| Topic | Bandwidth-dependent transformation of noise data from frequency into time domain and vice versa |
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| Authors | **Peter Bormann** (formerly GFZ German Research Centre for Geosciences, D-14473 Potsdam, Germany); E-mail: pb65@gmx.net<br>**Erhard Wielandt** (formerly Institute of Geophysics, University of Stuttgart, D-70184 Stuttgart, Germany); E-mail: e.wielandt@t-online.de |
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## 1 Aim

The exercise aims at:

- deepening the understanding and developing skills in using the related equations presented in section 4.1 of Chapter 4, Vol. 1;
- application of the conversion program **noisecon**, which can be downloaded by right mouse click on this program name in the NMSOP-2 content overview folder *Download Programs & Files*.;
- demonstrating that the various data presentations given in this Exercise and in Chapter 4 on signal and noise spectra or amplitudes in different kinematic units are in fact all compatible or – if not – that reasons for it can be given.

## 2 Fundamentals

The underlying fundamentals have been outlined in detail in the introduction to Chapter 4. In summary, the following should be remembered:

When a broadband signal is split up into narrower frequency bands with ideal band-pass filters, then

- the instantaneous amplitudes in the individual bands add up to the instantaneous amplitude of the broadband signal,
- the signal powers (or energies in case of *transient* signals) in the individual bands add up to the power (or energy) of the broadband signal,
- the RMS amplitudes in the individual bands DO NOT add up to the RMS amplitude of the broadband signal. The rms amplitude must be calculated from the total power.

**A specification of noise amplitudes without a definition of the bandwidth is meaningless!**

Also: Signal energy is the time-integral of power. Accordingly, transient signals have a finite energy while stationary (noise) signals have an infinite energy but a finite and, in the time average, constant power. Transient signals and stationary signals must therefore be treated differently. The spectrum of a transient signal cannot be expressed in the same units as the spectrum of a stationary signal. Earthquake spectra and noise spectra can, therefore, not be represented in the same plot, unless the conversion between the units is explained. Also, band-pass filtered amplitudes in different spectral ranges are comparable only when having been filtered with the same relative bandwidth (RBW). Note that in signal analysis the "power" of a signal is understood to be the mean square of its instantaneous amplitude. Physical power is proportional but not identical to what is called "power" in signal analysis - for example, the electric power is $W = U^2/R$, not $W = U^2$.

## 3 Data, relationships and programs

The exercises are based on data presented in Figs. 4.5 to 4.8 in Chapter 4, and the Figures 1 to 3 below.

