CHAPTER

5

Seismic Sensors and their Calibration

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5.1 Overview

There are two basic types of seismic sensors: inertial seismometers which measure ground motion relative to an inertial reference (a suspended mass), and strainmeters or extensometers which measure the motion of one point of the ground relative to another. Since the motion of the ground relative to an inertial reference is in most cases much larger than the differential motion within a vault of reasonable dimensions, inertial seismometers are generally more sensitive to earthquake signals. However, at very low frequencies it becomes increasingly difficult to maintain an inertial reference, and for the observation of low-order free oscillations of the Earth, tidal motions, and quasi-static deformations, strainmeters may outperform inertial seismometers. Strainmeters are conceptually simpler than inertial seismometers although their technical realization and installation may be more difficult (see IS 5.1). This Chapter is concerned with inertial seismometers only. For a more comprehensive description of inertial seismometers, recorders and communication equipment see Havskov and Alguacil (2002).

An inertial seismometer converts ground motion into an electric signal but its properties can not be described by a single scale factor, such as output volts per millimeter of ground motion. The response of a seismometer to ground motion depends not only on the amplitude of the ground motion (how large it is) but also on its time scale (how sudden it is). This is because the seismic mass has to be kept in place by a mechanical or electromagnetic restoring force. When the ground motion is slow, the mass will move with the rest of the instrument, and the output signal for a given ground motion will therefore be smaller. The system is thus a high-pass filter for the ground displacement. This must be taken into account when the ground motion is reconstructed from the recorded signal, and is the reason why we have to go to some length in discussing the dynamic transfer properties of seismometers.

The dynamic behavior of a seismograph system within its linear range can, like that of any linear time-invariant (LTI) system, be described with the same degree of completeness in four different ways: by a linear differential equation, the Laplace transfer function (see 5.2.2), the complex frequency response (see 5.2.3), or the impulse response of the system (see 5.2.4). The first two are usually obtained by a mathematical analysis of the physical system (the hardware). The latter two are directly related to certain calibration procedures (see 5.7.4 and 5.7.5) and can therefore be determined from calibration experiments where the system is considered as a "black box"(this is sometimes called an identification procedure). However, since all four are mathematically equivalent, we can derive each of them either from a knowledge of the physical components of the system or from a calibration experiment. The mutual relations between the "time-domain" and "frequency-domain" representations are illustrated in Fig. 5.1. Practically, the mathematical description of a seismometer is limited to a certain bandwidth of frequencies that should at least include the bandwidth of seismic signals. Within this limit then any of the four representations describe the system's response to arbitrary input signals completely and unambiguously. The viewpoint from which they differ is how efficiently and accurately they can be implemented in different signal-processing procedures.

In digital signal processing, seismic sensors are often represented with other methods that are efficient and accurate but not mathematically exact, such as recursive (IIR) filters. Digital signal processing is however beyond the scope of this section. A wealth of textbooks is available both on analog and digital signal processing, for example Oppenheim and Willsky (1983) for analog processing, Oppenheim and Schafer (1975) for digital processing, and Scherbaum (1996) for seismological applications.

The most commonly used description of a seismograph response in the classical observatory practice has been the "*magnification curve*", i.e. the frequency-dependent magnification of the ground motion. Mathematically this is the modulus (absolute value) of the complex frequency response, usually called the *amplitude response*. It specifies the steady-state harmonic responsivity (amplification, magnification, conversion factor) of the seismograph as a function of frequency. However, for the correct interpretation of seismograms, also the phase response of the recording system must be known. It can in principle be calculated from the amplitude response, but is normally specified separately, or derived together with the amplitude response from the mathematically more elegant description of the system by its *complex transfer function* or its *complex frequency response*.

While for a purely electrical filter it is usually clear what the amplitude response is - a dimensionless factor by which the amplitude of a sinusoidal input signal must be multiplied to obtain the associated output signal - the situation is not always as clear for seismometers because different authors may prefer to measure the input signal (the ground motion) in different ways: as a displacement, a velocity, or an acceleration. Both the physical dimension and the mathematical form of the transfer function depend on the definition of the input signal, and one must sometimes guess from the physical dimension to what sort of input signal it applies. The output signal, traditionally a needle deflection, is now normally a voltage, a current, or a number of counts.

Calibrating a seismograph means measuring (and sometimes adjusting) its transfer properties and expressing them as a complex frequency response or one of its mathematical equivalents. For most applications the result must be available as parameters of a mathematical formula, not as raw data; so determining parameters by fitting a theoretical curve of known shape to the data is usually part of the procedure. Practically, seismometers are calibrated in two steps.

The first step is an electrical calibration (see 5.7) in which the seismic mass is excited with an electromagnetic force. Most seismometers have a built-in calibration coil that can be connected to an external signal generator for this purpose. Usually the response of the system to different sinusoidal signals at frequencies across the system's passband (steady-state method, 5.7.4), to impulses (transient method, 5.7.5), or to arbitrary broadband signals (random signal method, 5.7.6) is observed while the absolute magnification or gain remains unknown. For the exact calibration of sensors with a large dynamic range such as those employed in modern seismograph systems, the latter method is most appropriate.

The second step, the determination of the absolute gain, is more difficult because it requires mechanical test equipment in all but the simplest cases (see 5.8). The most direct method is to calibrate the seismometer on a shake table. The frequency at which the absolute gain is measured must be chosen so as to minimize noise and systematic errors, and is often predetermined by these conditions within narrow limits. A calibration over a large bandwidth can not normally be done on a shake table. At the end of this Chapter we will propose some methods by which a seismometer can be absolutely calibrated without a shake table.

5.2 Basic theory

This section introduces some basic concepts of the theory of linear systems. For a more complete and rigorous treatment, the reader should consult a textbook such as by Oppenheim and Willsky (1983). Digital signal processing is based on the same concepts but the mathematical formulations are different for discrete (sampled) signals (see Oppenheim and Schafer, 1975; Scherbaum, 1996; Plešinger et al., 1996). Readers who are familiar with the mathematics may proceed to section 5.2.7.

5.2.1 The complex notation

A fundamental mathematical property of linear time-invariant systems such as seismographs (as long as they are not driven out of their linear operating range) is that they do not change the waveform of sinewaves and of exponentially decaying or growing sinewaves. The mathematical reason for this fact is explained in the next section. An input signal of the form

$$f(t) = e^{\sigma t}(a_1 \cos \omega t + b_1 \sin \omega t) \tag{5.1}$$

will produce an output signal

$$g(t) = e^{\sigma t}(a_2 \cdot \cos \omega t + b_2 \cdot \sin \omega t) \tag{5.2}$$

with the same σ and ω, but possibly different *a* and *b*. Note that ω is the angular frequency, which is 2π times the common frequency. Using Euler's identity

$$e^{j\omega t} = \cos \omega t + j \sin \omega t \tag{5.3}$$

and the rules of complex algebra, we may write our input and output signals as

$$f(t) = \Re[c_1 \cdot e^{(\sigma + j\omega)t}] \text{ and } g(t) = \Re[c_2 \cdot e^{(\sigma + j\omega)t}] \tag{5.4}$$

respectively, where $\Re[..]$ denotes the real part, and $c_1 = a_1 - jb_1$, $c_2 = a_2 - jb_2$. It can now be seen that the only difference between the input and output signal lies in the complex amplitude c, not in the waveform. The ratio c_2 / c_1 is the complex gain of the system, and for $\sigma = 0$, it is the value of the complex frequency response at the angular frequency ω. What we have outlined here may be called the engineering approach to complex notation. The sign $\Re[..]$ for the real part is often omitted but always understood.

The mathematical approach is slightly different in that real signals are not considered to be the real parts of complex signals but the sum of two complex-conjugate signals with positive and negative frequency:

$$f(t) = c_1 \cdot e^{(\sigma + j\omega)t} + c_1^* \cdot e^{(\sigma - j\omega)t} \tag{5.5}$$

where the asterisk * denotes the complex conjugate. The mathematical notation is slightly less concise, but since for real signals only the term with c_1 must be explicitly written down (the other one being its complex conjugate), the two notations become very similar. However, the c_1 term describes the whole signal in the engineering convention but only half of the signal in the mathematical notation! This may easily cause confusion, especially in the definition of power spectra. Power spectra computed after the engineer's method (such as the USGS Low Noise Model, see 5.5.1 and Chapter 4) attribute all power to positive frequencies and therefore have twice the power appearing in the mathematical notation.

5.2.2 The Laplace transformation

A signal that has a definite beginning in time (such as the seismic waves from an earthquake) can be decomposed into exponentially growing, stationary, or exponentially decaying sinusoidal signals with the *Laplace integral transformation*:

$$f(t) = \frac{1}{2\pi j} \int_{\sigma - j\infty}^{\sigma + j\infty} F(s)\, e^{st} ds\,, \qquad F(s) = \int_0^\infty f(t)\, e^{-st} dt \tag{5.6}$$

The first integral defines the inverse transformation (the synthesis of the given signal) and the second integral the forward transformation (the analysis). It is assumed here that the signal begins at or after the time origin. s is a complex variable that may assume any value for which the second integral converges (depending on $f(t)$, it may not converge when s has a negative real part). The Laplace transform $F(s)$ is then said to “exist” for this value of *s*. The real parameter σ which defines the path of integration for the inverse transformation (the first integral) can be arbitrarily chosen as long as the path remains on the right side of all singularities of $F(s)$ in the complex s plane. This parameter decides whether $f(t)$ is synthesized from decaying ($\sigma < 0$), stationary ($\sigma = 0$) or growing ($\sigma > 0$) sinusoidals (remember that the mathematical expression e^{st} with complex *s* represents a growing or decaying sinewave, and with imaginary *s* a pure sinewave).

The time derivative $\dot{f}(t)$ has the Laplace transform $s \cdot F(s)$, the second derivative $\ddot{f}(t)$ has $s^2 \cdot F(s)$, etc. Suppose now that an analog data-acquisition or data-processing system is characterized by the linear differential equation

$$c_2 \ddot{f}(t) + c_1 \dot{f}(t) + c_0 f(t) = d_2 \ddot{g}(t) + d_1 \dot{g}(t) + d_0 g(t) \tag{5.7}$$

where $f(t)$ is the input signal, $g(t)$ is the output signal, and the c_i and d_i are constants. We may then subject each term in the equation to a Laplace transformation and obtain

$$c_2 s^2 F(s) + c_1 s F(s) + c_0 F(s) = d_2 s^2 G(s) + d_1 s G(s) + d_0 G(s) \tag{5.8}$$

from which we get

$$G(s) = \frac{c_2 s^2 + c_1 s + c_0}{d_2 s^2 + d_1 s + d_0} F(s) \tag{5.9}$$

We have thus expressed the Laplace transform of the output signal by the Laplace transform of the input signal, multiplied by a known rational function of s. From this we obtain the output signal itself by an inverse Laplace transformation. This means, we can solve the differential equation by transforming it into an algebraic equation for the Laplace transforms. Of course, this is only practical if we are able to evaluate the integrals analytically, which is the case for a wide range of "mathematical" signals. Real signals must be approximated by suitable mathematical functions for a transformation. The method can obviously be applied to linear and time-invariant differential equations of any order. (Time-invariant means that the properties of the system, and hence the coefficients of the differential equation, do not depend on time.)

The rational function

$$H(s) = \frac{c_2 s^2 + c_1 s + c_0}{d_2 s^2 + d_1 s + d_0} \tag{5.10}$$

is the (Laplace) transfer function of the system described by the differential equation (5.7). It contains the same information on the system as the differential equation itself.

Generally, the transfer function H(s) of an LTI system is the complex function for which

$$G(s) = H(s) \cdot F(s) \tag{5.11}$$

with F(s) and G(s) representing the Laplace transforms of the input and output signals.

A rational function like *H(s)* in (5.10), and thus an LTU system, can be characterized up to a constant factor by its poles and zeros. This is discussed in section 5.2.6.

5.2.3 The Fourier transformation

Somewhat closer to intuitive understanding but mathematically less general than the Laplace transformation is the Fourier transformation

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{F}(\omega)\, e^{j\omega t} d\omega, \qquad \tilde{F}(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t} dt \tag{5.12}$$

The signal is here assumed to have a finite energy so that the integrals converge. The condition that no signal is present at negative times can be dropped in this case. The Fourier transformation decomposes the signal into purely harmonic (sinusoidal) waves $e^{j\omega t}$. The direct and inverse Fourier transformation are also known as a harmonic analysis and synthesis.

Although the mathematical concepts behind the Fourier and Laplace transformations are different, we may consider the Fourier transformation as a special version of the Laplace transformation for real frequencies, i.e. for $s = j\omega$. In fact, by comparison with Eq. (5.6), we see that $\tilde{F}(\omega) = F(j\omega)$, i.e. the Fourier transform for real angular frequencies ω is identical to the Laplace transform for imaginary $s = j\omega$. For practical purposes the two transformations are thus nearly equivalent, and many of the relationships between time-signals and their transforms (such as the convolution theorem) are similar or the same for both. The function $\tilde{F}(\omega)$ is called the complex frequency response of the system. Some authors use the name “transfer function” for $\tilde{F}(\omega)$ as well; however, $\tilde{F}(\omega) = F(j\omega)$ is not the same function as $F(\omega)$, so different names are appropriate. The distinction between $\tilde{F}(\omega)$ and $F(s)$ is essential when systems are characterized by their poles and zeros. These are equivalent but not identical in the complex s and ω planes, and it is important to know whether the Laplace or Fourier transform is meant. Usually, poles and zeros are given for the Laplace transform. In case of doubt, one should check the symmetry of the poles and zeros in the complex plane: those of the Laplace transform are symmetric to the real axis as in Fig. 5.2 while those of the Fourier transform are symmetric to the imaginary axis.

The absolute value $|\tilde{F}(\omega)|$ is called the amplitude response, and the phase of $\tilde{F}(\omega)$ the phase response of the system. Note that amplitude and phase do not form a symmetric pair; however a certain mathematical symmetry (expressed by the Hilbert transformation) exists between the real and imaginary parts of a rational transfer function, and between the phase response and the natural logarithm of the amplitude response.

The definition of the Fourier transformation according to Eq. (5.12) applies to continuous transient signals. For other mathematical representations of signals, different definitions must be used:

$$f(t) = \sum_{\nu=-\infty}^{\infty} b_\nu e^{2\pi j\nu t/T}, \quad b_\nu = \frac{1}{T}\int_0^T f(t)\, e^{-2\pi j\nu t/T} dt \tag{5.13}$$

for periodic signals $f(t)$ with a period T, and

$$f_k = \frac{1}{M}\sum_{l=0}^{M-1} c_l e^{2\pi jkl/M}, \quad c_l = \sum_{k=0}^{M-1} f_k e^{-2\pi jkl/M} \tag{5.14}$$

for time series f_k consisting of M equidistant samples (such as digital seismic data). We have noted the inverse transform (the synthesis) first in each case.

The Fourier integral transformation (Eq. (5.12)) is mainly an analytical tool; the integrals are not normally evaluated numerically because the discrete Fourier transformation Eq. (5.14) permits more efficient computations. Eq. (5.13) is the Fourier series expansion of periodic functions, also mainly an analytical tool but also useful to represent periodic test signals. The discrete Fourier transformation Eq. (5.13) is sometimes considered as being a discretized, approximate version of Eqs. (5.12) or (5.14) but is actually a mathematical tool in its own right: it is a mathematical identity that does not depend on any assumptions on the series f_k. Its relationship with the other two transformations, and especially the interpretation of the subscript l as representing a single frequency, do however depend on the properties of the original, continuous signal. The most important condition is that the bandwidth of the signal before sampling must be limited to less than half of the sampling rate f_s; otherwise the sampled series will not contain the same information as the original. The bandwidth limit $f_n = f_s/2$ is called the *Nyqvist frequency*. Whether we consider a signal as periodic or as having a finite duration (and thus a finite energy) is to some degree arbitrary since we can analyze real signals only for finite intervals of time, and it is then a matter of definition whether we assume the signal to have a periodic continuation outside the interval or not.

The Fast Fourier Transformation or FFT (see Cooley and Tukey, 1965) is a recursive algorithm to compute the sums in Eq. (5.14) efficiently, so it does not constitute a mathematically different definition of the discrete Fourier transformation.

5.2.4 The impulse response

A useful (although mathematically difficult) fiction is the Dirac “needle” pulse $\delta(t)$ (e.g. Oppenheim and Willsky, 1983), supposed to be an infinitely short, infinitely high, positive pulse at the time origin whose integral over time equals 1. It can not be realized, but its time-integral, the unit step function, can be approximated by switching a current on or off or by suddenly applying or removing a force. According to the definitions of the Laplace and Fourier transforms, both transforms of the Dirac pulse have the constant value 1. The amplitude spectrum of the Dirac pulse is “white” , this means, it contains all frequencies with equal amplitude. In this case Eq. (5.11) reduces to $G(s)=H(s)$, which means that the transfer function $H(s)$ is the Laplace transform of the impulse response $g(t)$. Likewise, the complex frequency response is the Fourier transform of the impulse response. All information contained in these complex functions is also contained in the impulse response of the system. The same is true for the step response, which is often used to test or calibrate seismic equipment.

