Topic	Take-off angle calculations for fault-plane solutions and reconstruction of nodal planes from the parameters of fault-plane solutions
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1 Aim

The exercise aims at making familiar with the calculation of the take-off angles AIN of seismic P-wave rays leaving the seismic source towards the seismic station. These angles are required for determining fault-plane solutions (FPS) from first-motion polarity readings (see EX 3.2). Take-off angles depend on the velocity model of the Earth, the source depth h and the epicentral distance Δ. at which the considered rays arrive at the Earth's surface. The AIN calculated in this exercise for a given event and a number of seismic stations at different Δ will then be checked whether they are consistent with the reported polarity readings and FPS calculated for this event by international agencies. For this you will reconstruct on a Lambert-Schmidt net projection the fault-plane traces from the reported nodal-plane parameters.

2 Data, models and procedure

When localizing near events by using HYPO71 or similar programs the values for both the azimuth (AZM) and for the take-off angles (AIN) of the rays leaving the source towards the considered stations are given in the localization output file. One can use them, together with the first motion polarity readings, straight forward for the determination of fault-plane solutions (see EX 3.2). When one intends to determine the fault-plane solution for seismic events published in the bulletins of the International Seismological Centre (ISC) one finds therein, besides data for polarity readings from the reporting stations (↑ or c for up and ↓ or d for down in short- or long-period instruments, respectively), only values for the azimuth (AZM) but not for the respective take-off angle (AIN). Figure 1 shows a typical portion of event-stations report from the ISC. Its header also gives the seismic moment tensor and fault-plane solutions calculated by various international data centers or agencies using different (sometimes automated) procedures. Values for AIN can be calculated by using the relationship

$$\sin AIN = (180/\pi) \times (v_P/r_h) \times p(\Delta, h). \qquad (1)$$

$v_P(h)$ is the P-wave velocity at the depth h (in km/s), r_o = 6371 km is the Earth´s radius and $r_h = r_o - h$. $p(\Delta, h) = dT/d\Delta$ is the ray parameter; it corresponds to the gradient of the travel-time curve at the point of observation on the Earth's surface (both in units s/deg) at the epicentral distance Δ (in degree) (see Fig. 2.27) and is a function of the hypocentral depth h (in km). The value of the ray parameter is identical with that of the horizontal component of the of the slowness vector. Tables 1 and 2 give the respective values $v_P(h)$ and $p(\Delta, h)$ for P waves.

Table 1 $v_P(h)$ according to the IASPEI91 velocity model (Kennett, 1991).

h (km)	v_P (km/s)	h (km)	v_P (km/s)	h (km)	v_P (km/s)
0	5.8000	120	8.0500	471	9.5650
20	5.8000	171	8.1917	571	9.9010
20	6.5000	210	8.3000	660	10.2000
35	6.5000	271	8.5227	660	10.7900
35	8.0400	371	8.8877	671	10.8192
71	8.0442	410	9.0300	760	11.0558
120	8.0500	410	9.3600		

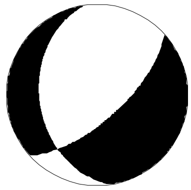
Table 2 Ray parameter p = dT/dΔ (= horizontal slowness component) of Pn, P and PKP_{df} first arrivals at the Earth´s surface as a function of hypocentral depth h according to IASPEI 1991 Seismological Tables (Kennett, 1991)

		p (in s/deg)			
Phase	**Δ (in deg)**	h = 0 km	h = 100 km	h = 300 km	h = 600 km
Pn (P)	2	13.75	12.90	7.91	4.01
	4	13.75	13.49	10.96	6.91
	6	13.74	13.58	11.95	8.60
	8	13.72	13.60	12.25	9.48
	10	13.70	13.59	12.26	9.90
	12	13.67	13.29	12.12	10.05
	14	13.64	12.91	11.03	10.06
	16	12.92	12.43	10.91	9.17
	18	12.33	10.97	10.73	9.10
P	20	10.90	10.81	10.50	9.02
	22	10.70	10.58	9.12	8.90
	24	9.14	9.11	9.03	8.83
	26	9.06	9.02	8.91	8.76
	28	8.93	8.90	8.83	8.66
	30	8.85	8.82	8.75	8.56
	32	8.77	8.74	8.65	8.45
	34	8.67	8.64	8.54	8.33
	36	8.56	8.52	8.42	8.21
	38	8.44	8.40	8.29	8.08
	40	8.30	8.26	8.16	7.95
	42	8.17	8.13	8.03	7.82
	44	8.03	7.99	7.89	7.69
	46	7.89	7.85	7.75	7.56
	48	7.55	7.71	7.61	7.42
	50	7.60	7.56	7.47	7.29
	52	7.46	7.42	7.33	7.15
	54	7.31	7.28	7.19	7.02
	56	7.17	7.13	7.05	6.88
	58	7.02	6.99	6.90	6.74
	60	6.88	6.84	6.76	6.61
	62	6.73	6.70	6.62	6.47

