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Detecting Site Resonant Frequency Using $HVSR$: Fourier versus Response Spectrum and the First versus the Highest Peak Frequency

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Abstract

In this investigation, we examine the uncertainties using the horizontal-to-vertical spectral ratio ($HVSR$) technique on earthquake recordings to detect site resonant frequencies at 207 KiK-net sites. Our results show that the scenario-dependence of response ($PSA$) spectral ratio could bias the estimates of resonant frequencies for sites having multiple significant peaks with comparable amplitudes. Thus, the Fourier spectrum ($FAS$) should be preferred in computing $HVSR$. For more than 80% of the investigated sites, the first peak (in the frequency domain) on the average $HVSR$ curve over multiple sites coincides with the highest peak. However, for sites with multiple peaks, the highest peak frequency ($f_p$) is less susceptible to the selection criteria of significant peaks and the extent of smoothing to spectrum than the first peak frequency ($f_0$). Meanwhile, in comparison to the surface-to-borehole spectral ratio ($SBSR$), $f_0$ tends to underestimate the predominant frequency (at which the largest amplification occurs) more than $f_p$. In addition, in terms of characterizing linear site response, $f_p$ shows a better overall performance than $f_0$. Based on these findings, we thus recommend seismic network operators to provide $f_p$ on the average $HVSR_{FAS}$ curve as a priority, ideally together with the average $HVSR_{FAS}$ curve in site characterization.
Introduction

Many previous investigations (e.g., Pitilakis et al., 2018; Derras et al., 2017; Stambouli et al., 2017; Zhao and Xu, 2013; Luzi et al., 2011; Zhu et al., 2019a) have shown that site resonant frequency (or period) has a better overall performance than the conventional 30 m time-averaged shear-wave velocity ($V_{S30}$) in depicting site response. Hence Zhu et al., (2019a) recommended using the resonant frequency as the primary predictor variable in site-effects models. Also, resonant frequency tops the list of mandatory site proxies recommended by the “Seismology and Earthquake Engineering Research Infrastructure Alliance for Europe: Networking databases of site and station characterization” (SERA-NA5) project to characterize recording stations.

One approach to obtain site resonant frequency is the horizontal-to-vertical spectral ratio (HVSR) approach. This technique gained momentum after a series of articles by Nakamura (1989) who very recently clarified a few misunderstandings about the so-called “Nakamura method” (Nakamura, 2019). Since then, innumerable investigations on implementing the HVSR method on either ambient noises or earthquake ground-motions sprung up, as reviewed by Bard (1995), Kudo et al. (2004) and many others. Regarding the theoretical aspects of the HVSR, we refer interested readers to reviews by, for instance, Bonnefoy-Claudet et al. (2006) and Lunedei and Malischewsky (2015), as well as works by Kawase et al. (2011) and many others.

Though exceptions can be found in literature, there is a general consensus that HVSR approach can reveal the fundamental resonant frequency of most sites but tends to underestimate the level of ground-motion amplification compared with the classic standard spectral ratio (SSR, Borcherdt, 1970; Lachet et al., 1996; Riepl et al., 1998, Duval et al., 2001; Haghshenas et al., 2008; Rong et al., 2017; and others). SSR requires a reference rock site (Steidl et al., 1996) which is often difficult
to find in practice, thus, the single-station HVSR technique is appealing, at least, in detecting site resonant frequency.

Alternatively, Tokeshi et al. (1996) proposed to use the phase of the one-sided autocorrelogram of the horizontal components (Fourier Phase Spectral Method, FPSM) to derive site resonant frequency. However, Parolai et al. (2001) later demonstrated that the FPSM technique had no advantage over the HVSR approach. Thus, we will focus this study on the HVSR method and its uncertainties in quantifying resonant frequency. In a preceding study (Zhu et al., 2019b), we compiled four sets of resonant frequency data derived by different teams using spectral ratio techniques at KiK-net stations and compared the resonant frequency measurements at the same site. We have found different levels of discrepancies in these data, which has motivated the present investigation.

