decreased (at the end of earthquake slip) at all patches according to

$$\dot{\gamma}_j = \dot{\gamma}_j \pm G^{\gamma}_{ij} V^{\gamma}_{ij},$$

$$\dot{\sigma}_i = \dot{\sigma}_i \pm G^{\sigma}_{ij} V^{\sigma}_{ij}.$$  

(5)

(6)

In contrast, the switch of a fault patch from state 0 into state 1 is assumed to have no effect on the stressing rates of other patches, hence its transfer in stressing rate is assumed to be negligible.

In agreement with experimental data, sliding resistance or fault strength is governed by rate- and state-dependent friction (Linker $\&$ Dieterich 1992; Scholz 1998)

$$\tau = \sigma [\mu_0 + A \ln(V/V^*) + B \ln(\theta V^*/D_c)].$$

(7)

with constants $\mu_0$, $A$, $B$, $V^*$, $D_c$, the slip velocity $V$ and the state parameter $\theta$. In particular, $D_c$ represents the characteristic slip distance over which $\theta$ evolves, with typical laboratory values on the order of 10 $\mu m$. The coupling of this constitutive relation to the aging law for the state evolution equation leads on each fault element to (Linker $\&$ Dieterich 1992)

$$\dot{\theta} = 1 - \frac{\theta V}{D_c} - \frac{\theta \dot{\sigma}}{B \sigma},$$

(8)

where $\alpha$ is a dimensionless constant with laboratory values typically in the range 0.25–0.5.

An analytical solution for the slip acceleration of a fault patch is given by Dieterich (2007):

$$V = \left[ \frac{1}{V_0} + \frac{H \sigma}{S} \right] \exp \left[ \frac{-S t}{A \sigma} \right] + \frac{H \sigma}{S},$$

with

$$H = \frac{B}{D_c} \frac{K}{\sigma};$$

(9)

where $V_0$ is the initial slip speed, $K = G_{ii}$ is the bulk stiffness of the cell and $S$ refers to the stressing rate, which is related to the modified Coulomb stress function

$$S = \tau + \mu' \sigma,$$

(10)

with the effective constant friction coefficient $\mu' = \tau/\sigma - \alpha$.

With this solution, the time required for each patch changing from one state to the next can be determined by the equations:

$$\Delta t(0 \to 1) : \Delta t = \frac{1}{S} \left[ \sigma \left( \mu_0 + (B - A) \ln \left( \frac{(\Delta t + \theta) V^*}{D_c} \right) - \tau \right) \right],$$

(11)

$$\Delta t(1 \to 2) : \Delta t = \frac{A \alpha}{S} \left[ \ln \left( \frac{S}{H_\sigma V_0} - 1 \right) - \ln \left( \frac{S}{H_\sigma V^*} - 1 \right) \right].$$

(12)

$$\Delta t(2 \to 0) : \Delta t = \frac{1 + d_v}{S} \left[ \sigma \left( \mu_0 + (B - A) \ln \left( \frac{V^*}{V^*_{eq}} \right) - \tau \right) \right],$$

(13)

where $V^*_{eq}$ is the slip velocity during an earthquake, which is assumed to be constant. The model accounts for dynamic overshoot seen in fully dynamic rupture simulations by means of parameter $d_v$, as discussed in detail by Richards-Dinger $\&$ Dieterich (2012). Here we used an overshoot value of 5 per cent.

During interseismic periods, all patches are in states 0 or 1. The first switch of a patch from state 1 into state 2 defines the onset of a new earthquake rupture with hypocentre defined by the patch location. Due to the change of the stressing rates during the sliding of this patch, other patches might switch to state 2 and the earthquake rupture grows. The earthquake ends when all patches are again in state 0 or 1, and eventually its moment magnitude is calculated.

### 2.2 Stochastic simulators

We test whether the occurrence of mainshocks in this deterministic model can be described by models with stochastic earthquake nucleation. Those simulators are based on input informations about the target magnitudes, stress interactions and loading. Stochastic simulators are numerically much more efficient and can be thus easily extended to account for off-fault background activity and aftershock triggering (see Section 3.4). In the following, the three tested stochastic simulators are briefly introduced.