Explicit expressions for the response of a linear system to impulses, steps, ramps and other simple waveforms can be obtained by evaluating the inverse Laplace transform over a suitable contour in the complex s plane, provided that the poles and zeros are known. The result, generally a sum of decaying complex exponential functions, can then be numerically evaluated with a computer or even a calculator. Although this is an elegant way of computing the response of a linear system to simple input signals with any desired precision, a warning is necessary: the numerical samples so obtained are not the same as the samples that would be obtained with an ideal digitizer. The digitizer must limit the bandwidth before sampling and therefore does not generate instantaneous samples but some sort of time-averages. For computing samples of band-limited signals, different mathematical concepts must be used (see Schuessler, 1981).

Specifying the impulse or step response of a system in place of its transfer function is not practical because the analytic expressions are cumbersome to write down and represent signals of infinite duration that can not be tabulated in full length.

5.2.5 The convolution theorem

Any signal may be understood as consisting of a sequence of pulses. This is obvious in the case of sampled signals, but can be generalized to continuous signals by representing the signal as a continuous sequence of Dirac pulses. We may construct the response of a linear system to an arbitrary input signal as a sum over suitably delayed and scaled impulse responses. This process is called a convolution:

$$g(t) = \int_0^\infty h(t')\, f(t-t')\, dt' = \int_0^\infty h(t-t')\, f(t')\, dt' \qquad (5.15)$$

Here *f*(*t*) is the input signal and *g*(*t*) the output signal while *h*(*t*) characterizes the system. We assume that the signals are causal (i.e. zero at negative time), otherwise the integration would have to start at $-\infty$. Taking $f(t) = \delta(t)$, i.e. using a single impulse as the input, we get $g(t) = \int h(t')\,\delta(t-t')\,dt' = h(t)$, so h(t) is in fact the impulse response of the system.

The response of a linear system to an arbitrary input signal can thus be computed either by convolution of the input signal with the impulse response in time domain, or by multiplication of the Laplace-transformed input signal with the transfer function, or by multiplication of the Fourier-transformed input signal with the complex frequency response in frequency domain.

Since instrument responses are often specified as a function of frequency, the FFT algorithm has become a standard tool to compute output signals. The FFT method assumes, however, that all signals are periodic, and is therefore mathematically inaccurate when this is not the case. Signals must in general be tapered to avoid spurious results. Fig. 5.1 illustrates the interrelations between signal processing in the time and frequency domains.

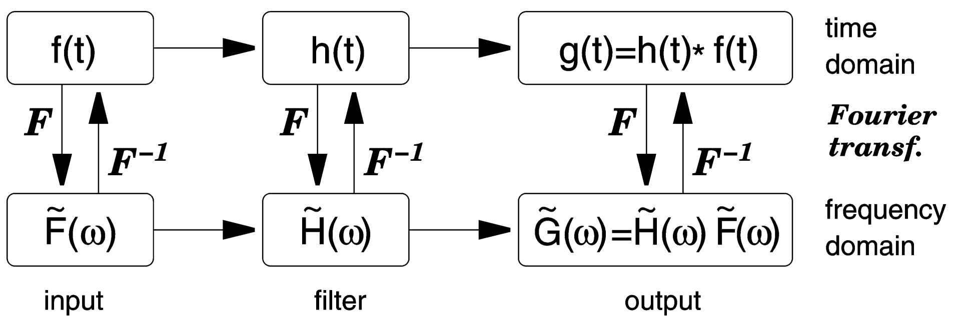

Fig. 5.1 Pathways of signal processing in the time and frequency domains. The asterisk between *f(t)* and *g(t)* indicates a convolution.

In digital processing, these methods translate into convolving discrete time series or transforming them with the FFT method and multiplying the transforms. For impulse responses with more than 100 samples, the FFT method is usually more efficient. The convolution method is also known as a FIR (finite impulse response) filtration. A third method, the recursive or IIR (infinite impulse response) filtration, is only applicable to digital signals; it is often preferred for its flexibility and efficiency although its accuracy requires special attention (see contribution by Scherbaum (1997) to the Manual web page under http://www.seismo.com).

5.2.6 Specifying a system

When $P(s)$ is a polynomial of s and $P(\alpha) = 0$, then $s = \alpha$ is called a zero, or a root, of the polynomial. A polynomial of order n has n complex zeros α_i, and can be factorized as $P(s) = p \cdot \prod (s - s_i)$ .Thus, the zeros of a polynomial together with the factor p determine the polynomial completely. Since our transfer functions $H(s)$ are the ratio of two polynomials as in Eq. (5.10), they can be specified by their zeros (the zeros of the numerator $G(s)$), their poles (the zeros of the denominator $F(s)$), and a gain factor (or equivalently the total gain at a given frequency). The whole system, as long as it remains in its linear operating range and does not produce noise, can thus be described by a small number of discrete parameters.

Transfer functions are usually specified according to one of the following concepts:

1. The real coefficients of the polynomials in the numerator and denominator are listed.

2. The denominator polynomial is decomposed into normalized first-order and second-order factors with real coefficients (a total decomposition into first-order factors would require complex coefficients). The factors can in general be attributed to individual modules of the system. They are preferably given in a form from which corner periods and damping coefficients can be read, as in Eqs. (5.31) to (5.33). The numerator often reduces to a gain factor times a power of s.

3. The poles and zeros of the transfer function are listed together with a gain factor. Poles and zeros must either be real or symmetric to the real axis, as mentioned above. When the numerator polynomial is s^m, then $s = 0$ is an m-fold zero of the transfer function, and the system is a high-pass filter of order m. Depending on the order n of the denominator and accordingly on the number of poles, the response may be flat at high frequencies ($n = m$), or the system may act as a low-pass filter there ($n > m$). The case $n < m$ can occur only as an approximation in a limited bandwidth because no practical system can have an unlimited gain at high frequencies.

In the header of the widely-used SEED-format data (see 10.4), the gain factor is split up into a normalization factor bringing the gain to unity at some normalization frequency in the passband of the system, and a gain factor representing the actual gain at this frequency. EX 5.5 contains an exercise in determining the response from poles and zeros. A program POL_ZERO (in BASIC) is also available for this purpose (see 5.9).

5.2.7 The transfer function of a WWSSN-LP seismograph

The long-period seismographs of the now obsolete WWSSN (Worldwide Standardized Seismograph Network) consisted of a long-period electrodynamic seismometer normally tuned to a free period of 15 sec, and a long-period mirror-galvanometer with a free period around 90 sec. (In order to avoid confusion with the frequency variable $s = j\omega$ of the Laplace transformation, we use the non-standard abbreviation „sec" for seconds in the present subsection.) The WWSSN seismograms were recorded on photographic paper rotating on a drum. We will now derive several equivalent forms of the transfer function for this system. In our example the damping constants are chosen as 0.6 for the seismometer and 0.9 for the galvanometer. Our treatment is slightly simplified. Actually, the free periods and damping constants are modified by coupling the seismometer and the galvanometer together; the above values are understood as being the modified ones.

As will be shown in section 5.2.9, Eq.(5.31), the transfer function of an electromagnetic seismometer (input: displacement, output: voltage) is

$$H_s(s) = Es^3 /(s^2 + 2s\omega_s h_s + \omega_s^2) \tag{5.16}$$

where $\omega_s = 2\pi / T_s$ is the angular eigenfrequency and h_s the numerical damping. (see EX 5.2 for a practical determination of these parameters.) The factor E is the generator constant of the electromagnetic transducer, for which we assume a value of 200 Vsec/m.

The galvanometer is a second-order low-pass filter and has the transfer function

$$H_g(s) = \gamma\omega_g^2 /(s^2 + 2s\omega_g h_g + \omega_g^2) \tag{5.17}$$

Here γ is the responsivity (in meters per volt) of the galvanometer with the given coupling network and optical path. We use a value of 393.5 m/V, which gives the desired overall magnification. The overall transfer function H_d of the seismograph is obtained in our simplified treatment as the product of the factors given in Eqs. (5.16) and (5.17):

$$H_d(s) = \frac{Cs^3}{(s^2 + 2s\omega_s h_s + \omega_s^2)(s^2 + 2s\omega_g h_g + \omega_g^2)} \tag{5.18}$$

The numerical values of the constants are $C = E\gamma\omega_g^2$ = 383.6/sec, $2\omega_s h_s$ = 0.5027/sec, ω_s^2 = 0.1755/sec^2, $2\omega_g h_g$ =0.1257/sec, and ω_g^2 = 0.00487/sec^2.

As the input and output signals are displacements, the absolute value $|H_d(s)|$ of the transfer function is simply the frequency-dependent magnification of the seismograph. The gain factor C has the physical dimension sec^{-1}, so $H_d(s)$ is in fact a dimensionless quantity. C itself is however not the magnification of the seismograph. To obtain the magnification at the angular frequency ω, we have to evaluate $M(\omega) = |H_d(j\omega)|$:

$$M(\omega) = \frac{C\omega^3}{\sqrt{(\omega_s^2 - \omega^2)^2 + 4\omega^2\omega_s^2 h_s^2}\sqrt{(\omega_g^2 - \omega^2)^2 + 4\omega^2\omega_g^2 h_g^2}} \tag{5.19}$$

Eq. (5.18) is a factorized form of the transfer function in which we still recognize the subunits of the system. We may of course insert the numerical constants and expand the denominator into a fourth-order polynomial

$$H_d(s) = 383.6s^3 /(s^4 + 0.6283s^3 + 0.2435s^2 + 0.0245s + 0.000855) \tag{5.20}$$

but the only advantage of this form would be its shortness.

The poles and zeros of the transfer function are most easily determined from Eq. (5.18). We read immediately that a triple zero is present at s = 0. Each factor $s^2 + 2s\omega_0 h + \omega_0^2$ in the denominator has the zeros

$$s_0 = \omega_0(-h \pm j\sqrt{1-h^2}) \qquad \text{for } h < 1$$

$$s_0 = \omega_0(-h \pm \sqrt{h^2 - 1}) \qquad \text{for } h \geq 1$$

so the poles of $H_d\,(s)$ in the complex s plane are (Fig. 5.2):

$$s_1 = \omega_s(-h_s + j\sqrt{1-h_s^2}) \qquad = -0.2513 + 0.3351j \quad [\sec^{-1}]$$
$$s_2 = \omega_s(-h_s - j\sqrt{1-h_s^2}) \qquad = -0.2513 - 0.3351j \quad [\sec^{-1}]$$
$$s_3 = \omega_g(-h_g + j\sqrt{1-h_g^2}) \qquad = -0.0628 + 0.0304j \quad [\sec^{-1}]$$
$$s_4 = \omega_g(-h_g - j\sqrt{1-h_g^2}) \qquad = -0.0628 - 0.0304j \quad [\sec^{-1}]$$

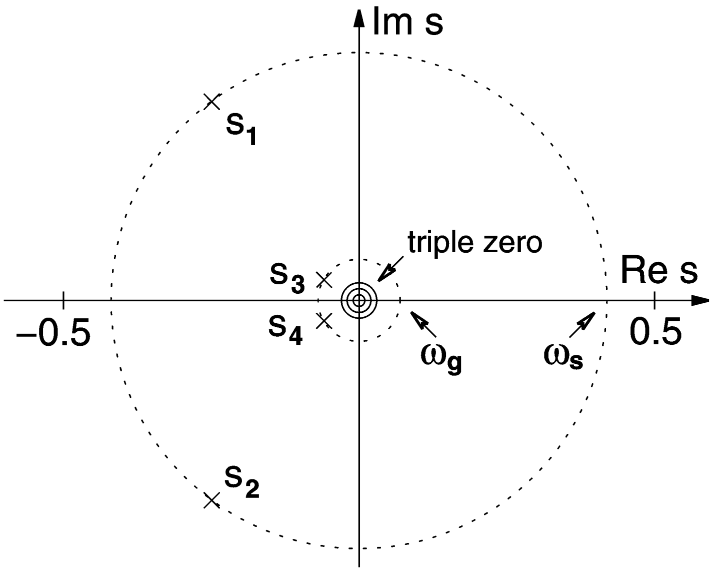

Fig. 5.2 Position of the poles of the WWSSN-LP system in the complex s plane**.**

In order to reconstruct H_d(s) from its poles and zeros and the gain factor, we write

$$H_d(s) = \frac{Cs^3}{(s-s_1)(s-s_2)(s-s_3)(s-s_4)}\,. \tag{5.21}$$

It is now convenient to pairwise expand the factors of the denominator into second-order polynomials:

$$H_d(s) = \frac{Cs^3}{(s^2 - s(s_1+s_2) + s_1 s_2)(s^2 - s(s_3+s_4) + s_3 s_4)}\,. \tag{5.22}$$

This makes all coefficients real because $s_2 = s_1^*$ and $s_4 = s_3^*$. Since $s_1 + s_2 = -2\omega_s h_s$, $s_1 s_2 = \omega_s^2$, $s_3 + s_4 = -2\omega_g h_g$, and $s_3 s_4 = \omega_g^2$, Eq. (5.22) is in fact the same as Eq. (5.18). We may of course also reconstruct $H_d(s)$ from the numerical values of the poles and zeros. Dropping the physical units, we obtain

$$H_d(s) = \frac{383.6 s^3}{(s^2 + 0.5027 s + 0.1755)(s^2 + 0.1257 s + 0.00487)} \qquad (5.23)$$

in agreement with Eq.(5.20).

Fig. 5.3 shows the corresponding amplitude response of the WWSSN seismograph as a function of frequency. The maximum magnification is 750 near a period of 15 sec. The slopes of the asymptotes are at each frequency determined by the dominant powers of s in the numerator and denominator of the transfer function. Generally, the low-frequency asymptote has the slope m (the number of zeros, here = 3) and the high-frequency asymptote has the slope m-n (where n is the number of poles, here = 4). What happens in between depends on the position of the poles in the complex s plane. Generally, a pair of poles s_1, s_2 corresponds to a second-order corner of the amplitude response with $\omega_0^2 = s_1 s_2$ and $2\omega_0 h = -s_1 - s_2$. A single pole at s_0 is associated with a first-order corner with $\omega_0 = s_0$. The poles and zeros however do not indicate whether the respective subsystem is a low-pass, high-pass, or band-pass filter. This does not matter; the corners bend the amplitude response downward in each case. In the WWSSN-LP system, the low-frequency corner at 90 sec corresponding to the pole pair s_1, s_2 reduces the slope of the amplitude response from 3 to 1, and the corner at 15 sec corresponding to the pole pair s_3, s_4 reduces it further from 1 to -1.

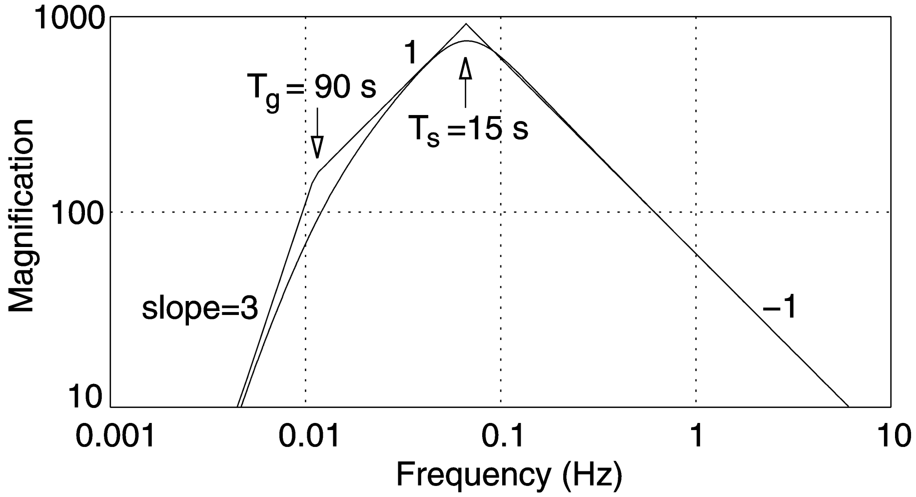

Fig. 5.3 Amplitude response of the WWSSN-LP system with asymptotes (Bode plot).

Looking at the transfer function H_s (Eq. (5.16)) of the electromagnetic seismometer alone, we see that the low-frequency asymptote has the slope 3 because of the triple zero in the numerator. The pole pair s_1, s_2 corresponds to a second-order corner in the amplitude response at ω_s which reduces the slope to 1. The resulting response is shown in a normalized form in the upper right panel of Fig. 5.6. As stated in section 5.2.6 under point 3, this case of $n<m$ can

only be an approximation in a limited bandwidth. In modern seismograph systems, the upper limit of the bandwidth is usually set by an analog or digital cut-off (anti-alias) filter.