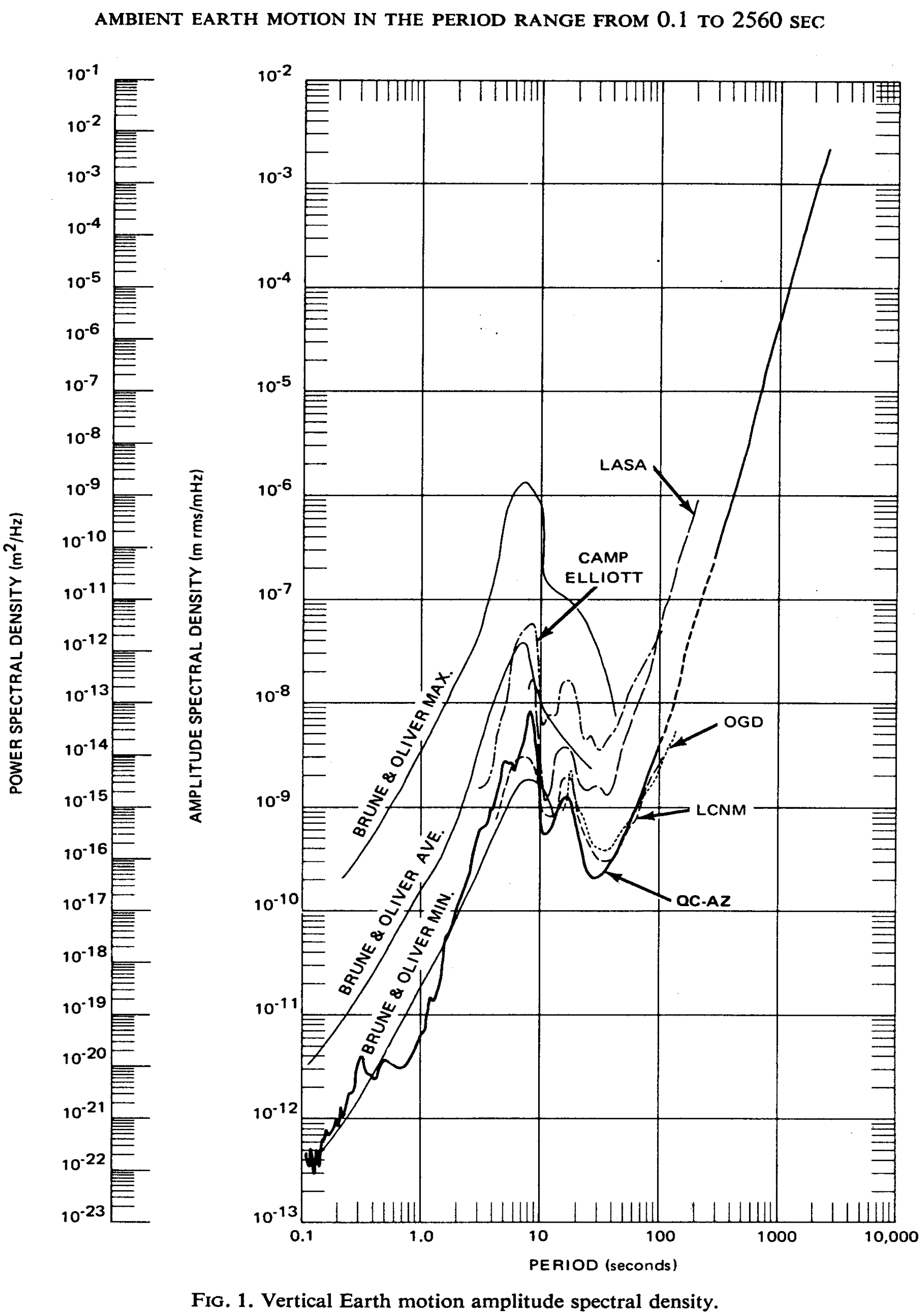


**Figure 1** Compilation of various noise amplitude and power spectral densities at various stations and according to the Brune and Oliver (1959) noise model as published by Fix (1972).

**Figure 2** Cut-out section of a record of the WWSSN-LP seismograph of strong secondary ocean microseisms caused by a winter storm over the Atlantic ocean, reproduced at original scale (30 mm = 1 minute). The magnification at the dominant period is about 400 times.