#### 2.2.1 Stochastic rate-state simulator (sRS)

A widely applied stochastic model version for rate- and state-dependent frictional earthquake nucleation has been introduced by Dieterich (1994). The main assumption of this sRS model is that a larger number of faults/sites exist in each volume, where earthquakes are nucleating independently of each other. Although important in realistic deterministic simulations, the locking of sites (state 0) and correlations due to coseismic ruptures are ignored in this stochastic model version. Despite this limitation, we test whether the application of the sRS model can still give some useful description of the mainshock recurrences.

In this model, the earthquake nucleation rate $R$ depends on the state variable $\gamma$, tectonic stressing rate $\dot{S}$ and the background seismicity rate $r$ according to

$$R = \frac{r}{\gamma \dot{S}};$$

(14)

where the evolution of the state variable is governed by

$$d\gamma = \frac{1}{A \sigma} [d\tau - \gamma d\dot{S}]$$

(Dieterich 1994; Dieterich $et$ $al.$ 2000). Based on this evolution equation, the time dependence can be explicitly calculated for stress histories consisting of coseismic stress steps and constant tectonic loading (Dieterich 1994). In particular, the state after a stress step $\Delta S$ at time $t = 0$ is given by (Hainzl $et$ $al.$ 2010)

$$\gamma(t) = [\gamma_0 e^{\frac{\Delta S}{\sigma}} - 1] e^{-\frac{\dot{S}}{\sigma}} + 1,$$

(16)

where the characteristic time $t_0$ is given by $t_0 = A \sigma / \dot{S}$ and $\gamma_0$ is the state variable just before the stress step.

In Monte Carlo simulations, the time $\Delta t$ to the next earthquake is calculated by means of the inverse method (Daley $\&$ Vere-Jones 2003), that is, by choosing a random value $u$ between 0 and 1 and

$$\Delta t(2 \to 0) : \Delta t = \frac{1 + d_v}{S} \left[ \sigma \left( \mu_0 + (B - A) \ln \left( \frac{V^*}{V^*_{eq}} \right) - \tau \right) \right],$$

(13)
solving numerically \( F(\Delta t) = u \), where the survivor function \( F \) is given by

\[
F(\Delta t) = \exp \left[ - \int_0^{\Delta t} R(t) \, dt \right] = \exp \left( -r \Delta t - \frac{\Delta S}{A \sigma} - \ln(\gamma_0) \right) \cdot \left[ \tilde{y}(\Delta t) \right]^{1 - u}. \tag{17}
\]

The next rupture then occurs at the fault segment with the smallest value \( \Delta t \).

### 2.2.2 Stress release (SR) model

The SR model is a stochastic version of elastic rebound theory, a classical explanation for the mechanism of earthquakes. For an isolated fault, the model has been introduced by Vere-Jones (1978). Its extension to fault systems is called coupled or linked SR model (Liu et al. 1999; Lu et al. 1999; Bebbington & Harte 2003; Kuehn et al. 2008; Jiang et al. 2011). It imposes no boundary conditions to the stress level \( S \) and just assumes that the earthquake nucleation rate, respectively, hazard function, is an exponential function of stress in each location, that is, \( R(t) = r \exp(\beta S(t)) \) with the two parameters \( r \) and \( \beta \). The ratio between the rates immediately after and before a stress step \( \Delta S \) is an exponential function of the stress change, \( \exp(\beta \Delta S) \), such as in the sRS model, where the ratio is \( \exp(\Delta S/A \sigma) \). Therefore, we set \( \beta = 1/A \sigma \) which leads to

\[
R = r \exp \left( \frac{S}{A \sigma} \right). \tag{18}
\]

In Monte Carlo simulations, the next failure time can be determined analytically by the inverse transform method according to

\[
\Delta t = t_0 \ln \left( \frac{1}{\gamma_0} \frac{\tilde{S}}{A \sigma} e^{-\frac{\gamma_0}{r} t_0} \right), \tag{19}
\]

where \( r \) is a random number between 0 and 1, and \( S_0 \) is the stress value after the last event.