In this paper, we first present this comparison of resonant frequencies from different teams and postulate the factors explaining these inconsistencies, followed by a description of the procedure of obtaining resonant frequency using the HVSR approach. Then we utilize the KiK-net recording network and investigate how different factors in the process affect the determination of resonant frequency, including using Fourier (FAS) spectral ratio or (5% damped) pseudo-spectral acceleration (PSA) ratio, the degree of smoothing to spectrum, the definition of significant peaks, and the choice of the first or the highest peak frequency when there are multiple peaks. Then we evaluate the efficacies of various resonant frequencies derived under different schemes in modelling liner site effects. Based on their performance, we finally determine the optimal scheme to derive site resonant frequency using HVSR.
Potential Factors Explaining Inconsistencies in Resonant Frequency Measurements

To determine the resonant frequency of a given site using spectral ratio methods (e.g., HVSIR) on earthquake data, one may utilize either the complete (from P-arrival to the end of S-coda) or partial (typically S-wave window) waveform with a signal-to-noise ratio above a certain level. Then one needs to compute the spectral ratios on either smoothed PSA or FAS of each recording and then obtain the spectral ratios averaged over all recordings at the site.

Next, from the average spectral ratio curve, one can pinpoint all significant peaks \( f_s \) which are often defined as local maxima points fulfilling certain clarity (e.g., larger than 2.0) and/or stability criteria (SESAME, 2004). Then, from these significant peaks, one can pick up either the first-peak frequency \( f_0 \) or the highest-peak frequency \( f_p \) as site resonant frequency. However, since there is a lack of a uniform procedure, measurements of resonant frequency are not without uncertainties as can be seen from four sets of empirically-derived resonant frequency data at KiK-net sites compiled by Zhu et al., (2019b) (Fig. 1).

These data sets were published by Wang et al. (2018) using HVSIR, by Wang (2017) and Pousse (2005) using surface-to-borehole spectral ratio (SBSR), and by Fujiwara et al. (2009) using radial-to-vertical spectral ratio (RVSR) approaches, respectively (Table 1). Complete waveforms of earthquake recordings were utilized except for Fujiwara et al., (2009) who used only S-coda waves (20 s after S-wave arrival). More details pertaining to the determination of resonant frequency are tabulated in Table 1. There are substantial inconsistencies among their results (Fig. 1), especially between those obtained by Pousse (2005) and Fujiwara et al. (2009).

The choice of PSA (Wang et al., 2018) or FAS (Pousse, 2005; Fujiwara et al., 2009; Wang, 2017) may contribute to the inconsistencies in these data (Fig. 1). As pointed out by Bora et al.
(2016), the response spectral ordinate at a high oscillator-frequency is not controlled by the Fourier spectral ordinate at the corresponding frequency but is determined by Fourier amplitudes in a rather broad frequency range. In addition to this peculiar property, response spectral ratios are scenario-dependent (Stafford et al., 2017), and this scenario-dependency is strong at high frequencies and is particularly evident for small earthquakes. Thus, frequency- and scenario-dependent differences exist between PSA- and FAS-based spectral ratios. However, many researchers used the PSA and FAS interchangeably (e.g., Molnar et al., 2015), which may be one source of discrepancy in resonant frequencies (Fig. 1).

Damping used in calculating a response spectrum (e.g., 5%) has an incidental smoothing effect. However, if FAS is utilized to compute spectral ratios, additional smoothing to FAS is required prior to dividing the horizontal component by its vertical counterpart (Table 1). Choice of smoothing techniques (e.g., the type of smoothing operator and the degree of smoothing) will affect the value of the spectral ratio and subsequently the qualification of significant peaks. Hence, inhomogeneous smoothing can also cause a mismatch in resonant frequency (Fig. 1).