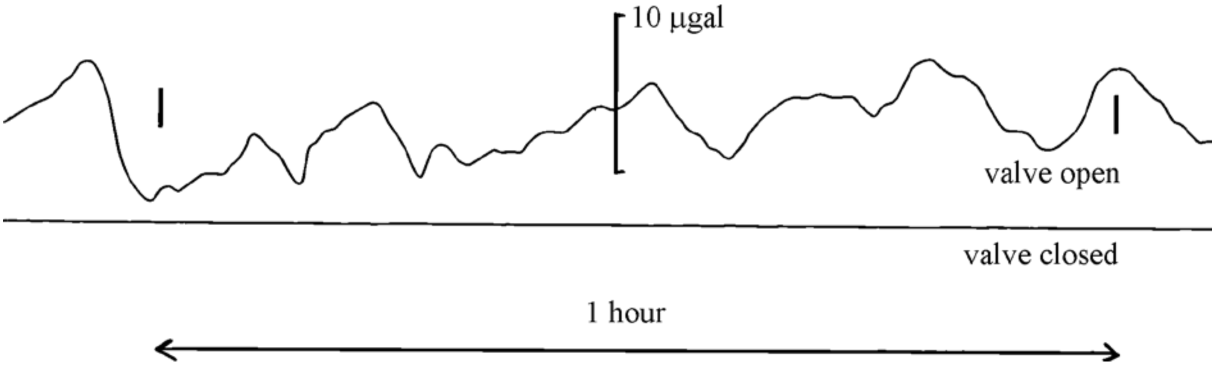


**Figure 3** Output signal of an STS1 seismometer with the vacuum bell valve open (upper trace) and closed (lower trace), respectively. The noise in the top trace is caused by changes in barometric pressure.

For manual solutions use the respective relationships given in Eqs. (4.4) to (4.17) of Chapter 4 and a pocket calculator with the required basic functions. Alternatively, you may use the program **noisecon**.

## 4 Tasks

**Task 1:** Determine the relative bandwidth (RBW) of an

a) 2-octave filter

b) 2/3-octave filter

c) 1/3-octave filter

d) 1/6-decade filter

by using Eq. (4.15) in Chapter 4 and

e) express the bandwidth of an 1/3-decade filter in terms of octaves

by using Eq. (4.17).

**Task 2:** Calculate for the noise maximum of the upper curve in Fig. 4.5 the corresponding RMS ground motion (velocity and displacement).

a) Estimate the velocity power maximum from Fig. 4.5 (Note the logarithmic scale!).

b) Give this value also in units of $(m/s)^2/Hz$.

c) Estimate the frequency $f_o$ related to this maximum.

d) Calculate the RMS-velocity amplitude $a_{vRMS}$ by considering Eqs. (4.15) and (4.16) and a relative bandwidth of 2/3 octaves.

e) Transform this RMS velocity amplitude determined under d) into the corresponding RMS displacement amplitude $a_{dRMS}$ considering Eq. (4.4).

**Task 3:** Transform the displacement power values of Fig. 4.6 at f = 1 Hz and f = 10 Hz in

a) units of $m^2/Hz$,

b) acceleration power values with units $(m/s^2)^2/Hz$ using Eq. (4.5),

c) the values determined under b) in units of dB referred to 1 $(m/s^2)^2/Hz$ according to Eq. 4.6) and

d) compare the result with the respective values in Fig. 4.7 for the New Low Noise Model (NLNM).

**Task 4:** Determine from Fig. 4.7 the respective ground acceleration power spectral density values of the NHNM in units of $(nm/s^2)^2/Hz$ for

a) f = 1 Hz ,

b) f = 0.1 Hz .

using Eq. (4.6)

**Task 5:** Select any period between 0.01 and 10,000 sec (e.g., T = 100s) and confirm that the presentations in Figs. 4.7 and 4.8 in Chapter 4 are equivalent when assuming a relative bandwidth of 1/6 decades as used in Fig. 4.8.

**Task 6:** Transform selected velocity PSD values given in the lower curve of Fig. 4.23 into acceleration PSD $\mathbf{P_a}$[dB] = 10 log ($P_a$ / 1 $(m/s^2)^2$/Hz) via Eq. (4.5) and compare them with the NLNM at

a) 1.5 Hz and

b) 10 Hz.

**Task 7:** Figure 1 is a historical compilation of observed low noise spectra, now made obsolete by the USGS New Low Noise Model (NLNM).

a) One of the two vertical scales has an incorrect label; both the name and the unit are incorrect. Try to correct it.

b) Compare the lowermost curve (QC-AZ) to the NLNM, for example by plotting the points of the NLNM curve for 30 s, 300 s and 3000 s into the figure and discuss the difference.

c) What would be the rms acceleration of the QC-AZ noise in a half-octave frequency band from 1 to (roughly) 1.4 Hz and in a half-octave period band from 25 to 35 seconds? Compare to figure 5.18 of the NLNM. Note that ½ octave and 1/6 decade are nearly the same.