#### 2.2.3 BRO

The BRO is also a stochastic version of elastic rebound theory, but with a fixed stress threshold for earthquake initiation. Earthquake nucleation is assumed to be instantaneous, if stress exceeds the threshold (Matthews et al. 2002). In the BRO model, each fault is loaded linearly in time with \( \dot{S} \) and a rupture occurs, if the stress exceeds the threshold \( \Delta t \). This value is equal to the stress drop due to rupturing at the same fault segment, \( \Delta \tau = -\Delta S_0 \), and related to the mean recurrence time \( T \) by \( \Delta \tau = \dot{S} T \). However, variability is considered by adding Gaussian noise (Brownian walk) to the loading rate in order to account for subscale processes like small earthquakes, aseismic stress release, spatial heterogeneities or pore pressure changes. Due to the central limit theorem, the Gaussian noise essentially accounts also for other types of fluctuations, because only the sum of stress changes influences the time of the next event (Zöller et al. 2008).

The standard deviation of the recurrence times is \( \sigma_T = \sigma_\tau \sqrt{\Delta \tau \tilde{S}^{-1.5}} \), where \( \sigma_\tau \) is the standard deviation of the noise. Thus, the coefficient of variation of the earthquake recurrence times at one fault is given by \( C_V = \sigma_T/T = \sigma_\tau/\sqrt{\Delta \tau \tilde{S}} = \sigma_\tau/(\sqrt{T \tilde{S}}) \) and hence \( \sigma_\tau \) is defined for given \( T \) and \( C_V \) values. An analytic formula for the probability density distribution of the recurrence times is provided in the case of an isolated fault by the BPT distribution

\[
f(t) = \sqrt{\frac{T^3}{2\pi \sigma_T^2 t^5}} \exp \left( -\frac{T(t - T)^2}{2\sigma_T^2 t} \right). \tag{20}
\]

In the case of fault interactions, the BRO model has to be iterated numerically.

### 3 SIMULATIONS

#### 3.1 Model parameters and fault network

The aim of this study is the comparison of the deterministic and stochastic simulators for a realistic fault system without detailed fitting of observed earthquake data. Thus, we use standard parameters related to the emergence of typically observed earthquake patterns. The chosen parameters are summarized in Table 1. Synthetic catalogues are simulated for a fault system adapted to the Lower Rhine Embayment, Germany, consisting of 16 major fault segments with normal faulting. Fig. 2 shows the analysed fault network in relation to the surface traces of known faults. For simplicity, we assumed rather steep dipping planes of 80 degrees to avoid intersections of planes in depth. The long-term slip rate (backslope) is assumed to be 0.1 mm per year on each fault. Faults are subdivided into patches of edge length 1.2 km and the stress interactions are calculated assuming standard elastic parameters (see Table 1).

The application of the stochastic simulators requires some input parameters related to the target earthquakes on each fault (see Table 2):

1. \( D \), the average mainshock slip on the corresponding fault.
2. It is calculated from the earthquake target magnitude \( M \) via

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho_i )</td>
<td>2750 kg m(^{-3})</td>
<td>Earth model (density and velocities)</td>
</tr>
<tr>
<td>( v_i )</td>
<td>3.3 km(^2) s(^{-1})</td>
<td>Normal pre-stress</td>
</tr>
<tr>
<td>( v_p )</td>
<td>5.7 km(^2) s(^{-1})</td>
<td>Shear pre-stress</td>
</tr>
<tr>
<td>( \sigma_{\text{tec}} )</td>
<td>120 MPa</td>
<td>Reference friction coefficient</td>
</tr>
<tr>
<td>( \tau_{\text{tec}} )</td>
<td>30 MPa</td>
<td>State parameter (eq. 8)</td>
</tr>
<tr>
<td>( \mu_0 )</td>
<td>0.65</td>
<td>State parameter (eq. 7)</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0.25</td>
<td>State parameter (eq. 7)</td>
</tr>
<tr>
<td>( A )</td>
<td>0.0025</td>
<td>Critical slip distance</td>
</tr>
<tr>
<td>( B )</td>
<td>0.0030</td>
<td>Slip velocity during rupture</td>
</tr>
<tr>
<td>( y^{\text{eq}} )</td>
<td>1 m s(^{-1})</td>
<td>Reference slip velocity</td>
</tr>
<tr>
<td>( y^{\nu} )</td>
<td>1 ( \mu ) m s(^{-1})</td>
<td>Dynamic overshoot parameter (eq. 12)</td>
</tr>
<tr>
<td>( d_{os} )</td>
<td>0.05</td>
<td>Parameter related to the target earthquakes on each fault (see Table 2):</td>
</tr>
</tbody>
</table>

### Table 1. Summary of the model parameters used for the deterministic simulations of the rate-
Figure 2. Analysed system of normal faults in Lower Rhine Embayment, Germany. Bold straight lines refer to the simplified fault geometry used for simulations, while thin lines refer to observed surface traces of the faults.