As we will see in section 5.2.9, the classification of a subsystem as a high-pass, band-pass or low-pass filter may be a matter of definition rather than hardware; it depends on the type of ground motion (displacement, velocity, or acceleration) to which it relates. We also notice that interchanging ω_s, h_s with ω_g, h_g will change the gain factor C in the numerator of Eq. (5.19) from $E\gamma\omega_g^2$ to $E\gamma\omega_s^2$ and thus the gain, but will leave the denominator and therefore the shape of the response unchanged. While the transfer function is insensitive to arbitrary factorization, the hardware may be quite sensitive, and certain engineering rules must be observed when a given transfer function is realized in hardware. For example, it would have been difficult to realize a WWSSN seismograph with a 15 sec galvanometer and a 90 sec seismometer; the restoring force of a Lacoste-type suspension can not be made small enough without becoming unstable.

Fig. 5.4 illustrates the impulse responses of the seismometer, the galvanometer, and the whole WWSSN-LP system. We have chosen a pulse of acceleration (or of calibration current) as the input, so the figure does not refer to the transfer function H_d of Eq. (5.18) but to $H_a = s^{-2} H_d$. H_a has a single zero at $s = 0$ but the same poles as H_d. The pulse was slightly broadened for a better graphical display (the δ pulse is not plottable). The output signal (d) is the convolution of the input signal to the galvanometer (b) with the impulse response (c) of the galvanometer. (b) itself is the convolution of the broadband impulse (a) with the impulse response of the seismometer. (b) is then nearly the impulse response of the seismometer, and (d) is nearly the impulse response of the seismograph.

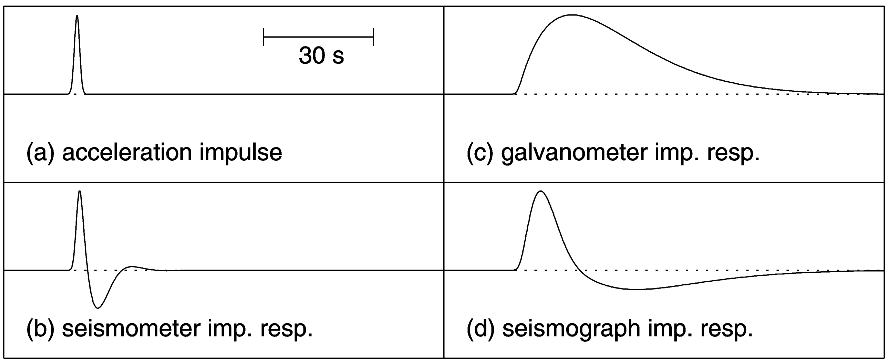

Fig. 5.4 WWSSN-LP system: Impulse responses of the seismometer, the galvanometer, and the seismograph. The input is an impulse of acceleration. The length of each trace is 2 minutes.

5.2.8 The mechanical pendulum

The simplest physical model for an inertial seismometer is a mass-and-spring system with viscous damping (Fig. 5.5).

We assume that the seismic mass is constrained to move along a straight line without rotation (i.e., it performs a pure translation). The mechanical elements are a mass of M kilograms, a spring with a stiffness S (measured in Newtons per meter), and a damping element with a constant of viscous friction R (in Newtons per meter per second). Let the time-dependent ground motion be $x(t)$, the absolute motion of the mass $y(t)$, and its motion relative to the ground $z(t) = y(t) - x(t)$. An acceleration $\ddot{y}(t)$ of the mass results from any external force $f(t)$ acting on the mass, and from the forces transmitted by the spring and the damper.

$$M\,\ddot{y}(t) = f(t) - S\,z(t) - R\,\dot{z}(t)\,. \tag{5.24}$$

Since we are interested in the relationship between $z(t)$ and $x(t)$, we rearrange this into

$$M\,\ddot{z}(t) + R\,\dot{z}(t) + S\,z(t) = f(t) - M\,\ddot{x}(t)\,. \tag{5.25}$$

We observe that an acceleration $\ddot{x}(t)$ of the ground has the same effect as an external force of magnitude $f(t) = -M\,\ddot{x}(t)$ acting on the mass in the absence of ground acceleration. We may thus simulate a ground motion $x(t)$ by applying a force $-M\,\ddot{x}(t)$ to the mass while the ground is not moving. The force is normally generated by sending a current through an electromagnetic transducer, but it may also be applied mechanically.

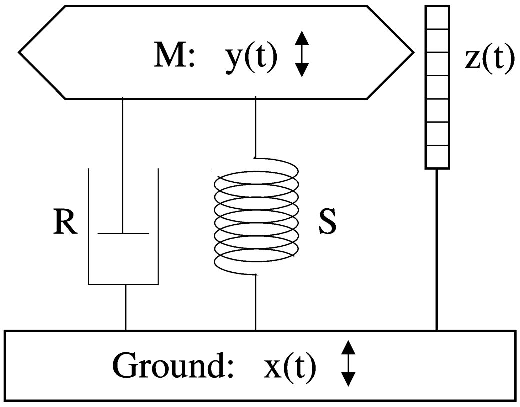

Fig. 5.5 Damped harmonic oscillator.

5.2.9 Transfer functions of pendulums and electromagnetic seismometers

According to Eqs.(5.7) and (5.8), Eq. (5.25) can be rewritten as

$$(s^2 M + sR + S)\,Z = F - s^2 M\,X \tag{5.26}$$

or

$$Z = (F/M - s^2 X)/(s^2 + sR/M + S/M)\,. \tag{5.27}$$

From this we can obtain directly the transfer functions $T_f = Z/F$ for the external force F and $T_d = Z/X$ for the ground displacement X. We arrive at the same result, expressed by the Fourier-transformed quantities, by simply assuming a time-harmonic motion $x(t) = \tilde{X}e^{j\omega t}/2\pi$ as well as a time-harmonic external force $f(t) = \tilde{F}e^{j\omega t}/2\pi$, for which Eq. (5.25) reduces to

$$(-\omega^2 M + j\omega R + S)\tilde{Z} = \tilde{F} + \omega^2 M\tilde{X} \tag{5.28}$$

or

$$\tilde{Z} = (\tilde{F}/M + \omega^2 \tilde{X}) / (-\omega^2 + j\omega R/M + S/M). \qquad (5.29)$$

While in mathematical derivations it is convenient to use the angular frequency $\omega = 2\pi f$ to characterize a sinusoidal signal of frequency f, and some authors omit the word „angular“ in this context, we reserve the term „frequency“ to the number of cycles per second.

By checking the behavior of $\tilde{Z}(\omega)$ in the limit of low and high frequencies, we find that the mass-and-spring system is a second-order high-pass filter for displacements and a second-order low-pass filter for accelerations and external forces (Fig. 5.6). Its corner frequency is $f_o = \omega_0 / 2\pi$ with $\omega_0 = \sqrt{S/M}$. This is at the same time the „eigenfrequency“ or „natural frequency“ with which the mass oscillates when the damping is negligible. At the angular frequency ω_0, the ground motion $\tilde{X}$ is amplified by a factor $\omega_0 M/R$ and phase shifted by $\pi/2$. The imaginary term in the denominator is usually written as $2\omega\omega_0 h$ where $h = R/(2\omega_0 M)$ is the numerical damping, i.e., the ratio of the actual to the critical damping. Viscous friction will no longer appear explicitly in our formulas; the symbol R will later be used for electrical resistance.

In order to convert the motion of the mass into an electric signal, the mechanical pendulum in the simplest case is equipped with an electromagnetic velocity transducer (see 5.3.7) whose output voltage we denote with $\tilde{U}$. We then have an electromagnetic seismometer, also called a geophone when designed for seismic exploration. When the responsivity of the transducer is E (volts per meter per second; $\tilde{U} = -Ej\omega\tilde{Z}$) we get

$$\tilde{U} = -j\omega E(\tilde{F}/M + \omega^2 \tilde{X}) / (-\omega^2 + 2j\omega\omega_0 h + \omega_0^2) \qquad (5.30)$$

from which, in the absence of an external force (i.e. $f(t) = 0, \tilde{F} = 0$), we obtain the frequency-dependent complex response functions

$$\tilde{H}_d(\omega) := \tilde{U} / \tilde{X} = -j\omega^3 E / (-\omega^2 + 2j\omega\omega_0 h + \omega_o^2) \qquad (5.31)$$

for the displacement,

$$\tilde{H}_v(\omega) := \tilde{U} / (j\omega\tilde{X}) = -\omega^2 E / (-\omega^2 + 2j\omega\omega_0 h + \omega_o^2) \qquad (5.32)$$

for the velocity, and

$$\tilde{H}_a(\omega) := \tilde{U} / (-\omega^2 \tilde{X}) = j\omega E / (-\omega^2 + 2j\omega\omega_0 h + \omega_0^2) \qquad (5.33)$$

for the acceleration.

With respect to its frequency-dependent response, the electromagnetic seismometer is a second-order high-pass filter for the velocity, and a band-pass filter for the acceleration. Its response to displacement has no flat part and no concise name. These responses (or, more precisely speaking, the corresponding amplitude responses) are illustrated in Fig. 5.6. IS 5.2 shows response curves for different subsystems of analog seismographs in more detail and EX 5.1 illustrates the construction of the simplified response curve (Bode diagram) of a now historical electronic seismograph.

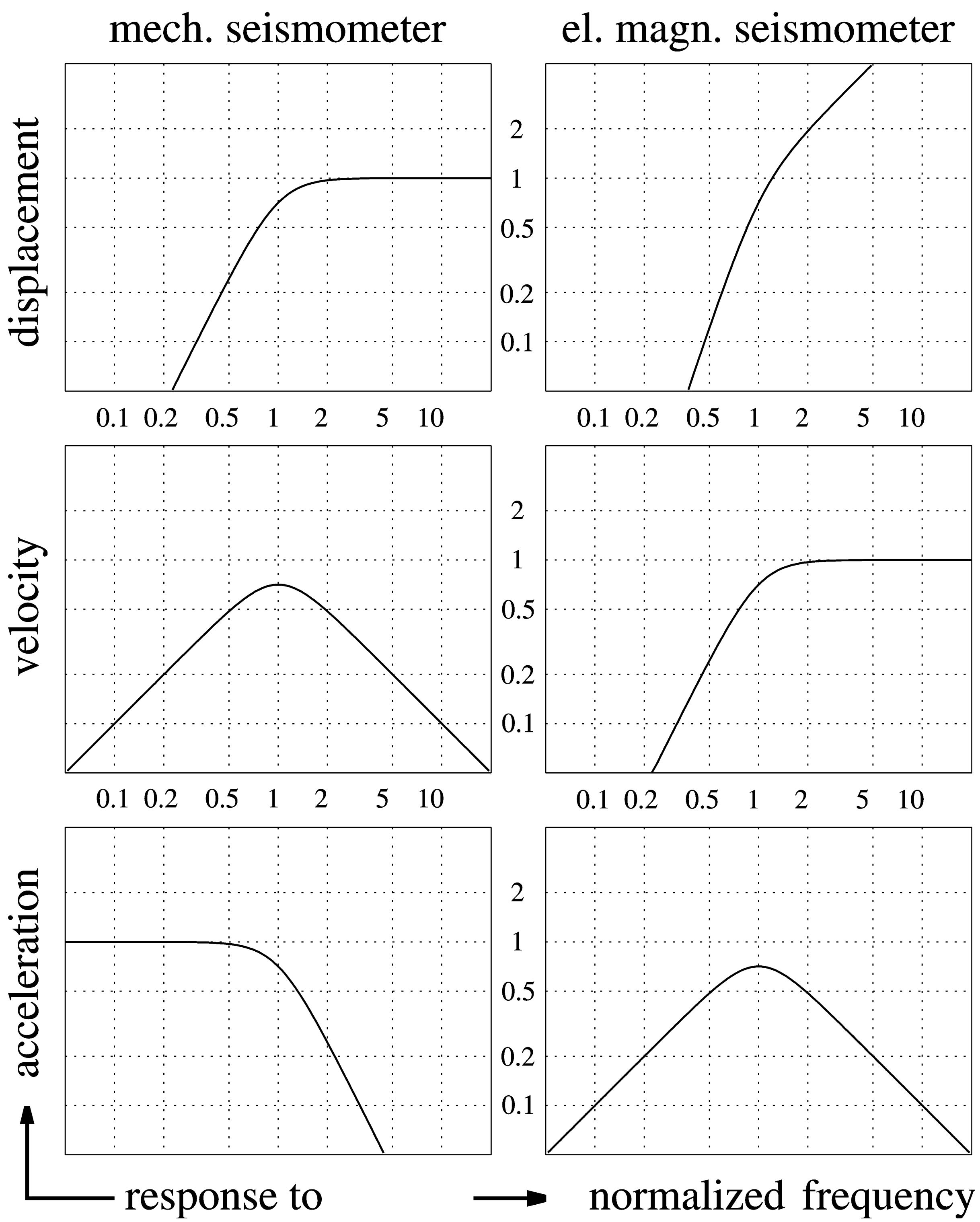

Fig. 5.6 Response curves of a mechanical seismometer (spring pendulum, left) and electro-dynamic seismometer (geophone, right) with respect to different kinds of input signals (displacement, velocity and acceleration, respectively). The normalized frequency is the signal frequency divided by the eigenfrequency (corner frequency) of the seismometer. All of these response curves have a second-order corner at the normalized frequency 1. In analogy to it, Fig. 5.26 shows the normalized step responses of second-order high-pass, band-pass and low-pass filters.

5.3 Design of seismic sensors

Although the mass-and-spring system of Fig. 5.5 is a useful mathematical model for a seismometer, it is incomplete as a practical design. The suspension must suppress five out of the six degrees of freedom of the seismic mass (three translational and three rotational) but the mass must still move as freely as possible in the remaining direction. This section discusses some of the mechanical concepts by which this can be achieved. In principle it is also possible to let the mass move in all directions and observe its motion with three orthogonally arranged transducers, thus creating a three-component sensor with only one suspended mass. Indeed, some historical instruments have made use of this concept. It is, however, difficult to minimize the restoring force and to suppress parasitic rotations of the mass when its translational motion is unconstrained. Modern three-component seismometers therefore have separate mechanical sensors for the three axes of motion.

5.3.1 Pendulum-type seismometers

Most seismometers are of the pendulum type, i.e., they let the mass rotate around an axis rather than move along a straight line (Fig. 5.7 to Fig. 5.10). The point bearings in our figures are for illustration only; most seismometers have crossed flexural hinges. Pendulums are not only sensitive to translational but also to angular acceleration. Since the rotational component in seismic waves is normally small, there is not much practical difference between linear-motion and pendulum-type seismometers. However, they may behave differently in technical applications or on a shake table where it is not uncommon to have noticeable rotations.

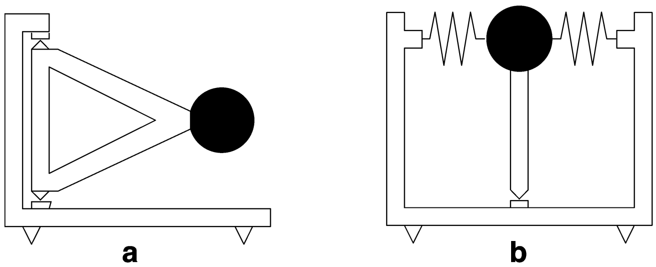

Fig. 5.7 (a) Garden-gate suspension; (b) Inverted pendulum.

For small translational ground motions the equation of motion of a pendulum is formally identical to Eq. (5.25) but z must then be interpreted as the angle of rotation. Since the rotational counterparts of the constants M, R, and S in Eq. (5.25) are of little interest in modern electronic seismometers, we will not discuss them further and refer the reader instead to the older literature, such as Berlage (1932) or Willmore (1979).

The simplest example of a pendulum is a mass suspended with a string or wire (like Foucault's pendulum). When the mass has small dimensions compared to the length ℓ of the string so that it can be idealized as a point mass, then the arrangement is called a mathematical pendulum. Its period of oscillation is $T = 2\pi\sqrt{\ell / g}$ where g is the gravitational acceleration. A mathematical pendulum of 1 m length has a period of nearly 2 seconds; for a period of 20 seconds the length has to be 100 m. Clearly, this is not a suitable design for a long-period seismometer.

5.3.2 Decreasing the restoring force

At low frequencies and in the absence of an external force, Eq. (5.25) can be simplified to $Sz = -M\ddot{x}$ and read as follows: a relative displacement of the seismic mass by $-\Delta z$ indicates a ground acceleration of magnitude

$$\ddot{x} = (S/M)\,\Delta z = \omega_0^{\;2}\Delta z = (2\pi/T_0)^2\,\Delta z \qquad (5.34)$$

where ω_0 is the angular eigenfrequency of the pendulum, and T_0 its eigenperiod. If Δz is the smallest mechanical displacement that can be measured electronically, then the formula determines the smallest ground acceleration that can be observed at low frequencies. For a given transducer, it is inversely proportional to the square of the free period of the suspension. A sensitive long-period seismometer therefore requires either a pendulum with a low eigenfrequency or a very sensitive transducer. Since the eigenfrequency of an ordinary pendulum is essentially determined by its size, and seismometers must be reasonably small, astatic suspensions have been invented that combine small overall size with a long free period.