Besides, in all these above studies, site resonant frequency is identified from significant peaks, but what qualifies a significant peak is defined rather subjectively. As can be seen from Table 1, the definition of significant peaks varies from study to study. This discretionary exercise contributes to the disparities among these resonant frequency data (Fig. 1). Also, after detecting all significant peaks on an average spectral ratio curve, either the first ($f_0$) or the highest ($f_p$) peak frequency is treated as site resonant frequency. Some studies (e.g., Hassani and Atkinson, 2018) utilized $f_p$ whereas others (e.g., Ghofrani et al., 2013 and 2014) opted for $f_0$ (Table 1). There is often a lack of an evidence-based justification to their choice of one instead of the other, which is also responsible for the inconsistency in site resonant frequency. Nevertheless, it is worth noting
that there are other factors that contribute to the deviations observed in Figure 1. For instance, resonant frequency based on SBSR can differ from that based on HVSR if an abrupt impedance contrast exists below the borehole sensor.

Identification of Resonant Frequency using HVSR at KiK-net stations

In a preceding investigation (Zhu et al., 2019a), we studied the linear site effects at 207 KiK-net sites (Table S1). Site amplification at each site was derived using the SBSR approach on earthquake recordings. Site data is displayed in Figure 2a. \( Z_{0.8} \) is the depth (in meters) to the isosurface having \( V_S = 800 \) m/s and is obtained from KiK-net velocity profiles. Downhole stations of these 207 sites are all installed in layers at least 100 m deep from the ground surface and having shear-wave velocity larger than 800 m/s. Thus, downhole recordings are not expected to be significantly contaminated by downgoing waves (Bonilla et al., 2002).

Zhu et al. (2019a) selected a total number of 1840 ground motions recorded by these stations between October 1997 and December 2011 from the KiK-net database processed by Dawood et al. (2016). The complete waveform from P-arrival was utilized (e.g., Thompson et al., 2012). The rupture distance \( (R_{rup}) \) of these selected ground motions was up to 400 km, and the moment magnitude \( (M_w) \) was in the range between 3.5 and 8.0 (Fig. 2b). All the selected records had a low-cut frequency of 0.24 Hz or lower and had a signal-to-noise ratio of no less than 3.0 in the usable frequency range. Selected seismograms were considered not being significantly affected by soil nonlinearity according to the threshold proposed by Fujimoto and Midorikawa (2006). Each of these 207 stations had recorded at least three such events. In the present investigation, we utilize the same dataset (Zhu et al., 2019a) to derive resonant frequencies. We refer readers to Zhu et al.
(2019a) for more details on data selection. The main steps of site frequency determination using HVSR technique are as follows:

i) Computing $FAS$ and $PSA$ of each time-series (complete waveform);

ii) Smoothing $FAS$ and $PSA$ using Konno-Ohmachi window (Konno and Ohmachi, 1998) with a smoothing coefficient $b=20$ (default), 30 and 40, respectively;

iii) Obtaining $HVSR(f)$ by dividing the geometrical mean of the two horizontal components ($NS$ and $EW$) of a certain recording by its vertical component ($V$):

$$
\log_{10}HVSR_{ij}(f) = 0.5\left[\log_{10}(H_{NS})_{ij}(f) + \log_{10}(H_{EW})_{ij}(f)\right] - \log_{10}(V)_{ij}(f)
$$

where $HVSR_{ij}$ is the horizontal-to-vertical Fourier or response spectral ratio of the ground motion recorded at station $j$ from event $i$; and $(H_{NS})_{ij}$, $(H_{EW})_{ij}$ and $(V)_{ij}$ are the Fourier or response spectrum of the $NS$, $EW$ and $V$ components, respectively;

iv) Deriving $\overline{HVSR}(f)$ by averaging $HVSR(f)$s over all recordings ($\geq 3$) at a certain site;

$$
\log_{10}\overline{HVSR}_j(f) = \frac{\sum_{i=1}^{n_j} \log_{10}HVSR_{ij}(f)}{n_j}
$$

$$
std_j(f) = \sqrt{\frac{\sum_{i=1}^{n_j} (\log_{10}HVSR_{ij}(f) - \log_{10}\overline{HVSR}_j(f))^2}{n_j}}
$$

where $\overline{HVSR}_j(f)$ and $std_j(f)$ are the mean $H/V$ spectral ratio and its standard deviation at station $j$, respectively; and $n_j$ is the number of recordings at station $j$;

v) Identifying all significant peaks ($f_s$) on $\overline{HVSR}_j(f)$;

vi) Picking up both the first ($f_0$) and the highest ($f_p$) peak frequencies from significant peaks (© Table S1, see Data and Resources).