Table 2. Target earthquakes (moment magnitude $M$, mean recurrence time $T$ and coefficient of variation $C_V$) as input for the stochastic simulators.

<table>
<thead>
<tr>
<th>Fault</th>
<th>$M$</th>
<th>$T$ (yr)</th>
<th>$C_V$</th>
</tr>
</thead>
<tbody>
<tr>
<td>S01</td>
<td>6.8</td>
<td>21 806</td>
<td>0.06</td>
</tr>
<tr>
<td>S02</td>
<td>6.8</td>
<td>22 181</td>
<td>0.05</td>
</tr>
<tr>
<td>S03</td>
<td>6.9</td>
<td>22 003</td>
<td>0.06</td>
</tr>
<tr>
<td>S04</td>
<td>7.0</td>
<td>23 711</td>
<td>0.07</td>
</tr>
<tr>
<td>S05</td>
<td>6.6</td>
<td>23 574</td>
<td>0.34</td>
</tr>
<tr>
<td>S06</td>
<td>6.6</td>
<td>20 884</td>
<td>0.17</td>
</tr>
<tr>
<td>S07</td>
<td>7.0</td>
<td>24 015</td>
<td>0.07</td>
</tr>
<tr>
<td>S08</td>
<td>6.9</td>
<td>23 130</td>
<td>0.06</td>
</tr>
<tr>
<td>S09</td>
<td>6.6</td>
<td>24 271</td>
<td>0.32</td>
</tr>
<tr>
<td>S10</td>
<td>7.0</td>
<td>23 928</td>
<td>0.07</td>
</tr>
<tr>
<td>S11</td>
<td>6.8</td>
<td>21 477</td>
<td>0.08</td>
</tr>
<tr>
<td>S12</td>
<td>6.4</td>
<td>31 505</td>
<td>0.47</td>
</tr>
<tr>
<td>S13</td>
<td>6.4</td>
<td>30 456</td>
<td>0.45</td>
</tr>
<tr>
<td>S14</td>
<td>6.7</td>
<td>23 768</td>
<td>0.26</td>
</tr>
<tr>
<td>S15</td>
<td>6.7</td>
<td>20 802</td>
<td>0.10</td>
</tr>
<tr>
<td>S16</td>
<td>7.1</td>
<td>25 587</td>
<td>0.07</td>
</tr>
</tbody>
</table>

$D = 10^{1.5M+9.1}/(\mu_s A)$, where $A$ is the fault area and $\mu_s$ the shear modulus.

(2) $T$, the mean recurrence time of the corresponding characteristic earthquake (mainshock). It is used to set the effective loading rate according to $\dot{S} = \Delta\tau/T$.

(3) $C_V$, the coefficient of variation of the recurrence times on each fault is also an input parameter in the case of the BRO model. In contrast to the BRO model, the $C_V$ is not an input but an output parameter of the sRS and SR simulators as result of the input values $\sigma$ and $r = 1/T$.

In the Monte Carlo simulations, stress interactions are implemented on a patch and fault basis:

(1) In patch-based simulations, a stochastic simulator is run for each patch separately, but it is assumed that all patches $j$ of a fault $n$ fail simultaneously, if an earthquake nucleates at one of its patches. Then the stress change on a patch $i$ due to rupture of fault $n$ is given by summation of all contributions related to patches $j$ of fault $n$ according to $\Delta S_{in} = (G_{ij} + \bar{f}G_{ij})D_n$.

(2) In the case of fault-based simulations, each fault is represented by only one simulator. The stress change on fault $k$ (consisting of patches with index $i$) due to slip on fault $n$ is given by the average stress change value on the receiver fault $\Delta S_{kn} = \langle \Delta S_{in} \rangle$.