The simplest astatic suspension is the "garden-gate" pendulum used in horizontal seismometers (Fig. 5.7a). The mass moves in a nearly horizontal plane around a nearly vertical axis. Its free period is the same as that of a mass suspended from the point where the plumb line through the mass intersects the axis of rotation (Fig. 5.8a). The eigenperiod $T_0 = 2\pi\sqrt{\ell / g \sin\alpha}$ is infinite when the axis of rotation is vertical (α =0), and is usually adjusted by tilting the whole instrument. This is one of the earliest designs for long-period horizontal seismometers.

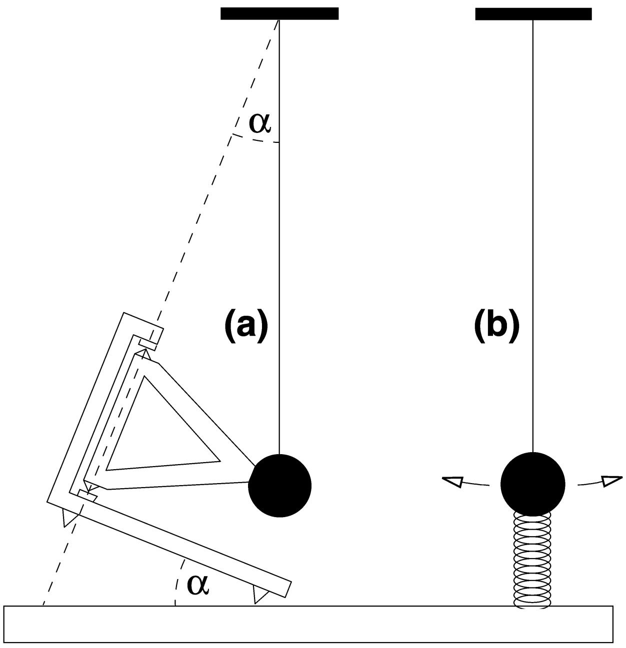

Fig. 5.8 Equivalence between a tilted "garden-gate" pendulum and a string pendulum. For a free period of 20 sec, the string pendulum must be 100 m long. The tilt angle α of a garden-gate pendulum with the same free period and a length of 30 cm is about 0.2°. The longer the period is made, the less stable it will be under the influence of small tilt changes. (b) Period-lengthening with an auxiliary compressed spring.

Another early design is the inverted pendulum held in stable equilibrium by springs or by a stiff hinge (Fig. 5.7b); a famous example is Wiechert's horizontal pendulum built around 1905.

An astatic spring geometry for vertical seismometers invented by LaCoste (1934) is shown in Fig. 5.9a. The mass is in neutral equilibrium and has therefore an infinite free period when three conditions are met: the spring is pre-stressed to zero length (i.e. the spring force is proportional to the total length of the spring), its end points are seen under a right angle from the hinge, and the mass is balanced in the horizontal position of the boom. A finite free period is obtained by making the angle of the spring slightly smaller than 90°, or by tilting the frame accordingly. By simply rotating the pendulum, astatic suspensions with a horizontal or oblique (Fig. 5.9b) axis of sensitivity can be constructed as well.

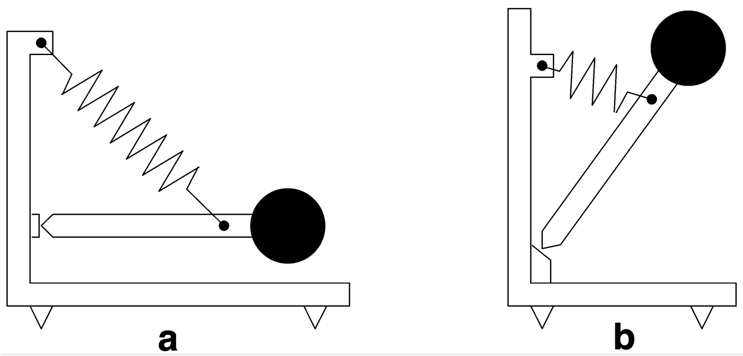

Fig. 5.9 LaCoste suspensions.

The astatic leaf-spring suspension (Fig. 5.10a, Wielandt, 1975), in a limited range around its equilibrium position, is comparable to a LaCoste suspension but is much simpler to manufacture. A similar spring geometry is also used in the triaxial seismometer Streckeisen STS2 (see Fig. 5.10b and DS 5.1). The delicate equilibrium of forces in astatic suspensions makes them susceptible to external disturbances such as changes in temperature; they are difficult to operate without a stabilizing feedback system.

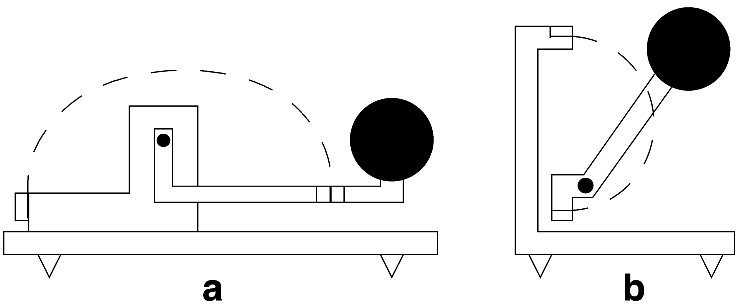

Fig. 5.10 Leaf-spring astatic suspensions.

Apart from genuinely astatic designs, almost any seismic suspension can be made astatic with an auxiliar spring acting normal to the line of motion of the mass and pushing the mass away from its equilibrium (Fig. 5.8b). The long-period performance of such suspensions, however, is quite limited. Neither the restoring force of the original suspension nor the destabilizing force of the auxiliary spring can be made perfectly linear (i.e. proportional to the displacement). While the linear components of the force may cancel, the nonlinear terms remain and cause the oscillation to become non-harmonic and even unstable at large amplitudes. Viscous and hysteretic behavior of the springs may also cause problems. The additional spring (which has to be soft) may introduce parasitic resonances. Modern seismometers do not use this concept and rely either on a genuinely astatic spring geometry or on the sensitivity of electronic transducers.

5.3.3 Sensitivity of horizontal seismometers to tilt

We have already seen (Eq. (5.25)) that a seismic acceleration of the ground has the same effect on the seismic mass as an external force. The largest such force is gravity. It is normally cancelled by the suspension, but when the seismometer is tilted, the projection of the vector of gravity onto the axis of sensitivity changes, producing a force that is in most cases undistinguishable from a seismic signal (Fig. 5.11). Undesired tilt at seismic frequencies may be caused by moving or variable surface loads such as cars, people, and atmospheric pressure. The resulting disturbances are a second-order effect in well-adjusted vertical seismometers but otherwise a first-order effect (see Rodgers, 1968; Rodgers, 1969). This explains why horizontal long-period seismic traces are always noisier than vertical ones. A short, impulsive tilt excursion is equivalent to a step-like change of ground velocity and therefore will cause a long-period transient in a horizontal broadband seismometer. For periodic signals, the apparent horizontal displacement associated with a given tilt increases with the square of the period (see also 5.8.1).

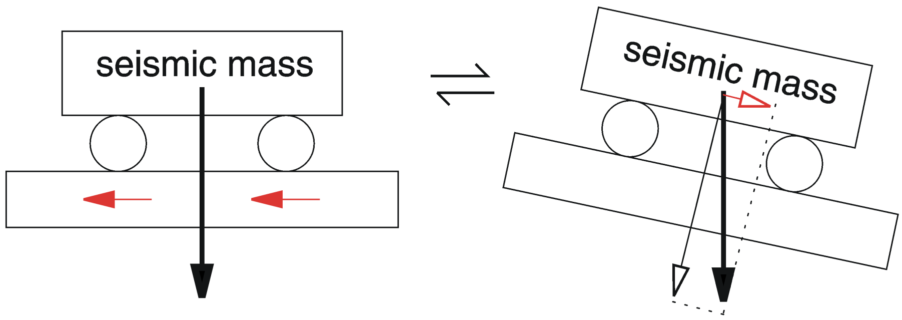

Fig. 5.11 The relative motion of the seismic mass is the same when the ground is accelerated to the left as when it is tilted to the right.

Fig. 5.12 illustrates the effect of barometrically induced ground tilt. Let us assume that the ground is vertically deformed by as little ± 1 µm over a distance of 3 km, and that this deformation oscillates with a period of 10 minutes. A simple calculation then shows that seismometers A and C see a vertical acceleration of $\pm 10^{-10}$ m/s² while B sees a horizontal acceleration of $\pm 10^{-8}$ m/s^2. The horizontal noise is thus 100 times larger than the vertical one. In absolute terms, even the vertical acceleration is by a factor of four above the minimum ground noise in one octave, as specified by the USGS Low Noise Model (see 5.5.1)

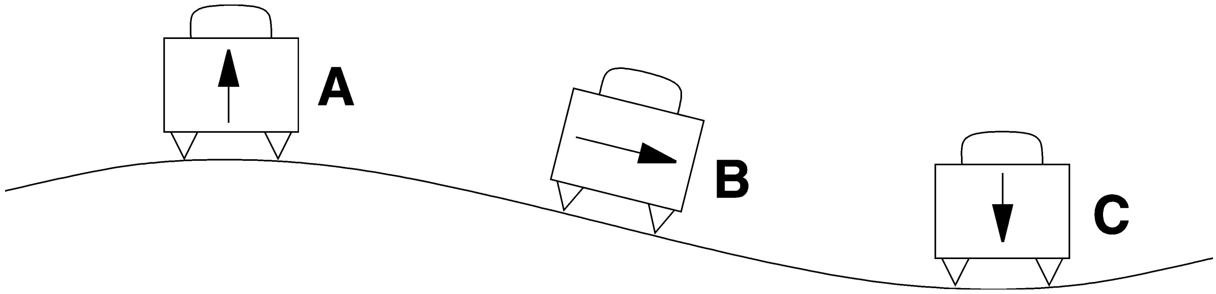

Fig. 5.12 Ground tilt caused by the atmospheric pressure is the main source of very-long-period noise on horizontal seismographs.

5.3.4 Direct effects of barometric pressure

Besides tilting the ground, the continuously fluctuating barometric pressure affects seismometers in at least three different ways: (1) when the seismometer is not enclosed in a hermetic housing, the mass will experience a variable buoyancy which can cause large disturbances in vertical sensors; (2) changes of pressure also produce adiabatic changes of temperature which affect the suspension (see the next subsection). Both effects can be greatly reduced by making the housing airtight or installing the sensor inside an external pressure jacket; however, then (3) the housing or jacket may be deformed by the pressure and these deformations may be transmitted to the seismic suspension as stress or tilt. While it is always worthwhile to protect vertical long-period seismometers from changes of the barometric pressure, it has often been found that horizontal long-period seismometers are less sensitive to barometric noise when they are not hermetically sealed. This, however, may cause other problems such as corrosion.

5.3.5 Effects of temperature

The equilibrium between gravity and the spring force in a vertical seismometer is disturbed when the temperature changes. Although thermally compensated alloys are available for springs, a self-compensated spring does not make a compensated seismometer. The geometry of the whole suspension changes with temperature; the seismometer must therefore be compensated as a whole. However, the different time constants involved prevent an efficient compensation at seismic frequencies. Short-term changes of temperature, therefore, must be suppressed by the combination of thermal insulation and thermal inertia. Special caution is required with seismometers where electronic components are enclosed with the mechanical sensor: these instruments heat themselves up when insulated and are then very sensitive to air drafts, so the insulation must at the same time suppress any possible air convection (see 5.5.3). Long-term (seasonal) changes of temperature do not interfere with the seismic signal (except when they cause convection in the vault) but may drive the seismic mass out of its operating range. Eq. (5.34) can be used to calculate the thermal drift of a vertical seismometer when the temperature coefficient of the spring force is formally assigned to the gravitational acceleration.

5.3.6 The homogeneous triaxial arrangement

In order to observe ground motion in all directions, a triple set of seismometers oriented towards North, East, and upward (Z) has been the standard for a century. However, horizontal and vertical seismometers differ in their construction, and it takes some effort to make their responses equal. An alternative way of manufacturing a three-component set is to use three sensors of identical construction whose sensitive axes are inclined against the vertical like the edges of a cube standing on its corner (Fig. 5.13), by an angle of arctan $\sqrt{2}$, or 54.7 degrees.

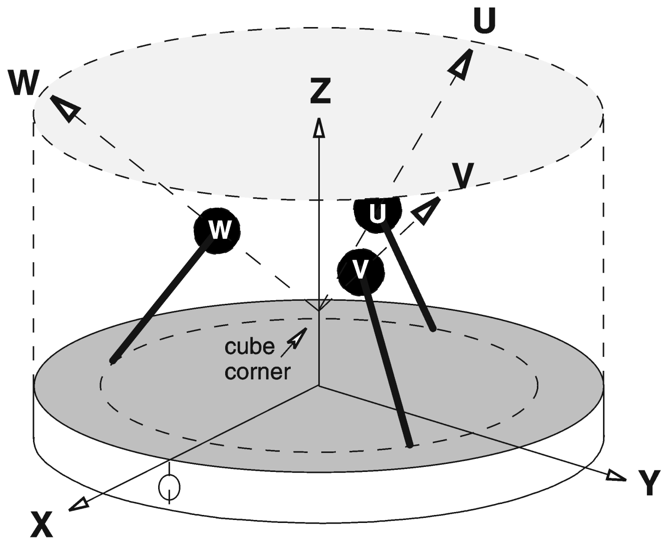

Fig. 5.13 The homogeneous triaxial geometry of the STS2 seismometer

At this time of writing, only one commercial seismometer, the Streckeisen STS2, makes use of this geometry, although it was not the first one to do so (see Gal´perin, 1955; Knothe, 1963; Melton and Kirkpatrick, 1970; Gal'perin, 1977). Since most seismologists want finally to see the conventional E, N and Z components of motion, the oblique components U, V, W of the STS2 are electrically recombined according to

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \frac{1}{\sqrt{6}} \begin{pmatrix} -2 & 1 & 1 \\ 0 & \sqrt{3} & -\sqrt{3} \\ \sqrt{2} & \sqrt{2} & \sqrt{2} \end{pmatrix} \begin{pmatrix} U \\ V \\ W \end{pmatrix} \tag{5.35}$$

The X axis of the STS2 seismometer is normally oriented towards East; the Y axis then points North. Noise originating in one of the sensors of a triaxial seismometer will appear on all three outputs (except for Y being independent of U). Its origin can be traced by transforming the X, Y and Z signals back to U, V and W with the inverse (transposed) matrix. Disturbances affecting only the horizontal outputs are unlikely to originate in the seismometer and are, in general, due to tilt. Disturbances of the vertical output only may be related to temperature, barometric pressure, or electrical problems affecting all three sensors in the same way as an unstable power supply.

5.3.7 Electromagnetic velocity sensing and damping

The simplest transducer both for sensing motions and for exerting forces is an electromagnetic (electrodynamic) device where a coil moves in the field of a permanent magnet, as in a loudspeaker (Fig. 5.14). The motion induces a voltage in the coil; a current flowing in the coil produces a force. From the conservation of energy it follows that the responsivity of the coil-magnet system as a force transducer, in Newtons per Ampere, and its responsivity as a velocity transducer, in Volts per meter per second, are identical. The units are in fact the same (remember that 1Nm = 1Joule = 1VAs). When such a velocity transducer is loaded with a resistor, thus permitting a current to flow, then according to Lenz's law it generates a force, opposing the motion. This effect is used to damp the mechanical free oscillation of passive seismic sensors (geophones and electromagnetic seismometers).

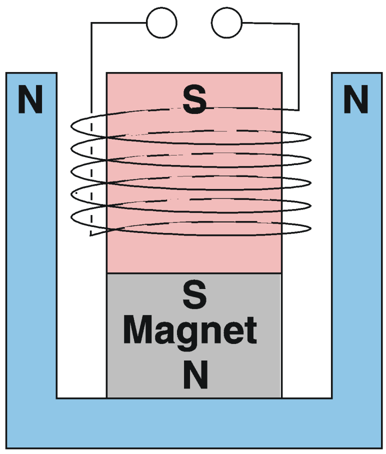

Fig. 5.14 Electromagnetic velocity and force transducer.

We have so far treated the damping of passive sensors as if it were a viscous effect in the mechanical receiver. Actually, only a small part h_m of the damping is due to mechanical causes. The main contribution normally comes from the electromagnetic transducer which is suitably shunted for this purpose. Its contribution is

$$h_{el} = E^2 / 2M\omega_0 R_d \qquad (5.36)$$

where R_d is the total damping resistance (the sum of the resistances of the coil and of the external shunt). The total damping h_m+h_{el} is preferably chosen as $1/\sqrt{2}$, a value that defines a second-order Butterworth filter characteristic, and gives a maximally flat response in the passband (such as the velocity-response of the electromagnetic seismometer in Fig. 5.6).