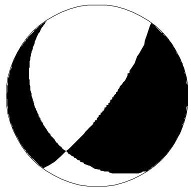
Table 2: cont.

		p (in s/deg)			
Phase	**Δ (in deg)**	h = 0 km	h = 100 km	h = 300 km	h = 600 km
P	64	6.59	6.55	6.48	6.33
	66	6.44	6.41	6.33	6.19
	68	6.30	6.27	6.19	6.05
	70	6.15	6.12	6.05	5.91
	72	6.00	5.97	5.90	5.77
	74	5.86	5.83	5.76	5.63
	76	5.71	5.68	5.61	5.49
	78	5.56	5.53	5.46	5.34
	80	5.40	5.38	5.31	5.20
	82	5.25	5.22	5.16	5.04
	84	5.09	5.07	5.01	4.90
	86	4.94	4.92	4.85	4.72
	88	4.74	4.72	4.69	4.65
	90	4.66	4.65	4.64	4.61
	92	4.61	4.61	4.60	4.57
	94	4.58	4.57	4.55	4.51
	96	4.52	4.51	4.49	4.44
	98	4,45	4.44	4.44	4.44
P_{diff}	100-144	4.44	4.44	4.44	4.44
PKP_{df}	114	1.92	1.92	1.92	1.92
	116-122	1.91	1.91	1.91	1.91
	124-126	1.90	1.90	1.90	1.90
	130	1.88	1.88	1.88	1.88
	136	1.84	1.84	1.84	1.83
	140	1.80	1.79	1.79	1.78
	142	1.76	1.76	1.76	1.75
	144	1.73	(1.72)	(1.72)	(1.71)
	146	1.68	1.68	1.67	1.66
	148	1.63	1.62	1.62	1.60
	150	1.57	1.56	1.55	1.54
	152	1.49	1.49	1.48	1.47
	154	1.42	1.41	1.40	1.39
	156	1.33	1.33	1.32	1.30
	158	1.24	1.23	1.23	1.21
	160	1.14	1.14	1.13	1.11
	162	1.04	1.03	1.03	1.01
	164	0.93	0.93	0.92	0.91
	166	0.82	0.82	0.81	0.80
	168	0.71	0.70	0.70	0.69
	170	0.59	0.59	0.58	0.58
	172	0.47	0.47	0.47	0.47
	174	0.36	0.36	0.35	0.35
	176	0.24	0.24	0.24	0.23
	178	0.12	0.12	0.12	0.12
	180	0.00	0.00	0.00	0.00

NEIC Moment-tensor solution: s23, scale 10^{17} Nm; M_{rr}-3.05; $M_{\theta\theta}$-0.97; $M_{\phi\phi}$4.03; $M_{r\theta}$-2.51; $M_{r\phi}$-1.95; $M_{\theta\phi}$2.71. Depth 272km; Principal axes: T 6.09, Plg17°, Azm117°; N -136, Plg27°, Azm216°; P -4.73, Plg57°, Azm358°; Best double couple: M_o5.4x10^{17}Nm; NP1: ϕ_s172°, δ36°, λ-140°. NP2: ϕ_s48°, δ68°, λ-60°.



*HRVD 05^d 13^h 24^m $15^s.7\pm0^s.2$, 39°.10N±°.02x15°.39E±°.02, h$295^{km}\pm8^{km}$, Centroid moment-tensor solution. Data used: GDSN; LP body waves: s50, c**; Half duration: $1^s.9$. Moment tensor: Scale 10^{17}Nm; M_{rr}-2.17±.06; $M_{\theta\theta}$-1.97±.10; $M_{\phi\phi}$4.14±.09; $M_{r\theta}$-3.51±.09; $M_{r\phi}$--3.29±.09; $M_{\theta\phi}$0.01±.09. Principal Axes: T 5.83, Plg27°, Azm103°; N 0.32, Plg30°, Azm210°; P -6.15, Plg48°, Azm339°. Best Double couple: M_o6.0x10^{17}Nm, NP1: ϕ_s146°, δ33°, λ-157°. NP2: ϕ_s37°, δ78°, λ-60°.*



ISC **$05^d13^h24^m11^s.4$**$\pm0^s$.13, **39.16**$\pm0^s$.16x**15°.18E**±°.014, **h290^{km}**$\pm1.3^{km}$, **(h286km$\pm2.7^{km}$:pP-P), n757, σ1^s.04**/729, **Mb5.7**/107, **119C-155D, Southern Italy.**

OVO	**Vesuviano**	**1.77**	**340**	↑*i***P**	**13 24 57.2**	**+1.5**
MCT	**Mte Cammarata**	**1.95**	**219**	**P**	**13 24 57.7**	**+0.6**
FG4	**Candela**	**1.99**	**8**	**P**	**13 24 58.2**	**+0.9**
MEU	**Monte Lauro**	**2.07**	**186**	**dP**	**13 24 56.8**	**-1.3**
PZI	**Palazzolo**	**2.14**	**186**	e**P**	**13 24 57**	**-1.7**
FAI	**Favara**	**2.21**	**213**	**dP**	**13 24 59.5**	**+0.1**
MSC	**Monte Massico**	**2.23**	**336**	↑*i***P**	**13 25 01.1**	**+1.6**
SGG	**Gregorio Matese**	**2.30**	**345**	↑*i***P**	**13 25 01.9**	**+1.8**

Figure 1 Typical section of an ISC bulletin (left) with NEIC (National Earthquake Information Center) and Harvard University (HRVD) moment-tensor fault-plane solutions (right) for the Italy deep earthquake (h = 286 km) of Jan. 05, 1994. Columns 3 to 5 of the bulletin give the following data: 3 - epicentral distance in degrees, 4 - azimuth AZM in degrees, 5 - phase code and polarity.