Table 3 summarizes all $\Delta S_{in}$ values for the analysed fault network.

The BRO simulations are only performed on fault basis, because the coefficient of variation is only available for each fault, but not for its individual patches. In contrast, sRS and SR simulations can be applied in both cases, only the background rate $r$ has to be reset from $1/T$ to $N/T$ in the case of the fault simulations, where $N$ is the number of patches of the specific fault. In all cases, we run simulations of $10^8$ target events. The first 10 per cent of the catalogues are removed as it was done in the deterministic model simulations to account for transient effects.

3.2 Isolated fault simulations

We first run deterministic fault simulations of $10^7$ yr for each of the 16 faults separately. This enables us to study the general fault dynamics and to define the target mainshock magnitudes as well as the parameter $T$ (and $C_V$ in the case of BRO) for the stochastic fault simulators. For identification of the characteristic events, we search for the magnitude threshold providing strongest quasi-periodic behaviour. For each fault, we analyse the coefficient of variation, $C_V = \sigma/T$, for events above a given magnitude threshold. A clear minimum is observed at magnitude values separating the Gutenberg–Richter type magnitude distribution from the characteristic earthquakes that rupture almost the whole fault segment. A typical example is shown in Fig. 3. Note that, the $b$ value of the Gutenberg–Richter distributed small magnitude events is around two in the deterministic simulations and thus significantly larger.
1. In addition, the coefficient of variation for events larger than the value typically observed for regional seismicity ($b \approx 1$). However, the observations usually include a large number of faults of varying length scales which inhibits a simple comparison with single fault simulations. A summary of all target events for the different LRE faults is provided in Table 2. For two examples, Figs 4(a) and (b) show the full recurrence time distributions of these target events in the deterministic simulations in comparison to the correspondent results for the three stochastic simulators. For all faults, an overview is shown in Fig. 5(a), where the mean and the interval between the 16 and 84 per cent quantiles (corresponding to ± one standard deviation in the case of a Gaussian distribution) are compared for all isolated faults.

The results indicate the sRS and SR simulators can both rather well reproduce the recurrence time distributions of the largest earthquakes in the deterministic simulations, only based on the input information of the mean recurrence time and the mean magnitude. In particular, the widths of the distributions are well approximated without accounting for the small and intermediate magnitude earthquakes and the details of mainshock ruptures occurring in the deterministic simulations. Note that the result for the BRO simulator is of no interest in the case of isolated faults, because the coefficient of variation related to the width of the distribution is an input parameter of this simulator.

### 3.3 Fault system simulations

In the case of fault system simulations, all input parameters of the models and the target events remain fixed, but stresses do not only change by loading and stress redistribution within one fault but also in response to stress interactions between faults. This can lead to corupturing, clock advance or delay of earthquakes on adjacent fault segments. For the same target events as before, Figs 4(c) and (d) show the full recurrence time distributions for the two examples, while Fig. 5(b) provides a summary for all faults. We find that for most of the faults, the forecasts of the stochastic sRS and SR models are approximating the mean recurrence time (which is not fixed any more) and the width of the distribution of the deterministic simulations. The peak values of the distributions are found to be significantly reduced for the patch-based simulations (sRS and SR), while the effect of fault coupling is only minimal in the case of the BRO simulator. Anyway, the recurrence time distributions of some faults cannot be sufficiently reproduced by any of the stochastic simulations. Strong deviations are particularly observed in Fig. 5(b) for faults S02, S04 and S06. These faults are among the two fault pairs with highest interfault patch coupling as shown in Table 4, where fault pairs according to their maximum stress coupling between the 16 and 84 per cent quantiles (corresponding to ± one standard deviation in the case of a Gaussian distribution) are compared for all isolated faults.