5.3.8 Electronic displacement sensing

At very low frequencies, the output signal of electromagnetic transducers becomes too small to be useful for seismic sensing. One then uses active electronic transducers where a carrier signal, usually in the audio frequency range, is modulated by the motion of the seismic mass. The basic modulating device is an inductive or capacitive half-bridge. Inductive half-bridges are detuned by a movable magnetic core. They require no electric connections to the moving part and are environmentally robust; however their sensitivity appears to be limited by the granular nature of magnetism. Capacitive half-bridges (Fig. 5.15) are realized as three-plate capacitors where either the central plate or the outer plates move with the seismic mass. Their sensitivity is limited by the ratio between the electrical noise of the demodulator and the electrical field strength; which is typically a hundred times better than that of the inductive type. The comprehensive paper by Jones and Richards (1973) on the design of capacitive transducers still represents state-of-the-art in all essential aspects.

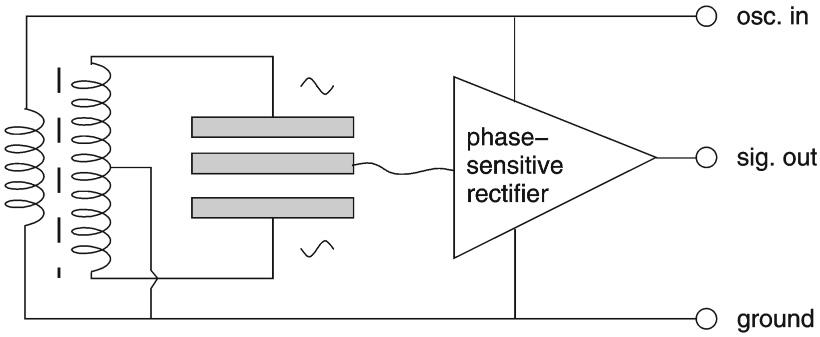

Fig. 5.15 Capacitive displacement *transducer* (Blumlein bridge).

5.4 Force-balance accelerometers and seismometers

5.4.1 The force-balance principle

In a conventional passive seismometer, the inertial force produced by a seismic ground motion deflects the mass from its equilibrium position, and the displacement or velocity of the mass is then converted into an electric signal. This principle of measurement is now used for short-period seismometers only. Long-period or broadband seismometers are built according to the force-balance principle. The inertial force is compensated (or 'balanced') with an electrically generated force so that the seismic mass moves as little as possible; of course some small motion is still required because otherwise the inertial force could not be observed. The feedback force is generated with an electromagnetic force transducer or ‘forcer’ (Fig. 5.14). The electronic circuit (Fig. 5.16) is a servo loop, as in an analog chart recorder.

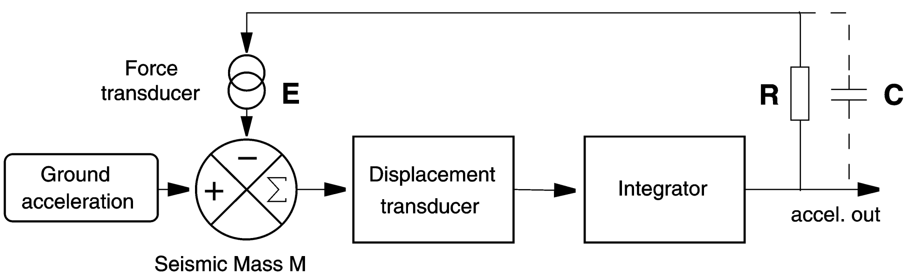

Fig. 5.16 Feedback circuit of a force-balance accelerometer (FBA). The motion of the mass is controlled by the sum of two forces: the inertial force due to ground acceleration, and the negative feedback force. The electronic circuit adjusts the feedback force so that the forces very nearly cancel each other.

A servo loop is most effective when it contains an integrator, in which case the offset of the mass is exactly nulled in the time average (in a chart recorder, the difference between the input signal and a voltage indicating the pen position, is nulled). Due to unavoidable delays in the feedback loop, force-balance systems have a limited bandwidth; however, at frequencies where they are effective, they force the mass to move with the ground by generating a feedback force strictly proportional to ground acceleration. When the force is proportional to the current in the transducer, then the current, the voltage across the feedback resistor R, and the output voltage are all proportional to ground acceleration. Thus we have converted the acceleration into an electric signal without depending on the precision of a mechanical suspension.

The response of a force-balance system is approximately inverse to the gain of the feedback path. It can be easily modified by giving the feedback path a frequency-dependent gain. For example, if we make the capacitor C large so that it determines the feedback current, then the gain of the feedback path increases linearly with frequency and we have a system whose responsivity to acceleration is inverse to frequency and thus flat to velocity over a certain passband. We will look more closely at this option in section 5.4.3.

5.4.2 Force-balance accelerometers

Fig. 5.16 without the capacitor *C* represents the circuit of a force-balance accelerometer (FBA), a device that is widely used for earthquake strong-motion recording, for measuring tilt, and for inertial navigation. By equating the inertial and the electromagnetic force, it is easily seen that the responsivity (the output voltage per ground acceleration) is

$$U_{out} / \ddot{x} = MR / E \tag{5.37}$$

where *M* is the seismic mass, *R* the total resistance of the feedback path, and *E* the responsivity of the forcer (in N /A). The conversion is determined by only three passive components of which the mass is error-free by definition (it defines the inertial reference), the resistor is a nearly ideal component, and the force transducer very precise because the motion is small. Some accelerometers do not have a built-in feedback resistor; the user can insert a resistor of his own choice and thus select the gain. The responsivity in terms of current per acceleration is simply $I_{out} / \ddot{x} = M / E$.

FBAs work down to zero frequency but the servo loop becomes ineffective at some upper corner frequency f_0 (usually a few hundred to a few thousand Hz), above which the arrangement acts like an ordinary inertial displacement sensor. The feedback loop behaves like an additional stiff spring; the response of the FBA sensor corresponds to that of a mechanical pendulum with the eigenfrequency f_0, as is schematically represented in the left panels of Fig. 5.6.

5.4.3 Velocity broadband seismometers

For broadband seismic recording with high sensitivity, an output signal proportional to ground acceleration is unfavorable. At high frequencies, sensitive accelerometers are easily saturated by traffic noise or impulsive disturbances. At low frequencies, a system with a response flat to acceleration generates a permanent voltage at the output as soon as the suspension is not completely balanced. The system would soon be saturated by the offset voltage resulting from thermal drift or tilt. What we need is a band-pass response in terms of acceleration, or equivalently a high-pass response in terms of ground velocity, like that of a normal electromagnetic seismometer (geophone, right panels in Fig. 5.6) but with a lower corner frequency.

The desired velocity broadband (VBB) response is obtained from the FBA circuit by adding paths for differential feedback and integral feedback (Fig. 5.17). A large capacitor *C* is chosen so that the differential feedback dominates throughout the desired passband. While the feedback current is still proportional to ground acceleration as before, the voltage across the capacitor *C* is a time integral of the current, and thus proportional to ground velocity. This voltage serves as the output signal. The output voltage per ground velocity, i.e. the apparent generator constant E_{app} of the feedback seismometer, is

$$E_{app} = V_{out} / \dot{x} = M / EC \,. \tag{5.38}$$

Again the response is essentially determined by three passive components. Although a capacitor with a solid dielectric is not quite as ideal a component as a good resistor. the response is still linear and very stable.

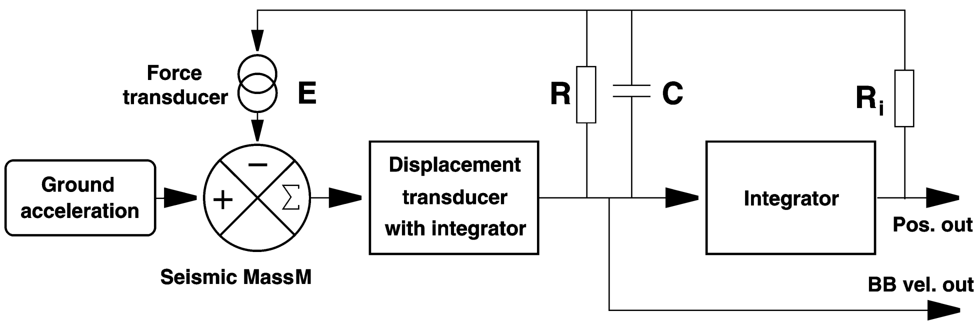

Fig. 5.17 Feedback circuit of a VBB (velocity-broadband) seismometer. As in Figure 5.16, the seismic mass is the summing point of the inertial force and the negative feedback force.

The output signal of the second integrator is normally accessible at the ,,mass position" output. It does not indicate the actual position of the mass but indicates where the mass would go if the feedback were switched off. ”Centering" the mass of a feedback seismometer has the effect of discharging the integrator so that its full operating range is available for the seismic signal. The mass-position output is not normally used for seismic recording but is useful as a state-of-health diagnostic, and is used in some calibration procedures.

The relative strength of the integral feedback increases at lower frequencies while that of the differential feedback decreases. These two components of the feedback force are of opposite phase (- $\pi/2$ and $\pi/2$ relative to the output signal, respectively). At certain low frequency, the two contributions are of equal strength and cancel each other out. This is the lower corner frequency of the closed-loop system. Since the closed-loop response is inverse to that of the feedback path, one would expect to see a resonance in the closed-loop response at this frequency. However, the proportional feedback remains and damps the resonance; the resistor R acts as a damping resistor. At lower frequencies, the integral feedback dominates over the differential feedback, and the closed-loop response to ground velocity decreases with the square of the frequency. As a result, the feedback system behaves like a conventional electromagnetic seismometer and can be described by the usual three parameters: free period, damping, and generator constant. In fact, electronic broadband seismometers, even if their actual electronic circuit is more complicated than presented here, follow the simple theoretical response of electromagnetic seismometers more closely than those ever did.

As far as the response is concerned, a force-balance circuit as described here may be seen as a means to convert a moderately stable short- to medium-period suspension into a stable electronic long-period or very-long-period seismometer. The corner period may be increased by a large factor, for example 24-fold (from 5 to 120 sec) in the STS2 seismometer or even 200-fold (from 0.6 to 120 sec) in a version of the CMG3. But this factor says little about the performance of the system. Feedback does not reduce the instrumental noise; a large extension of the bandwidth is useless when the system is noisy. According to Eq. (5.34), short-period suspensions must be combined with extremely sensitive transducers for a satisfactory sensitivity at long periods.

At some high frequency, the loop gain falls below unity. This is the upper corner frequency of the feedback system which marks the transition between a response flat to velocity and one flat to displacement. A well-defined and nearly ideal behavior of the seismometer, as at the lower corner frequency, should not be expected here both because the feedback becomes ineffective and because most suspensions have parasitic resonances slightly above the electrical corner frequency (otherwise they could have been designed for a larger bandwidth). The detailed response at the high-frequency corner, however, rarely matters since the upper corner frequency is usually outside the passband of the recorder. Its effect on the transfer function in most cases can be modeled as a small, constant delay (a few milliseconds) over the whole VBB passband.

5.4.4 Other methods of bandwidth extension

The force-balance principle permits the construction of high-performance, broadband seismic sensors but is not easily applicable to geophone-type sensors because fitting a displacement transducer to these is difficult. Sometimes it is desirable to broaden the response of an existing geophone without a mechanical redesign.

The simplest solution is to send the output signal of the geophone through a filter that removes its original response (this is called an inverse filtration) and replaces it by some other desired response, preferably that of a geophone with a lower eigenfrequency. The analog, electronic version of this process would only be used in connection with direct visible recording; for all other purposes, one would implement the filtration digitally as part of the data processing. Suitable filter algorithms are contained in seismic software packages, as listed in 5.9.

Alternatively, the bandwidth of a geophone may be enlarged by strong damping. This does not enhance the gain outside the passband but rather reduces it inside the passband; nevertheless, after appropriate amplification, the net effect is an extension of the bandwidth towards longer periods. Strong damping is obtained by connecting the coil to a preamplifier whose input impedance is negative. The total damping resistance, which is otherwise limited by the resistance of the coil (Eq. (5.36)), can then be made arbitrarily small. The response of the over-damped geophone is flat to acceleration around its free period. It can be made flat to velocity by an approximate (band-limited) integration. This technique is used in the Lennartz Le-1d and Le-3d seismometers (see DS 5.1) whose electronic corner period can be up to 40 times larger than the mechanical one. Although these are not strictly force-balance sensors, they take advantage of the fact that active damping (which is a form of negative feedback) greatly reduces the relative motion of the mass.

5.5 Seismic noise, site selection and installation

Electronic seismographs can be designed for any desired magnification of the ground motion. A practical limit, however, is imposed by the presence of undesired signals which must not be magnified so strongly as to obscure the record. Such signals are usually referred to as noise and may be of seismic, instrumental, or environmental origin. Seismic noise is treated in Chapter 4. Instrumental self-noise may have mechanical and electronic sources and will be discussed in the next section. Here we focus on those general aspects of site selection and of seismometer installation aimed at the reduction of environmental noise. For technical details on site selection as well as vault, tunnel and borehole installations see Chapter 7.

5.5.1 The USGS low-noise model

The USGS low-noise model (see Peterson, 1993, Fig. 5.18) is a graphical and numerical representation of the lowest vertical seismic noise levels observed worldwide, and is extremely useful as a reference for the quality of a site or of an instrument. Its origin and properties are discussed in Chapter 4.

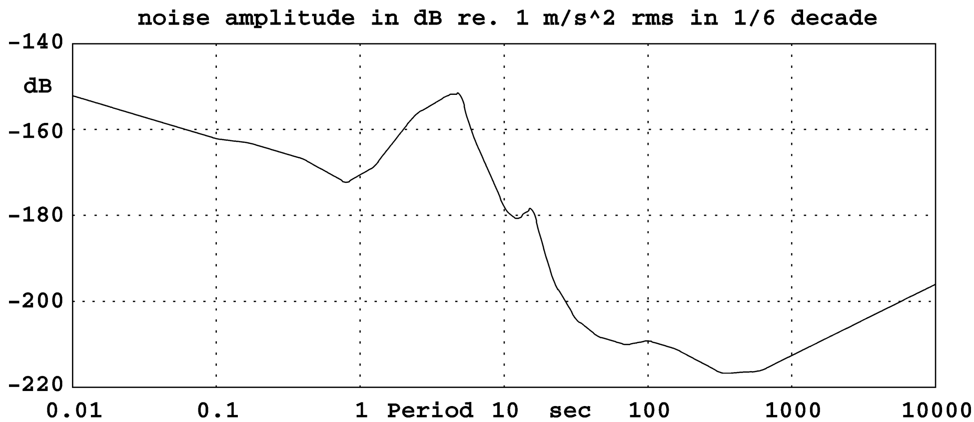

Fig. 5.18 The USGS New Low Noise Model (NLNM), here expressed as RMS amplitude of ground acceleration in a constant relative bandwidth of one-sixth decade.

5.5.2 Site selection

Site selection for a permanent station is always a compromise between two conflicting requirements: infrastructure and low seismic noise. The noise level depends on the geological situation and on the proximity of sources, some of which are usually associated with the infrastructure. A seismograph installed on solid basement rock can be expected to be fairly insensitive to local disturbances while one sitting on a thick layer of soft sediments will be noisy even in the absence of identifiable sources. As a rule, the distance from potential sources of noise, such as roads and inhabited houses, should be very much larger than the thickness of the sediment layer. Broadband seismographs can be successfully operated in major cities when the geology is favourable; in unfavourable situations, such as in sedimentary basins, only deep mines (4.3.2 and 7.4.3) and boreholes (7.4.5) may offer acceptable noise levels.

Obviously, most sites have a noise level above the Low Noise Model, some of them by a large factor. This factor, however, is not uniform over time or over the seismic frequency band. At short periods (< 2 s), a noise level within a factor of 10 of the NLNM may be considered very good in most areas. Short-period noise at most sites is predominantly man-made and somewhat larger in the horizontal components than in the vertical. At intermediate periods (2 to 20 s), marine microseisms dominate. They have similar amplitudes in the horizontal and vertical directions and have large seasonal variations. In winter they may be 50 dB above the NLNM. At longer periods, the vertical ground noise is often within 10 or 20 dB of the NLNM even at otherwise noisy stations. The horizontal long-period noise may nevertheless be horrible at the same station due to tilt-gravity coupling (see 5.3.3). It may be larger than vertical noise by a factor of up to 300, the factor increasing with period. Therefore, a site can be considered as favourable when the horizontal noise at 100 to 300 sec is within 20 dB (i.e., a factor of 10 in amplitude) above the vertical noise. Tilt may be caused by traffic, wind, or local fluctuations of the barometric pressure. Large tilt noise is sometimes observed on concrete floors when an unventilated cavity exists underneath; the floor then acts like a membrane. Such noise can be identified by its linear polarization and its correlation with the barometric pressure. Even on an apparently solid foundation, the long-period noise often correlates with the barometric pressure (see Beauduin et al., 1996). If the situation can not be remedied otherwise, the barometric pressure should be recorded with the seismic signal and used for a correction. An example is shown in Fig. 2.21. For very-broadband seismographic stations, barometric recording is generally recommended.