Not every extremum point on a $\overline{HVSR}_j(f)$ curve qualifies as a significant peak ($f_s$). Following
the SESAME (2004) guideline, in step (v), a local extremum point is only regarded as \( f_i \) if it passes one or more of the following criteria:

\[
C1 \quad \log_{10} \frac{HVSRI_j(f_s)}{HVSRI_j(f)} > 0.3;
\]

\[
C2 \quad f \text{ can be found within both } [f_s/4, f_s] \text{ and } [f_s, 4f_s] \text{ with } \log_{10} \left[ \frac{HVSRI_j(f_s)}{HVSRI_j(f)} \right] > 0.3;
\]

\[
C3 \quad \text{Standard deviation of } \log_{10} \frac{HVSRI_{ij}(f_s)}{HVSRI_{ij}(f)} \text{ lower than a frequency dependent threshold } \theta(f_s), \text{ where } \theta(f_s) = 0.48 \text{ for } f_s < 0.2 \text{ Hz}; \theta(f_s) = 0.40 \text{ for } 0.2 \leq f_s < 0.5 \text{ Hz}; \theta(f_s) = 0.30 \text{ for } 0.5 \leq f_s < 1.0 \text{ Hz}; \theta(f_s) = 0.25 \text{ for } 1.0 \leq f_s < 2.0 \text{ Hz}; \text{ and } \theta(f_s) = 0.20 \text{ for } f_s \geq 2.0 \text{ Hz}.
\]

According to the criterion/criteria applied in qualifying a significant peak, we define the following notation to differentiate the resonant frequencies or periods identified subsequently:

\( f_{p1}, f_{p2} \) and \( f_{p3} \) – highest significant-peak frequencies under criterion/criteria \( C1, C1&2, \) and \( C1\sim3, \) respectively;

\( T_{p1}, T_{p2} \) and \( T_{p3} \) – resonant periods, reciprocals of \( f_{p1}, f_{p2} \) and \( f_{p3}, \) respectively;

\( f_{01}, f_{02} \) and \( f_{03} \) – first significant-peak frequencies under criterion/criteria \( C1, C1&2, \) and \( C1\sim3, \) respectively;

\( T_{01}, T_{02} \) and \( T_{03} \) – resonant periods, reciprocals of \( f_{01}, f_{02} \) and \( f_{03}, \) respectively.

**Response or Fourier Spectral Ratio?**

Response spectral ratios show scenario-dependency (Stafford et al., 2017) and thus could give a different resonant frequency from Fourier spectral ratios. Figure 3a compares the highest peak
frequencies derived from Fourier \((f_{p1,FAS})\) and response \((f_{p1,PSA})\) spectral ratios when only criterion \(C1\) is applied to select significant peaks. There are 201 sites having both \(f_{p1,FAS}\) and \(f_{p1,PSA}\). At 11 out of 201 sites (or 5%), the two highest peak frequencies deviate significantly \(|\log_{10}(f_{p1,FAS}/ f_{p1,PSA})| > 0.3\). Outliers in Figure 3a often correspond to sites at which there are multiple significant peaks with comparable peak values. When the screening criterion \(C2\) is also applied (results are not shown here for brevity), the number of sites having both \(f_{p2,FAS}\) and \(f_{p2,PSA}\) reduces to 97. However, the deviation between these two resonant frequencies lessens slightly. Significant deviation only occurs at 4 out of 97 (or 4%) sites.