By analysing the waiting time distributions between adjacent on-fault mainshocks, the problem becomes even more clear. For four examples, Fig. 6 shows the probability density function of the time which has to be elapsed after a mainshock on one fault until the occurrence of the next mainshock on the other fault. While the stochastic simulators can approximate the distribution in the cases with moderate stress interactions rather well, they fail in the case of strong coupling. For strong positive coupling, both sRS and SR predicts an Omori-type decay of the waiting time distribution (Borovkov & Bebbington 2003) as shown in Fig. 6(c) for waiting time distributions between fault S04 and S06. Instead, the observed distribution in the deterministic simulations shows a gap. This can be explained
Figure 4. Recurrence time distribution for the characteristic earthquakes on two exemplary faults in the case of (a–b) isolated and (c–d) coupled faults.

by corupturing of both faults at the same time due to the dynamic rupture propagation in the deterministic simulations. If a major earthquake ruptures one fault, it often can jump to the other fault due to the stress concentrations at the rupture tips. The stochastic simulators do not account for such spatiotemporal correlations of the stress field and thus underestimate the temporal coupling of the two faults in the case of strong positive stress interactions. On the other hand, strong negative interactions lead to a delay of the ruptures at the coupled fault. In the patch-based simulations, the heterogeneous patch-to-patch coupling result in a number of patches with positive stress changes even though the average stress change on the fault is strongly negative. These local heterogeneities lead to a number of short waiting times in the stochastic simulations, which are not occurring in the deterministic simulations. Here, either they are ruptured instantaneously with the mainshock on the adjacent fault or only smaller magnitude events are triggered on the coupled fault, which do not grow into a large event, that is, characteristic fault rupture, if they occur in an overall unloaded fault. In this case, a fault-based model, such as the BRO model in Fig. 6(d), is better suited, because stresses are assumed to be reduced everywhere on the coupled fault and thus no short-term triggering is likely. On the other hand, fault-based models fail to reproduce the broad distribution of waiting times for the case of moderately negative-coupled faults (Fig. 6b), because of the mean field approach.

Overall, we find that the two patch-based simulators sRS and SR behave very similar. However, the SR model is numerically more efficient, because the next event time can be determined analytically (see eq. 20). In the following, we thus use SR simulations to test the impact of taking additionally off-fault seismicity into account.

3.4 Adding off-fault seismicity

The stochastic simulators can easily be extended to account for off-fault seismicity. Regional seismicity can be usually well-described by constant background activity plus aftershock triggering (Zhuang et al. 2011, 2012). Thus, we take a constant, uniformly distributed background activity into account. In particular, background events are assumed to occur spontaneously in the simulations with a Gutenberg–Richter frequency–magnitude distribution, \(10^{a-bM}\), in a box enclosing the faults with a 50-km buffer region. We simulate background events in the magnitude range between 4 and 6.4, where the maximum magnitude is chosen to be equivalent to the minimum on-fault magnitude in the fault network. The parameters for the yearly rate are chosen according to estimates for the LRE region, \(a = 2.9\) and \(b = 1\) (Schmedes et al. 2005). In addition, we simulated for all on-fault and background events potential aftershock triggering according to the ETAS model (Ogata 1988). In particular, the rate of \(M \geq 4\) aftershocks is given by

\[
R_a(x, y, t) = \sum_{i: t_i < t} f(M_i) g(t - t_i) h(x, y)
\]

\[= \sum_{i: t_i < t} K 10^{a(M_i-4)} (c + t - t_i)^{-p} (d^2 + r^2)^{-q}. \quad (21)
\]

Here, \(r\) is the distance to the nearest point of the earthquake rupture with magnitude \(M_i\). We set the parameters of the distance decay to typical values of \(d = 0.1\) km and \(q = 1.5\). Similarly, standard values are assumed for the constants of the Omori–Utsu law, namely \(c = 0.01\) d and \(p = 1.1\), where aftershock occurrences are limited to the first 10 yr after the mainshock. The productivity parameters are...
set to fulfill the empirical Båth law stating that the largest aftershock has on average a magnitude 1.2 less than the mainshock magnitude \( M \). Therefore, we set \( a = b = 1 \) and \( K \) is defined by the condition that, on average, one aftershock with magnitude larger or equal to \( M - 1.2 \) is triggered. The maximum possible magnitude of aftershocks related to an on-fault mainshock has been set to be 0.5 less the magnitude of the characteristic on-fault event, while it is set to 6.4 otherwise. The focal mechanisms of off-fault events are randomly selected from all on-fault mechanisms in the case of background events, while it is set to the mechanism of the corresponding fault in the case of aftershocks. In both case, variability is added by assuming random deviations of strike, dip and rake according to Gaussian distributions with standard deviation of 10°.