Besides ground noise, environmental conditions must be considered. An aggressive atmosphere may cause corrosion, wind and short-term variations of temperature may induce noise, and seasonal variations of temperature may exceed the manufacturer’s specifications for unattended operation. Seismometers must be protected against these conditions, sometimes by hermetic containers as described in the next subsection. As a precaution, cellars and vaults should be checked for signs of flooding.

5.5.3 Seismometer installation

We will briefly describe the installation of a portable broadband seismometer inside a building, vault, or cave. First, we mark the orientation of the sensor on the floor. This is best done with a geodetic gyroscope, but a magnetic compass will do in most cases. The magnetic declination must be taken into account. Since a compass may be deflected inside a building, the direction should be taken outside and transferred to the site of installation. A laser pointer may be useful for this purpose. When the magnetic declination is unknown or unpredictable (such as at high latitudes or in volcanic areas), the orientation should be determined with a sun compass.

To isolate the seismometer from stray currents, small glass or perspex plates should be cemented to the ground under its feet. Then the seismometer is installed, tested, and wrapped with a thick layer of thermally insulating material. The type of material does not matter very much; alternate layers of fibrous material and heat-reflecting blankets are probably most effective. The edges of the blankets should be taped to the floor around the seismometer.

Electronic seismometers produce heat and may induce convection in any open space inside the insulation; it is therefore important that the insulation has no gap and fits the seismometer tightly. Another method of insulation is to surround the seismometer with a large box which is then filled with fine styrofoam seeds. For a permanent installation under unfavourable environmental conditions, the seismometer should be enclosed in a hermetic container. A problem with such containers (as with all seismometer housings), however, is that they cause tilt noise when they are deformed by the barometric pressure. Essentially three precautions are possible: (1) either the base-plate is carefully cemented to the floor, or (2) it is made so massive that its deformation is negligible, or (3) a "warp-free" design is used, as described by Holcomb and Hutt (1992) for the STS1 seismometers (see DS 5.1). Also, some fresh desiccant (silicagel) should be placed inside the container, even into the vacuum bell of STS1 seismometers. Fig. 5.19 illustrates the shielding of the STS2 seismometers (see DS 5.1) in the German Regional Seismic Network (GRSN).

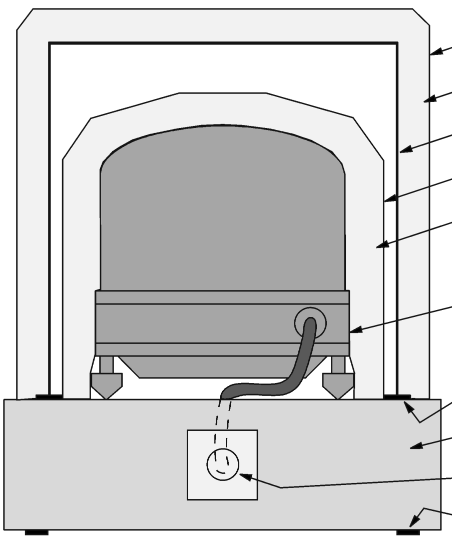

Fig. 5.19 The STS2 seismometer of the GRSN inside its shields.

Installation procedures for broadband seismometers are proposed in sub-Chapter 7.4 as well as on the web sites of the GeoForschungsZentrum Potsdam under http://www.gfz-potsdam.de/geofon/index.html (click on How to get a well-performing VBB Station?) and of the Seismological Lab, University of California at Berkeley: http://www.seismo.berkeley.edu/seismo/bdsn/instrumentation/guidelines.html.

5.5.4 Magnetic shielding

Broadband seismometers are to some degree sensitive to magnetic fields since all thermally compensated spring materials are slightly magnetic. This may be noticeable when the seismometers are operated in industrial areas or in the vicinity of dc-powered railway lines. Magnetic interferences are definitely suspect when the long-period noise follows a regular time table. Shields can be manufactured from permalloy (μ metal) but they are expensive and of limited efficiency. An active compensation is often preferable. It may consist of a three-component fluxgate magnetometer that senses the field near the seismometer, an electronic driver circuit in which the signal is integrated with a short time constant (a few milliseconds), and a three-component set of Helmholtz coils which compensate changes of the magnetic field. The permanent geomagnetic field should not be compensated; the resulting offsets of the fluxgate outputs can be compensated electrically before the integration, or with a small permanent magnet mounted near the fluxgate.

5.6 Instrumental self-noise

All modern seismographs use semiconductor amplifiers which, like other active (power-dissipating) electronic components, produce continuous electronic noise whose origin is manifold but ultimately related to the quantisation of the electric charge. Electromagnetic transducers, such as those used in geophones, also produce thermal electronic noise (resistor noise). The contributions from semiconductor noise and resistor noise are often comparable, and together limit the sensitivity of the system. Another source of continuous noise, the Brownian (thermal) motion of the seismic mass, may be noticeable when the mass is very small (less than a few grams). Presently, however, manufactured seismometers have sufficient mass to make the Brownian noise negligible against electronic noise and we will therefore not discuss it here. Seismographs may also suffer from transient disturbances originating in slightly defective semiconductors or in the mechanical parts of the seismometer when subject to stresses. The present section is mainly concerned with identifying and measuring instrumental noise.

5.6.1 Electromagnetic short-period seismographs

Electromagnetic seismometers and geophones are passive sensors whose self-noise is of purely thermal origin and does not increase at low frequencies as it does in active (power-dissipating) devices. Their output signal level , however, is comparatively low, so a low-noise preamplifier must be inserted between the geophone and the recorder (Fig. 5.20). Unfortunately the noise of the preamplifier does increase at low frequencies and limits the overall sensitivity. We will call this combination an electromagnetic short-period seismograph or , EMS for short. It is now rarely used for long-period or broadband recording because of the superior performance of feedback instruments.

The sensitivity of an EMS is normally limited by amplifier noise(see Fig. 5.20). However, this noise does not depend on the amplifier alone but also on the impedance of the electromagnetic transducer (which can be chosen within wide limits). Up to a certain impedance the amplifier noise voltage is nearly constant, but then it increases linearly with the impedance, due to a noise current flowing out of the amplifier input. On the other hand, the signal voltage increases with the square root of the impedance. The best signal-to-noise ratio is therefore obtained with an optimum source impedance defined by the corner between voltage and current noise, which is different for each type of amplifier and also depends on frequency. Vice versa, when the transducer is given, the amplifier must be selected for low noise at the relevant impedance and frequency.

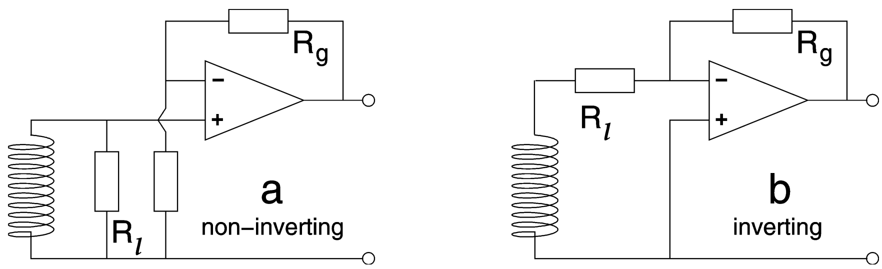

Fig. 5.20 Two alternative circuits for an EMS preamplifier with a low-noise op-amp. The non-inverting circuit is generally preferable when the damping resistor R_l is much larger than the coil resistance and the inverting circuit when it is comparable or smaller. However, the relative performance also depends on the noise specifications of the op-amp. The gain is adjusted with R_g.

The electronic noise of an EMS can be predicted when the technical data of the sensor and the amplifier are known. Semiconductor noise increases at low frequencies; amplifier specifications must apply to seismic rather than audio frequencies. In combination with a given sensor, the noise can then be expressed as an equivalent seismic noise level and compared to real seismic signals or to the NLNM (Fig. 5.18). As an example, Fig. 5.21 shows the self-noise of one of the better seismometer-amplifier combinations. It resolves minimum ground noise between 0.1 and 10 s period. Discussions and more examples are found in Riedesel et al. (1990) and in Rodgers (1992, 1993 and 1994). The result is easily summarized:

> Most well-designed seismometer-amplifier combinations resolve minimum ground noise up to 6 or 8 s period, that is, to the microseismic peak. A few of them may make it to about 15 s; they marginally resolve the secondary microseismic peak. To resolve minimum ground noise up to 30 s is hopeless, as is obvious from Fig. 5.21. Ground noise falls and electronic noise rises so rapidly beyond a period of 20 s that the cross-over point can not be substantially moved towards longer periods. Of course, at a reduced level of sensitivity, restoring long-period signals from short-period sensors may make sense, and the long-period surface waves of sufficiently large earthquakes may well be recorded with short-period electromagnetic seismometers.

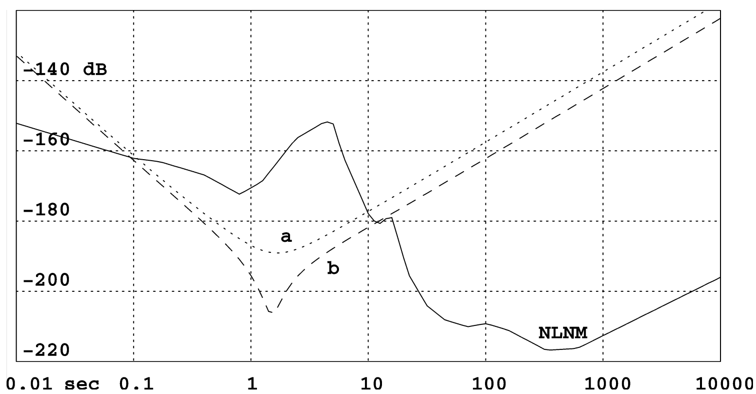

Fig. 5.21 Electronic self-noise of the input stage of a short-period seismograph. The EMS is a Sensonics Mk3 with two 8 kOhm coils in series and tuned to a free period of 1.5 s. The amplifier is the LT1012 op-amp. The curves a and b refer to the circuits of Fig. 5.20. NLNM is the USGS New Low Noise Model (Fig. 5.18). The ordinate gives rms noise amplitudes in dB relative to 1 m/s^2 in 1/6 decade.

Amplifier noise can be observed by locking the sensor or tilting it until the mass is firmly at a stop, or by substituting it with an ohmic resistor that has the same resistance as the coil. If these manipulations do not significantly reduce the noise, then obviously the seismograph does not resolve seismic noise. However, this is only a test, not a way to precisely measure the electronic self-noise. A locked sensor or a resistor do not exactly represent the electric impedance of the unlocked sensor.

5.6.2 Force-balance seismometers

Force-balance sensors can not be tested for instrumental noise with the mass locked. Their self-noise can thus only be observed in the presence of seismic signals and seismic noise. Although seismic noise is generally a nuisance in this context, natural signals may also be useful as test signals. Marine microseisms should be visible on any sensitive seismograph whose seismometer has a free period of one second or longer; they normally form the strongest continuous signal on a broadband seismograph. However, their amplitude exhibits large seasonal and geographical variations.

For broadband seismographs at quiet sites, the tides of the solid Earth are a reliable and predictable test signal. They have a predominant period of slightly less than 12 hours and an amplitude in the order of 10^{-6} m/s^2. While normally invisible in the raw data, they may be extracted by low-pass filtration with a corner frequency of about 1 mHz. For this purpose it is helpful to have the original data available with a sampling rate of 1 per second or less. By comparison with the predicted tides, the gain and polarity of the seismograph may be checked. A seismic broadband station that records Earth's tides is likely to be up to international standards.

For a quantitative determination of the instrumental noise, two instruments must be operated side by side (see Holcomb, 1989; Holcomb, 1990). One can then determine the coherency between the two records and assume that coherent noise is seismic and incoherent noise is instrumental. This works well if the reference instrument is known to be a good one, but the method is not safe. The two instruments may respond coherently to environmental disturbances caused by barometric pressure, temperature, the supply voltage, magnetic fields, vibrations, or electromagnetic waves. Nonlinear behaviour (intermodulation) may produce coherent but spurious long-period signals. When no good reference instrument is available, the test should be done with two sensors of a different type, in the hope that they will not respond in the same way to non-seismic disturbances.

The analysis for coherency is somewhat tricky in detail. When the transfer functions of both instruments are precisely known, it is in fact theoretically possible to measure the seismic signal and the instrumental noise of each instrument separately as a function of frequency. Alternatively, one may assume that the transfer functions are not so well known but the reference instrument is noise-free; in this case the noise and the relative transfer function of the other instrument can be determined. As with all statistical methods, long time series are required for reliable results. We offer a computer program UNICROSP (see 5.9) for the analysis.

5.6.3 Transient disturbances

Most new seismometers produce spontaneous transient disturbances, i.e., quasi miniature earthquakes caused by stresses in the mechanical components. Although they do not necessarily originate in the spring, their waveform at the output seems to indicate a sudden and permanent (step-like) change in the spring force. Long-period seismic records are sometimes severely degraded by such disturbances. The transients often die out within months or years; if they do not, and especially when their frequency increases, corrosion must be suspected. Manufacturers try to mitigate the problem with a low-stress design and by aging the components or the finished seismometer (by extended storage, vibrations, or heating and cooling cycles). It is sometimes possible to virtually eliminate transient disturbances by hitting the pier around the seismometer with a hammer, a procedure that is recommended in each new installation.

5.7 Calibration

5.7.1 Electrical and mechanical calibration

The calibration of a seismograph establishes knowledge of the relationship between its input (the ground motion) and its output (an electric signal), and is a prerequisite for a reconstruction of the ground motion. Since precisely known ground motions are difficult to generate, one makes use of the equivalence between ground accelerations and external forces on the seismic mass (Eq. (5.25)), and calibrates seismometers with an electromagnetic force generated in a calibration coil. If the factor of proportionality between the current in the coil and the equivalent ground acceleration is known, then the calibration is a purely electrical measurement. Otherwise, the missing parameter - either the transducer constant of the calibration coil, or the responsivity of the sensor itself - must be determined from a mechanical experiment in which the seismometer is subject to a known mechanical motion or a tilt. This is called an absolute calibration. Since it is difficult to generate precise mechanical calibration signals over a large bandwidth, one does not attempt normally to determine the complete transfer function in this way.

The present section is mainly concerned with the electrical calibration although the same methods may also be used for the mechanical calibration on a shake table (see 5.8.1). Specific procedures for the mechanical calibration without a shake table are presented in 5.8.2 and 5.8.3.

5.7.2 General conditions

Calibration experiments are disturbed by seismic noise and tilt and should therefore be carried out in a basement room. However, the large operating range of modern seismometers permits a calibration with relatively large signal amplitudes, making background noise less of a problem than one might expect. Thermal drift is more serious because it interferes with the long-period response of broadband seismometers. For a calibration at long periods, seismometers must be protected from draft and allowed sufficient time to reach thermal equilibrium. Visible and digital recording in parallel is recommended. Recorders themselves must be absolutely calibrated before they can serve to calibrate seismometers. The input impedance of recorders as well as the source impedance of sensors should be measured so that a correction can be applied for the loss of signal in the source impedance.

5.7.3 Calibration of geophones

Some simple electrodynamic seismometers (geophones) have no calibration coil. The calibration current must then be sent through the signal coil. There it produces an ohmic voltage in addition to the output signal generated by the motion of the mass. The undesired voltage can be compensated in a bridge circuit (see Willmore, 1959); the bridge is zeroed with the seismic mass locked or at a stop. When the calibration current and the output voltage are digitally recorded, it is more convenient to use only a half-bridge (Fig. 5.22) and to compensate the ohmic voltage numerically. The program CALEX (see 5.9.2) has provisions to do this automatically.

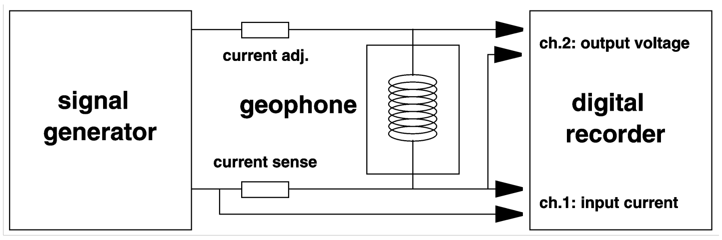

Fig. 5.22 Half-bridge circuit for calibrating electromagnetic seismometers

An alternative method has been proposed by Rodgers et al. (1995). A known direct current through the signal coil is interrupted and the resulting transient response of the seismometer recorded. The generator constant is then determined from the amplitude of the pulse.

Electrodynamic seismometers whose seismic mass moves along a straight line require no mechanical calibration when the size of the mass is known. The electromagnetic part of the numerical damping is inversely proportional to the total damping resistance (Eq.(5.36)); the factor of proportionality is $E^2/2M\omega_0$, so the generator constant E can be calculated from electrical calibrations with different resistive loads (Fig. 5.23).