**Task 8:** Assess the noise level of the microseism storm in Figure 2 with respect to the NLNM

a) Determine the range of periods of the microseisms.

b) Estimate the bandwidth of the microseisms, their center frequency $f_o$ and RBW.

c) Estimate the displacement $a_{RMS}$ from the *average peak amplitudes* (which are about $1.25a_{RMS}$). The magnification of the record is about 400 at $f_o$.

d) Transform the displacement $a_{RMS}$ into acceleration $a_{RMS}$ and $\mathbf{P_a}$ [dB].

e) Compare with Fig. 4.7 and discuss possible differences.

**Taks 9:** Compare the noise level for the acceleration records of an STS1 seismometer shown in Figure 3 and compare it with the NLNM. Note that 1 gal = $10^{-2}$ m/s$^2$.

a) Estimate $a_{RMS}$ from the *average peak amplitudes* in Figure 3, upper trace.

b) Estimate the upper limit of $a_{RMS}$ for the lower trace in Figure 3.

c) Estimate the periods and bandwidth of the noise in Figure 3.

d) Compare the $a_{RMS}$ for the upper and the lower trace with the NLNM presentation in Fig. 4.8.

e) Discuss the differences.

# 5 Solutions

**Note:** The errors in eye readings of the required parameters from the diagrams may be 10 to 30 %. Therefore, it is acceptable if your solutions differ from the ones given below in the same order or by about 1 to 3 dB in power. In case of larger deviations check your readings and calculations. Also: all power values given below in dB relate to the respective units in Fig. 4.7.

**Task 1:**

a) 1.5

b) 0.466

c) 0.231

d) 0.386

e) 1.1 octaves

**Task 2:**

a) $7\times10^{-8}$ (cm/s)$^2$/Hz

b) $7\times10^{-12}$ (m/s)$^2$/Hz

c) 0.16 Hz

d) $a_{vRMS} \approx 7\times10^{-7}$ m/s

e) $a_{dRMS} \approx 7\times10^{-7}$ m

**Task 3:**

a) $2\times10^{-18}$ m$^2$/Hz at 1 Hz and $1.5\times10^{-22}$ m$^2$/Hz at 10 Hz

b) $3.12\times 10^{-15}$ (m/s$^2$)$^2$/Hz at 1 HZ and $2.3\times 10^{-15}$ (m/s$^2$)$^2$/Hz at 10 Hz

c) - 145 dB for 1 Hz and -146 dB for 10 Hz

d) The noise power at this site is for the considered frequencies about 20 dB higher than for the NLNM.

**Task 4:** a) and b) ≈ -117 dB, i.e., $\approx 2\times10^6$ $(nm/s^2)^2$/Hz ;

**Task 5:** For T = 100 s we get from Fig. 4.7 $\mathbf{P_a}$ [dB] = -185 dB. With RBW = 0.3861 for 1/6 octave bandwidth and f = 0.01 Hz we calculate with Eq. (4.16) $a_{RMS}^2 = 1.1 \times 10^{-21}$ $(m/s^2)^2$ which is about –210 dB, in agreement with Fig. 4.8.

**Task 6:** a) $\mathbf{P_a}$ ≈ -153 db, 16 dB above the NLNM

b) $\mathbf{P_a}$ ≈ -153 db, 15 dB above the NLNM

**Task 7:**

a) The amplitude-density scale in Figure 1 is inapplicable to noise and cannot be related to the power-density scale, which is correct.

b) It is convenient to express the amplitudes in decibels relative to 1 $m^2$/Hz. For example, $10^{-13}$ $m^2$/Hz corresponds to –130 dB, $2*10^{-13}$ $m^2$/Hz to –127 dB, $5*10^{-13}$ $m^2$/Hz to –123 dB, $10^{-12}$ $m^2$/Hz to –120 dB. In these units we obtain:

At a period of 30 s, the NLNM is at –157 dB while QC-AZ is at –163 dB
At a period of 300 s, the NLNM is at –120 dB while QC-AZ is at –100 dB
At a period of 3000 s, the NLNM is at –59 dB while QC-AZ is at –17 dB

Obviously, the authors of figure 1 were able to resolve minimum ground noise at 30 s but not at much longer periods where their instruments were too noisy.

c) Half-octave band around 1.2 Hz (0.83 s): QC-AZ has a displacement power density of $10^{-20}$ $m^2$/Hz. Multiply by the fourth power of the angular frequency, 2π*1.2 Hz, to obtain the approximate power density of acceleration, $3.2*10^{-17}$ $m^2/s^4$/Hz. Multiply with the bandwidth of 0.4 Hz to obtain the total power, $1.3*10^{-17}$ $m^2/s^4$. Take the square root to obtain the rms amplitude, $3.6*10^{-9}$ $m/s^2$. The power can also be expressed as $10*\log(1.3*10^{-17})$ dB = –169 dB relative to 1 $m^2/s^4$. One would attribute the same number of decibels to the rms amplitude, understanding that decibels always refer to power. The NLNM has –172 dB.

For the half-octave band around 27 s (0.037 Hz), the bandwidth is 0.011 Hz and the rms acceleration $4.2*10^{-11}$ $m/s^2$, which can also be expressed as –207 dB. The NLNM has –200 dB.

**Task 8:** a) The periods of the microseisms in Figure 2 vary between T = 7 s (for the smaller amplitudes) and T = 10 s (for the largest amplitudes).

b) From this upper and lower period follows with Eq. (4.13) $n \approx 0.5$ octaves or $m \approx 1/6$ decade, a center frequency of $f_o \approx 0.119$ Hz ($T_o \approx 8.4$ s) and an RBW of $\approx 0.36$

c) Maximum double trace amplitudes of the microseisms range between about 6 and 3 mm, average about 4.5 mm, corresponding to a "true" *average peak* ground *amplitude* of about $5.6 \times 10^{-6}$ m and thus to a displacement $a_{RMS} \approx 4.5 \times 10^{-6}$ m.

d) The acceleration $a_{RMS} \approx 2.5 \times 10^{-6}$ m for $f_o \approx 0.119$ Hz and $\mathbf{P_a} \approx -98$ dB.

e) $\mathbf{P_a} \approx -98$ dB for this microseismic storm is close to the power at the NHNM peak around T = 5 s (-96.5 dB) but about 15 dB higher than the NHNM values at $T \approx 8$ s. Thus, the record corresponds to a really strong microseism storm.

**Task 9:** a) From Figure 3, upper trace, the estimated *average peak amplitude* is about 2.5 μgal and thus $a_{RMS}$ about $2 \times 10^{-8}$ m/s$^2$.

b) The related upper limit of about 1/100th of a), i.e., $a_{RMS} < 2 \times 10^{-10}$ m/s$^2$.

c) The periods of the noise in Figure 3 range between roughly 180 s and 750 s. This corresponds to a bandwidth of about 2 octaves or an RBW of 1.5.

d) The $a_{RMS}$ for the open valve corresponds to – 154 dB, that for the closed valve to < –194 dB.

e) Taking into account that Fig. 4.8 was calculated for 1/6 decade bandwidth only but the bandwidth of the considered noise signals being about 3 times larger we have to assume an about 5 dB higher noise level in Fig. 4.8. Therefore, for periods < 30 s the barometric pressure noise is surely well below the NLNM when the sensor operates in a vacuum. A higher resolution of the record with the vacuum bell valve closed would be required in order to determine the noise level distance to the NLNM for T > 30 s.

## References

Brune, J. N., and Oliver, J. (1959). The seismic noise of the Earth´s surface. *Bull. Seism. Soc. Am.*, **49**, 349-353.

Fix, J. E. (1972). Ambient Earth motion in the period range from 0.1 to 2560 sec. *Bull. Seism. Soc. Am.*, **62**, 1753-1760.