Table 3 gives respective selected data from the ISC bulletin for five seismic stations at different epicentral distances (Δ) and azimuth (AZM) for the Italy earthquake shown in Figure 1. The polarity readings correspond to the first P (Δ < 100 °) or PKP (Δ > 110 °) onsets.

Table 3

STA	**Δ** (deg)	**AZM** (deg)	**POL**	**v_P(h)** km/s	**v_P/r_h** (s^{-1})	**p(Δ,h)** (s/deg)	**AIN** (deg)	**AZMc** (deg)	**AINc** (deg)
SGG	2.30	345	+						
KHC	10.03	354	-						
BTH	12.25	294	+						
ZAK	60.02	48	-						
PAE	154.8	324	-						

3 Tasks

Task 1:
For the data given in Table 3, calculate the missing values in the blank columns for $v_P(h)$, v_P/r_h, $p(\Delta, h)$ and AIN using Table 1 and 2 and assuming an approximate focal depth for the recorded event of h = 300 km. Interpolate linearly as a first approximation.

Task 2:
Decide whether your ray has left the upper or lower half of the focal sphere and whether or not you need to calculate AINc and/or AZMc according to Figs. 3.28 and 3.29 in Chapter 3. Complete Table 3 accordingly.

Task 3:
Use the values given in Figure 1 for ϕ and δ for the nodal planes NP1 and NP2 of the NEIC fault-plane solution. Reconstruct both nodal (fault) planes using the *Lambert-Schmidt net* (Fig. 3.27b) by applying the procedure inverse to the one described in the Exercise : Determination of fault-plane solutions (EX 3.2). Compare your nodal-plane pattern with that of the NEIC "beach-ball" solution (Figure 1 upper right).

Task 4:
Find the corresponding equatorial plane to your NP1 and NP2 and mark the locations of the P and T axes on the focal sphere. Draw the P and T vectors towards and from the center of the net, determine their azimuth (Azm) and plunge (Plg) [equivalent to dip, measured from the horizontal]. Compare your respective values with those given by NEIC in Figure 1.

Task 5:
Use the values that you calculated for the P-wave take-off angle AINc and ray azimuth AZMc to all 5 stations in Table 3 and mark the point where the ray penetrates the focal sphere and indicate the respective polarity. Check whether they fall into the proper T and P quadrants and whether the short-period polarity readings given in Table 3 are consistent with the fault-plane solution published by the NEIC which is based on long-period waveform data.

4 Solutions and discussions

Table 4 Solutions for **Task 1**.

STA	**Δ** (deg)	**AZM** (deg)	**POL**	**v_P(h)** km/s	**v_P/r_h** (s^{-1})	**p(Δ,h)** (s/deg)	**AIN** (deg)	**AZMc** (deg)	**AINc** (deg)
SGG	2.30	345	+	8.6286	$1.4213\times$	8.368	(137.0)	165	43.0
KHC	10.03	354	-		10^{-3}	12.258	86.5		
BTH	12.25	294	+			11.984	77.4.1		
ZAK	60.02	48	-			6.759	33.4		
PAE	154.8	324	-			1.368	6.4		

Task 2:

Note: In the case of deep earthquakes the values of the ray parameter (and thus slowness) may increase with Δ, e.g., in Table 2 for h = 300 km up to Δ = 10°. This corresponds to seismic rays that leave the source upwards! Consequently, the value AIN = 43.0° calculated with Equation (1) for station SGG corresponds, according to the definition given in Fig. 3.28, in fact to an angle of 180° - AIN = 137.0°. Accordingly, AZMc and AINc for the equivalent lower hemisphere projection of this ray are 345°-180° = 165° and 43.0°, respectively.

Task 3:

Your manually drawn fault-plane solutions should look very similar to that of the NEIC solution in Figure 1 upper right.

Task 4:

Your manually re-constructed values for Azm and Plg of the P and T axes should agree with the NEIC solution within a few degrees (<10°). If not, check your drawing of the three planes, of the related P and T axis and the measured angles.

Task 5:

The short-period polarity data used in this exercise are consistent with the fault-plane solution published by the NEIC which is based on long-period waveform data. All your polarities should fall properly into quadrants of either observed compressional or dilatational P-wave first motions.

References (see References under Miscellaneous in Volume 2)