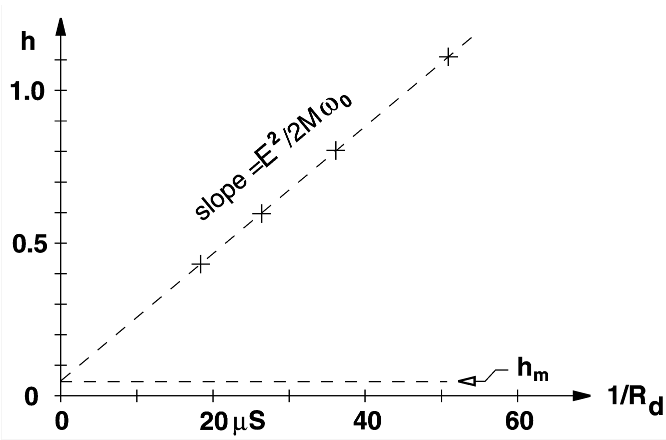

Fig. 5.23 Determining the generator constant from a plot of damping versus total damping resistance $R_d = R_{coil} + R_{load}$. The horizontal units are microsiemens (reciprocal Megohms).

5.7.4 Calibration with sinewaves

With a sinusoidal input, the output of a linear system is also sinusoidal, and the ratio of the two signal amplitudes is the absolute value of the transfer function. An experiment with sine-waves therefore permits an immediate check of the transfer function, without any a-priori knowledge of its mathematical form and without waveform modeling. This is often the first step in the identification of an unknown system (see EX 5.3 and 5.4). A computer program, however, would be required for deriving a parametric representation of the response from the measured values. A calibration with arbitrary signals, as described later, is more straightforward for this purpose.

When only analog equipment is available, the calibration coil or the shake table should be driven with a sinusoidal test signal and the input and output signals recorded with a chart recorder or an X-Y recorder. On the latter, the signals can be plotted as a Lissajous ellipse (Fig. 5.24) from which both the amplitude ratio and the phase can be read with good accuracy (see Mitronovas and Wielandt, 1975). For the calibration of high-frequency geophones, an oscilloscope may be used in place of an X-Y-recorder. The signal period should be measured with a counter or a stop watch because the frequency scale of sine-wave generators is often inaccurate.

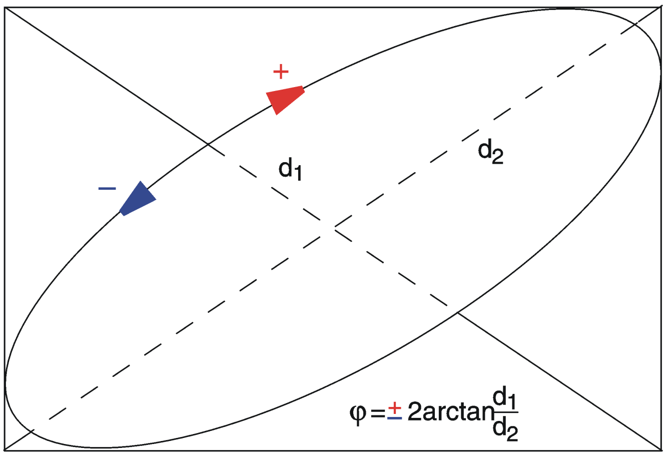

Fig. 5.24 Measuring the phase between two sine-waves with a Lissajous ellipse.

The accuracy of the graphic evaluation depends on the purity of the sine-wave. A better accuracy, of course, can be obtained with a numerical analysis of digitally recorded data. By fitting sine-waves to the signals, amplitudes and phases can be extracted for just one precisely known frequency at a time; distortions of the input signal don't matter. For best results, the frequency should be fitted as well, the fit should be computed for an integer number of cycles, and offsets should be removed from the data. A computer program „SINFIT" is offered for this purpose (see 5.9).

Eigenfrequency f_0 and damping h of electromagnetic and most other seismometers can be determined graphically with a set of standard resonance curves on double-logarithmic paper. (an empty sheet of such paper is contained in EX 5.1). The measured amplitude ratios are plotted as a function of frequency f on the same type of paper and overlain with the standard curves (Fig. 5.25). The desired quantities can be read directly. The method is simple but not very precise.

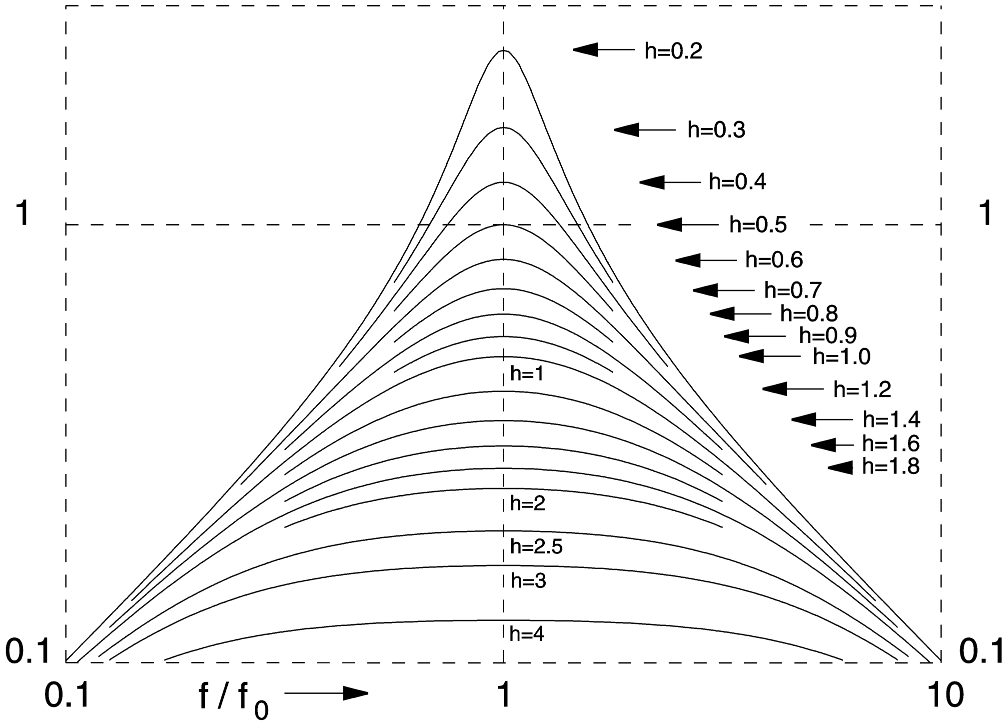

Fig. 5.25 Normalized resonance curves.

5.7.5 Step response and weight-lift test

The simplest, but only moderately accurate, calibration method is to observe the response of the system to a step input. It can be generated by switching on or off a current through the calibration coil, or by applying or removing a constant mechanical force on the seismic mass, usually by lifting a weight. Horizontal sensors used to be calibrated with a V-shaped thread attached to the mass at one end, to a fixed point at the other end, and to the test weight at half length. The thread was then burned off for a soft release.

The step-response experiment can be used both for a relative and an absolute calibration; when applicable, it is probably the simplest method for the latter. Using a known test weight *w* and knowing the seismic mass *M*, we also know the test signal: it is a step in acceleration whose magnitude is *w*/*M* times gravity (times a geometry factor when the force is applied through a thread). In case of a rotational pendulum, a correction factor must be applied when the force does not act at the center of gravity. The method has lost its former importance because the seismic mass of modern seismometers is not easily accessible, and the correction factor for rotational motion is rarely supplied by the manufacturers.

Interestingly, in the case of a simple electromagnetic seismometer with linear motion and a known mass, not even a calibration coil or the insertion of a test mass are required for an absolute calibration. A simple experiment where a step current is sent through the signal coil of the undamped sensor can supply all parameters of interest: the generator constant *E*, the free period, and the mechanical damping. The method is described in Chapter 4 of the old MSOP (see Willmore, 1979) and in EX 5.2. An alternative method is proposed in section 5.7.3.

In the context of relative calibration, the step-response method is still useful as a quick and intuitive test, and has the advantage that it can be evaluated by hand. Software like PREPROC or CALEX covers the step response as well (see 5.9). Fig. 5.26 shows the characteristic step responses of second-order high-pass, band-pass, and low-pass filters with $1/\sqrt{2}$ of critical damping. The amplitude responses of these systems were shown in the left column of Fig. 5.6.

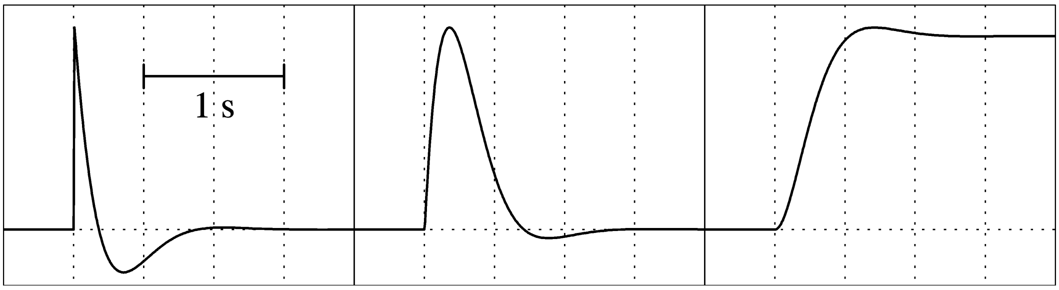

Fig. 5.26 Normalized step responses of second-order high-pass, band-pass and low-pass filters. The respective transfer functions are the same as in Fig. 5.6, left column. Compare also Fig. 5.4 which shows slightly smoothed impulse responses.

Each response is a strongly damped oscillation around its asymptotic value. With the specified damping, the systems are Butterworth filters, and the amplitude decays to $e^{-\pi}$ or 4.3% within one half-wave. The ratio of two subsequent amplitudes of opposite polarity is known as the overshoot ratio. It can be evaluated for the numerical damping h: when x_i and x_{i+n} are two (peak-to-peak) amplitudes n periods apart, with integer or half-integer n, then

$$\frac{1}{h^2} = 1 + \left(\frac{2\pi n}{\ln x_i - \ln x_{i+n}} \right)^2 \qquad (5.39)$$

The free period, in principle, can also be determined from the impulse or step response of the damped system but should be measured preferably without electrical damping so that more oscillations can be observed. A system with the free period T_0 and damping h oscillates with the period $T_0 / \sqrt{1-h^2}$ and the overshoot ratio $\exp(-\pi h / \sqrt{1-h^2})$. The determination of seismometer parameters from the step response is also explained in EX 5.2.

5.7.6 Calibration with arbitrary signals

In most cases, the purpose of calibration is to obtain the parameters of an analytic representation of the transfer function. Assuming that its mathematical form is known, the task is to determine its parameters from an experiment in which both the input and the output signals are known. Since only a signal that has been digitally recorded is known with some accuracy, both the input and the output signal should be recorded with a digital recorder. As compared to other methods where a predetermined input signal is used and only the output signal is recorded, recording both signals has the additional advantage of eliminating the transfer function of the recorder from the analysis.

Calibration is a classical inverse problem that can be solved with standard least-squares methods. The general solution is schematically depicted in Fig. 5.27. A computer algorithm (filter 1) is implemented that represents the seismometer as a filter and permits the computation of its response to an arbitrary input. An inversion scheme (3) is programmed around the filter algorithm in order to find best-fitting filter parameters for a given pair of input and output signals. The purpose of filter 2 is explained below. The sensor is then calibrated with a test signal (4) for which the response of the system is sensitive to the unknown parameters but which is otherwise arbitrary. When the system is linear, parameters determined from one test signal will also predict the response to any other signal.

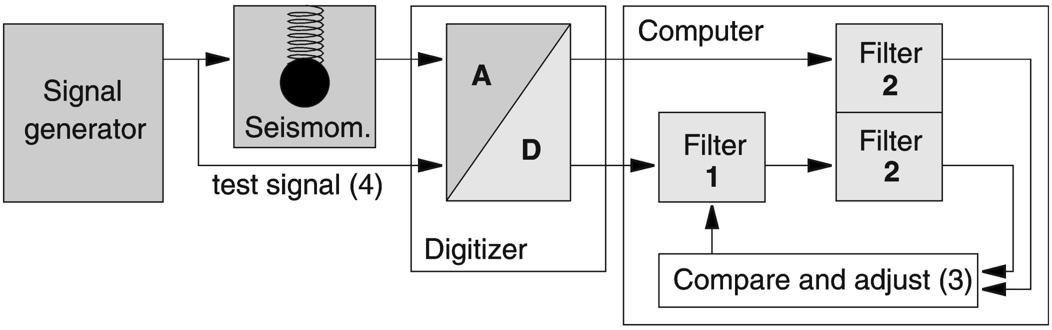

Fig. 5.27 Block diagram of the CALEX procedure. Storage and retrieval of the data are omitted from the figure.

When the transfer function has been correctly parameterized and the inversion has converged, then the residual error consists mainly of noise, drift, and nonlinear distortions. At a signal level of about one-third of the operating range, typical residuals are 0.03% to 0.05% rms for force- balance seismometers and $\geq$ 1% for passive electrodynamic sensors.

The approximation of a rational transfer function with a discrete filtering algorithm is not trivial. For the program CALEX (see 5.9) we have chosen an impulse-invariant recursive filter (see Schuessler, 1981). This method formally requires that the seismometer has a negligible response at frequencies outside the Nyqvist bandwidth (see 5.2.3) of the recorder, a condition that is severely violated by most digital seismographs; but this problem can be circumvented with an additional digital low-pass filtration (filter 2 in Fig. 5.27) that limits the bandwidth of the simulated system. Signals from a typical calibration experiment are shown in Fig. 5.28. A sweep as a test signal permits the residual error to be visualized as a function of time or frequency; since essentially only one frequency is present at a time, the time axis may as well be interpreted as a frequency axis. An exercise with the CALEX program is contained in EX 5.4.

With an appropriate choice of the test signal, other methods like the calibration with sinewaves step functions, random noise or random telegraph signals, can be duplicated and compared to each other. An advantage of the CALEX algorithm is that it makes no use of special properties of the test signal, such as being sinusoidal, periodic, step-like or random. Therefore, test signals can be short (a few times the free period of the seismometer) and can be generated with the most primitive means, even by hand (you may turn the dial of a sinewave generator by hand, or even produce the test signal with a battery and a potentiometer). A breakout box or a special cable, however, may be required for feeding the calibration signal into the digital recorder.

Some other routines for seismograph calibration and system identification are contained in the PREPROC software package (see Plešinger et al., 1996). An overview of identification software which has also been made publicly available on the Internet is given by Plešinger (1998).

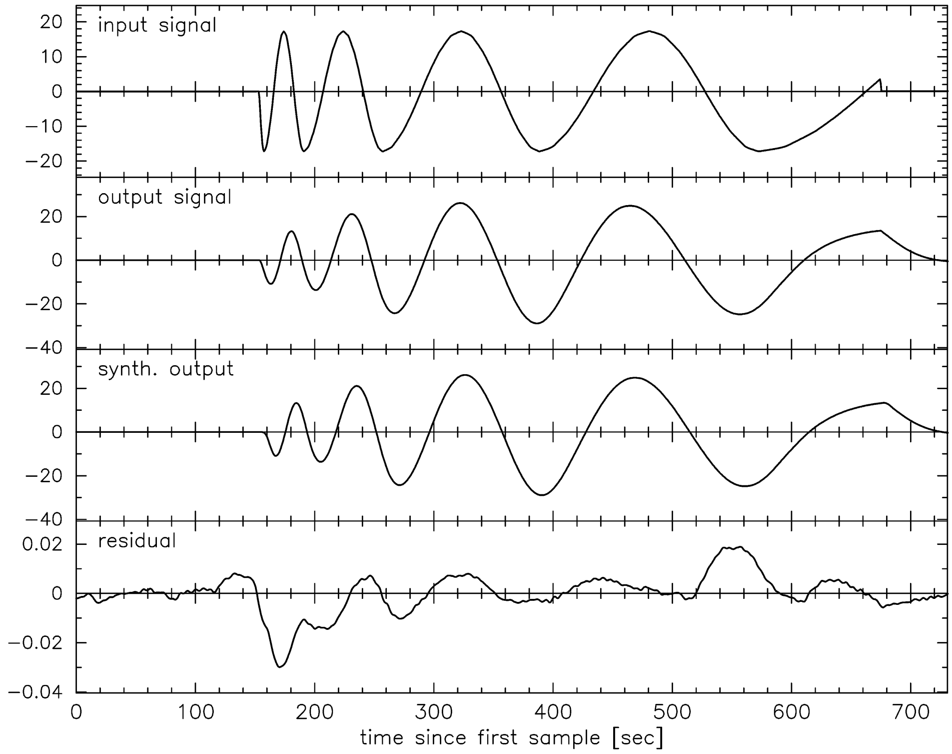

Fig. 5.28 Electrical calibration of an STS2 seismometer with CALEX. Traces from top to bottom: input signal (a sweep with a total duration of 10 min); observed output signal; modeled output signal; residual. The rms residual is 0.05 % of the rms output.