Examples of the spatial distribution of the SR model simulations with off-fault activity are shown in Fig. 7 for the three time periods of \( 10^3 \), \( 10^4 \) and \( 10^5 \) yr. While the seismicity is dominated by off-fault activity on the 1000 yr timescale, some faults light up due to the mainshock-triggered aftershocks on the \( 10^4 \)-yr period. However, only on the \( 10^5 \) yr timescale, all active faults become clearly visible.

While the off-fault seismicity dominates the visual impression due to its large quantity, the recurrence times of the on-fault events are not strongly affected by the off-fault activity. This is shown in Fig. 8, where the recurrence time distribution of fault S01 is presented for the cases with and without off-fault activity. Although off-fault activity leads to some delay in the mean recurrence time in the case of simulations of the isolated fault, the recurrence time distribution remains almost the same in the case of simulations of the coupled fault system. The reason is that off-fault events for single-fault simulations are assumed to have on average the same mechanism as the characteristic on-fault event which leads, on average, to an additional unloading of the fault by off-fault events. In the fault system simulations, the off-fault mechanisms are heterogeneous and therefore aftershocks have only a minor net effect on the stress budget.

### 4 DISCUSSION

Our analysis shows that stochastic simulators can provide reasonable estimates of earthquake recurrences in the case of a weak to intermediate coupling strength between faults. For strong coupling, spatiotemporal correlations of the stress field enables dynamic ruptures to jump between faults. These effects are not taken into account in the stochastic simulators. However, also fully deterministic forecasts of fault jumps are difficult. In particular, quasi-dynamic simulations—as applied in this study—are also not adequate because of the ignored effects of dynamic waves, which can be decisive whether or not ruptures can bridge step-overs. Thus, fully dynamic rupture models should be favourable, but those models depend crucially on the detailed knowledge of fault geometry as well as material properties (Finzi & Langer 2012). Because these informations are usually not all available, a stochastic treatment to estimate fault interactions remains necessary anyway. One solution could be to introduce exponential fault jump probabilities (Shaw & Dieterich 2007) in the described stochastic simulators, which has to be tested in future applications.

In contrast to strong fault coupling, the effect of smaller magnitude events is well covered by the stochastic simulators. In the deterministic simulations, small to intermediate magnitude events lead to unloading in subareas of the fault as well as to stressing of adjacent areas, and thus to a roughening of the stress field. This roughening disturbs the loading cycle of the characteristic fault ruptures and results in deviations from pure periodic recurrences. The stochastic component of the simulators seems to appropriately account for these effects as can be seen by the generally good agreement of the widths of the recurrence time distributions (Fig. 5).
Figure 6. Four examples of the distributions of waiting times between earthquake nucleation on one fault and the next rupture on another fault. Note that, the average stress interactions are moderate for both upper cases (0.09 MPa for S01 → S02 and −0.26 MPa for S01 → S03), while they are strong in the lower cases (1.01 MPa for S04 → S06 and −0.93 MPa for S16 → S15).

Figure 7. Snapshots of $M \geq 4$ earthquake occurrences on different timescales in the case of stochastic simulations with off-fault activity.

5 CONCLUSION
Stochastic models of earthquake occurrences are an important tool for seismic hazard assessment. However, models for earthquake recurrences on specific faults are not directly constrained by observations and rely mainly on theoretical considerations, proxy data sets like recurrences of small repeaters or uncertain palaeo-earthquake reconstructions. Two well-known stochastic implementations of the elastic rebound theory are the SR and the BRO, which involve at least two free model parameters. In particular, the latter model is currently used for time-dependent seismic hazard assessments in California and Japan (Stein et al. 2006; Field et al. 2009). However, the impact of fault interactions on the recurrence time distribution is not clear and systematic comparisons of stochastic models with more reliable physics-based deterministic simulations incorporating realistic stress interactions and earthquake nucleation are widely missed so far.

For the case of the fault system in the Lower Rhine Embayment, Germany, we now performed a systematic test of deterministic and stochastic simulations, where both model classes are based on
Figure 8. Comparison of recurrence times in the stochastic stress release simulations without and with off-fault seismicity for mainshocks on fault S01.

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