5.7.7 Calibration of triaxial seismometers

In a triaxial seismometer such as the Streckeisen STS2 (Fig. 5.13), transfer functions in a strict sense can only be attributed to the individual U,V,W sensors, not to the X,Y,Z outputs. Formally, the response of a triaxial seismometer to arbitrary ground motions is described by a nearly diagonal 3 x 3 matrix of transfer functions relating the X,Y,Z output signals to the X,Y,Z ground motions. (This is also true for conventional three-component sets if they are not perfectly aligned; only the composition of the matrix is slightly different.) If the U,V,W sensors are reasonably well matched, the effective transfer functions of the X,Y,Z channels have the traditional form and their parameters are weighted averages of those of the U,V,W sensors. The X,Y,Z outputs, therefore, can be calibrated as usual. For the simulation of horizontal and vertical ground accelerations via the calibration coils, each sensor must receive an appropriate portion of the calibration current. For the vertical component, this is approximately accomplished by connecting the three calibration coils in parallel. For the horizontal components and also for a more precise excitation of the vertical, the calibration current or voltage must be split into three individually adjustable and invertible U,V,W components. These are then adjusted so that the test signal appears only at the desired output of the seismometer.

It is also possible to calibrate the U, V, and W sensors separately - the Z output may be used for this purpose - and then to average the U, V, W transfer functions or parameters with a matrix whose elements are the squares of those of the matrix in Eq. (5.35):

$$\begin{pmatrix} T_X \\ T_Y \\ T_Z \end{pmatrix} = \frac{1}{6} \begin{pmatrix} 4 & 1 & 1 \\ 0 & 3 & 3 \\ 2 & 2 & 2 \end{pmatrix} \begin{pmatrix} T_U \\ T_V \\ T_W \end{pmatrix} \tag{5.40}$$

Eqs. (5.35) and (5.40) are only approximate since they assume the mechanical alignment to be perfect. Actually the resistor network that determines the matrix in Eq. (5.35) has been adjusted in each instrument so as to compensate for slight misalignments of the U, V and W sensors. The difference between the nominal and the actual matrix, however, can be ignored in the context of calibration.

5.7.8 Calibration against a reference sensor

Using ground noise or other seismic signals, an unknown sensor can be calibrated against a known one by operating the two sensors side by side (Pavlis and Vernon, 1994). As a method of relative (frequency-response) calibration, the method is limited to a frequency band where suitable seismic signals well above the instrumental noise level are present and spatially coherent between the two instruments. However, when the frequency response of the unknown sensor can be measured electrically, then its absolute gain may be determined quite accurately with this method. The two responses should be digitally equalized before the amplitudes are compared.

In a similar way, the orientation of a three-component borehole seismometer may be determined by comparison with a reference instrument at the surface.

5.8 Procedures for the mechanical calibration

5.8.1 Calibration on a shake table

Using a shake table is the most direct way of obtaining an absolute calibration. In practice, however, precision is usually poor outside a frequency band roughly from 0.5 to 5 Hz. At higher frequencies, a shake table loaded with a broadband seismometer may develop parasitic resonances, and inertial forces may cause undesired motions of the table. At low frequencies, the maximum displacement and thus the signal-to-noise ratio may be insufficient, and the motion may be non-uniform due to friction or roughness in the bearings. Still worse, most shake tables do not produce a purely translational motion but also some tilt. This has two undesired side-effects: the angular acceleration may be sensed by the seismometer, and gravity may be coupled into the seismic signal (see 5.3.3). Tilt can be catastrophic for the horizontal components at long periods since the error increases with the square of the signal period. One might think that a tilt of 10 µrad per mm of linear motion should not matter; however, at a period of 30 s, such a tilt will induce seismic signals twice as large as those originating from the linear motion. At a period of 1 s, the effect of the same tilt would be negligible. Long-period measurements on a shake table, if possible at all, require extreme care.

Although all calibration methods mentioned in the previous section are applicable on a shake table, the preferred method would be to record both the motion of the table (as measured with a displacement transducer) and the output signal of the seismometer, and to analyse these signals with CALEX or equivalent software (see 5.9). Depending on the definition of active and passive parameters, one might determine only the absolute gain (responsivity, generator constant) or any number of additional parameters of the frequency response.

5.8.2 Calibration by stepwise motion

The movable tables of machine tools like lathes and milling machines, and of mechanical balances, can replace a shake table for the absolute calibration of seismometers. The idea is to place the seismometer on the table, let it come to equilibrium, then move the table manually by a known amount and let it rest again. The apparent "ground" motion can then be calculated by inverse filtration of the output signal and compared with the known mechanical displacement. Since the calculation involves triple integrations, offset and drift must be carefully removed from the seismic trace. The main contribution to drift in the apparent horizontal „ground" velocity comes from tilt associated with the motion of the table. With the method subsequently described, it is possible to separate the contributions of displacement and tilt from each other so that the displacement can be reconstructed with good accuracy. This method of calibration is most convenient because it uses only normal workshop equipment; the inherent precision of machine tools and the use of relatively large displacements eliminate the problem of measuring small mechanical displacements. A FORTRAN program named DISPCAL is available for the evaluation (see 5.9).

The precision of the method depends on avoiding two main sources of error:

1 - The restitution of ground displacement from the seismic signal (a process of inverse filtration) is uncritical for broadband seismometers but requires a precise knowledge of the transfer function for short-period seismometers. Instruments with unstable parameters (such as electromagnetic seismometers) must be electrically calibrated while installed on the test table. However, once the response is known, the restitution of absolute ground motion is no problem even for a geophone with a free period of 0.1 s.

2 - The effect of tilt can only be removed from the displacement signal when the motion is sudden and short. The tilt is unknown during the motion, and is integrated twice in the calculation of the displacement. So the longer the interval of motion, the larger the effect the unknown tilt will be on the displacement signal. Practically, the motion may last about one second on a manually-operated machine tool, and about a quarter-second on a mechanical balance. It may be repeated at intervals of a few seconds.

Static tilt before and after the motion produces linear trends in the velocity which are easily removed. The effect of tilt during the motion, however, can be removed only approximately by interpolating the trends before and after the motion. The computational evaluation consists in the following major steps (Fig. 5.29):

1) the trace is deconvolved with the velocity transfer function of the seismometer;
2) the trace is piecewise detrended so that it is close to zero in the motion-free intervals; interpolated trends are removed from the interval of motion;

3) the trace is integrated;
4) The displacement steps are measured and compared to the actual motion.

In principle, a single step-like displacement is all that is needed. However, the experiment takes so little time that it is convenient to produce a dozen or more equal steps, average the results, and do some error statistics. On a milling machine or lathe, it is recommended to install some mechanical device that stops the motion after each full turn of the spindle. On a balance, the table is repeatedly moved from stop to stop. The displacement may be measured with a micrometer dial or determined from the motion of the beam (Fig. 5.30). From the mutual agreement between a number of different experiments, and from the comparison with shake-table calibrations, we estimate the absolute accuracy of the method to be better than 1%.

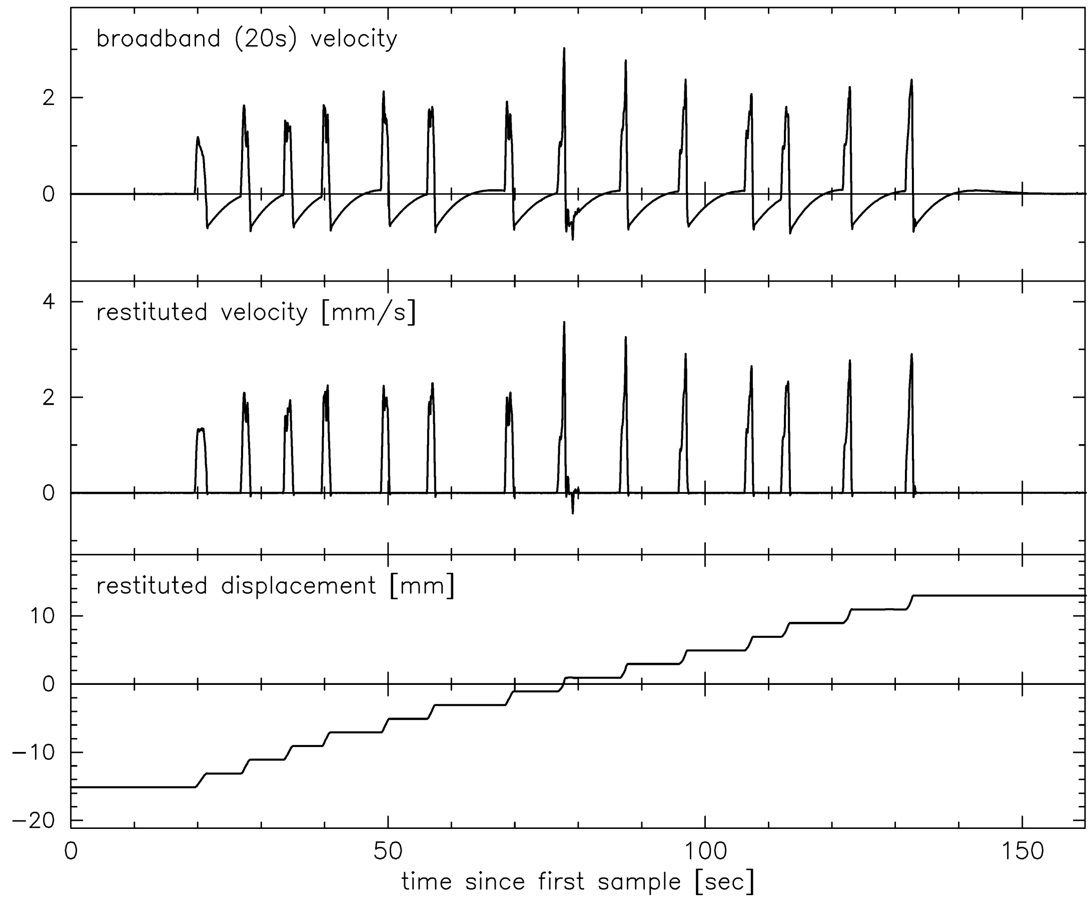

Fig. 5.29 Absolute mechanical calibration of an STS1-BB (20s) seismometer on the table of a milling machine, evaluated with DISPCAL. The table was manually moved in 14 steps of 2 mm each (one full turn of the dial at a time). Traces from top to bottom: recorded BB output signal; restored and de-trended velocity; restored displacement.

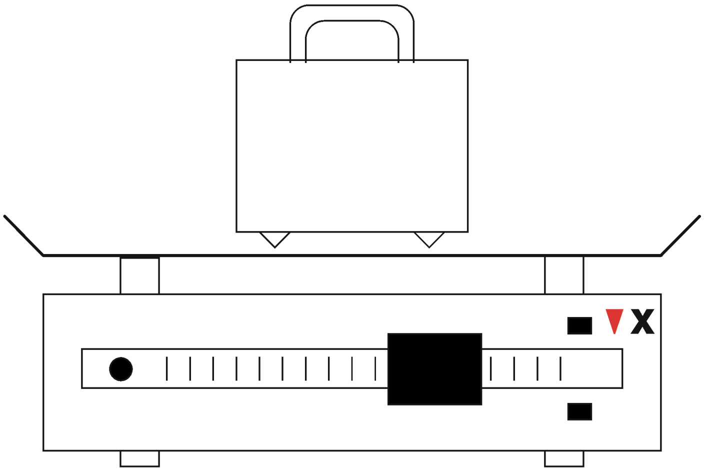

Fig. 5.30 Calibrating a vertical seismometer on a mechanical balance. When a mass of w_1 grams at some point X near the end of the beam is in balance with w_2 grams on the table or compensated with a corresponding shift of the sliding weight, then the motion of the table is by a factor w_1/w_2 smaller than the motion at X.

5.8.3 Calibration with tilt

Accelerometers can be statically calibrated on a tilt table. Starting from a horizontal position, the fraction of gravity coupled into the sensitive axis equals the sine of the tilt angle. (A tilt table is not required for accelerometers with an operating range exceeding $\pm 1g$; these are simply turned over.). Force-balance seismometers normally have a mass-position output which is a slowly responding acceleration output. With some patience, this output can likewise be calibrated on a tilt table; the small static tilt range of sensitive broadband seismometers, however, may be inconvenient. The transducer constant of the calibration coil is then obtained by sending a direct current through it and comparing its effect with the tilt calibration.

Finally, by exciting the coil with a sine-wave whose acceleration equivalent is now known, the absolute calibration of the broadband output is obtained. The method is not explained in more detail here because we propose a simpler method. Anyway, seismometers of the homogeneous-triaxial type can not be calibrated in this way because they do not have X,Y,Z mass-position signals.

The method which we propose (for horizontal components only; program TILTCAL) is similar to what was described under 5.8.2, but this time we excite the seismometer with a known step of tilt, and evaluate the recorded output signal for acceleration rather than displacement. This is simple: the difference between the drift rates of the de-convolved velocity trace before and after the step equals the tilt-induced acceleration; no baseline interpolation is involved. In order to produce repeatable steps of tilt, it is useful to prepare a small lever by which the tilt table or the seismometer can quickly be tilted back and forth by a known amount. The tilt may exceed the static operating range of the seismometer; then one has to watch the output signal and reverse the tilt before the seismometer comes to a stop.

5.9 Free software

Source codes of several computer programs mentioned in the text can be downloaded from the FTP sites given here (last update: August 2001). Some of these programs are used in IS 5.2 and EX 5.1 through EX 5.5. They are stand-alone programs for calibrating and testing seismometers and do not form a package for general seismic processing such as SAC, SEISMIC UNIX, PITSA, or PREPROC (see below). Wherever appropriate, test data, auxiliary files, and read.me files with detailed instructions are included. The Fortran programs do not produce graphic output but some of them generate data files in ASCII format from which the signals can be plotted.

5.9.1 Programs by J. Bribach in Turbo Pascal:

The CALIBRAT package consists of three programs:

- RESPONSE calculates the response function of a complete signal chain from seismometer/geophone via preamplifier and filter stages to analog or digital recorder. This response is represented as Amplitude/Phase Plot versus frequency (Bode Diagram) or as Poles and Zeros.

- CALISEIS calculates missing seismometer parameters by step response, and it designs the electronic scheme of the preamplifier stage as well as the calibration inputs to seismometer and preamplifier.

- SEISFILT designs single and complex electronic filter stages.

A short program description can be found in Volume 2 (see PD 5.1). The complete software can be downloaded from ftp.gfz-potsdam.de/pub/home/dss/brib/calibrat under the file names calibrat.zip (containing the programmes and sources) and Calibrat.doc (containing the complete program description).

5.9.2 Programs by E. Wielandt in Fortran:

- CALEX: Determines parameters of the transfer function of a seismometer from the response to an arbitrary input signal (both of which must have been digitally recorded). The transfer function is implemented in the time domain as an impulse-invariant recursive filter. Parameters represent the corner periods and damping constants of subsystems of first and second order.

- DISPCAL: Determines the generator constant of a horizontal or vertical seismometer from an experiment where the seismometer is moved stepwise on the table of a machine tool or a mechanical balance. Another, more automated version of the program is available as DISPCAL1.

- TILTCAL: Determines the generator constant of a horizontal seismometer from an experiment where the seismometer is stepwise tilted.

- SINFIT: fits sine-waves to a pair of sinusoidal signals and determines their frequency and the relative amplitude and phase.

- UNICROSP: Estimates seismic and instrumental noise separately from the coherency of the output signals of two seismometers.

- NOISECON: converts noise specifications into all kind of standard and non standard units and compares them to the USGS New Low Noise model (see Peterson, 1993). Interactive program available in BASIC, FORTRAN, C and as a Windows 95 - Executable .

Program descriptions of the above are enclosed in Volume 2 as PD 5.2 through PD 5.7 and PD 4.1, respectively. (see the table of contents). The programs can be obtained from ftp://ftp.geophys.uni-stuttgart.de/pub/ew (141.58.73.149). Two auxiliary programs used in the exercises – WINPLOT and POL_ZERO – are also available from this site.

5.9.3 Free seismic software packages from other sources

- SAC: http://www.llnl.gov/sac/SAC_Info_Install/Availability.html
- SEISMIC UNIX: http://www.cwp.mines.edu/cwpcodes/index.html
- PITSA: http://www.uni-potsdam.de/u/Geowissenschaft/Software/haupt_software.html
- PREPROC: ftp://orfeus.knmi.nl/pub/software/mirror/preproc/index.html

If you can not find these websites, try
http://www.seismolinks.com/Software/Seismological.htm
http://orfeus.knmi.nl/other.services/software.links.shtml

Acknowledgments

Three careful reviews by the Editor of the NMSOP, Peter Bormann, and suggestions by Axel Plešinger and Jens Havskov have significantly improved the clarity and completeness of this text. A shorter version of this text, with some advanced topics added, has been submitted for publication in part A of the International Handbook of Earthquake and Engineering Seismology (Lee et al., 2002).