Figure 3b displays the ratios of \(\overline{HVSR}_{PSA}(f)\) to \(\overline{HVSR}_{FAS}(f)\) at the 207 KiK-net sites in log10 units. It confirms that the response spectral ratios are close to Fourier spectral ratios at low frequencies whereas the former are apparently larger than the latter at high frequencies (Bora et al., 2016). Thus, for sites with multiple significant peaks, especially those with one of the significant frequencies in a relatively high-frequency range, using the response and Fourier spectral ratios could lead to different estimates of resonant frequency. To elaborate on this issue, we choose a KiK-net site IWTH16 (Fig. 4a) as an example. To obtain its resonant frequency, a total number of 144 seismograms recorded at this station is selected with a PGA in the range between 0.05 and 0.2 m/s\(^2\). Magnitude-distance distribution is displayed in Figure 4b.

Figure 5a and b depict the \(HVSR(f)\)s of these 144 recordings at IWTH16, as well as their average \((\overline{HVSR}(f)),\) bold solid line) and average ± one standard deviation (dashed lines) using Fourier and response spectra, respectively. Dots correspond to the highest peak frequencies \((f_{p1})\) of each individual curve under the screening criterion \(C1\). The true value of \(f_{p1}\) of this site is defined from \(\overline{HVSR}(f)\). There is more than one significant peak on \(\overline{HVSR}(f)\), but the one at 0.38 Hz is dominant in both Figure 5a and b. The highest peaks of each individual Fourier spectral ratio curve
cluster in frequency bands [0.2, 0.6] and [0.6, 2.0]. In contrast, for response spectral ratio curves, there are also the highest peaks in ranges [2.0, 8.0] and [8.0, 20].

However, resonant frequency is defined as the peak frequency of the average HVSR curve instead of the average of peak frequencies of each individual curve (Ghofrani and Atkinson, 2014). Meanwhile, the average HVSR curve is often based on a number (at least three) of randomly selected ground-motion recordings. To mimic this process, we select $N (N \geq 3)$ spectral ratio curves randomly from Figure 5 and then determine $f_{p1}$ from their average. For each $N$, we repeat the identification process for 100 times. The distributions of the identified $f_{p1}$ using Fourier and response spectra are shown in Figure 6a and b, respectively.

For the minimum number $N=3$, using FAS, the true site resonant frequency (0.38 Hz) is correctly identified 88 out of 100 times (Fig. 6a) whereas using PSA, $f_{p1}$ is only correctly identified 62 out of 100 times (Fig. 6b). Averaging HVSRs over more records can further curtail the randomness. Nine records are needed to have a 100% correct identification of $f_{p1}$ for IWTH16 when FAS is used (Fig. 6a). In comparison, PSA requires 59 recordings to achieve the same level of certainty (Fig. 6b). Thus, for sites having multiple significant peaks with comparable peak values, utilizing response spectrum is more likely to have biased estimates of resonant frequency than using Fourier spectrum. This can be attributed to the scenario-dependence of response spectral ratios, especially at high frequencies (Stafford et al., 2017). In contrast, Fourier spectral ratios show little such dependency (Zandieh and Pezeshk, 2011). The nonlinear feature of response spectral ratios also explains the larger scatter at relatively high frequencies ($> 2.5$ Hz) in Figure 5b than that in Figure 5a.

Therefore, for sites with multiple comparable peaks, the Fourier spectral ratio should be preferred over the response spectral ratio in the determination of site resonant frequency. We thus
use FAS to compute spectral ratios hereafter. However, for other sites, i.e. those with a single clear peak or with more than one peak having distinct amplitudes, utilizing either Fourier or response spectrum makes little difference.

The First or the Highest Peak Frequency?

Susceptibilities of the First and the Highest Peak Frequencies

Significant peaks were defined differently in previous researches (Table 1). To investigate the influence of peak definition, we thus compare the site resonant frequencies derived using only the screening criterion $C1$ with these obtained using $C1$, $C2$ and $C3$ in Figure 7. Figure 7a suggests that the definition of significant peaks has a remarkable impact on the first peak frequency ($f_0$). 56 out of 169 (or 33%) sites have $f_0$ significantly affected. When more screening criteria are applied, many significant peaks identified under a less strict condition will be disqualified, resulting in more no-peak sites but less multiple-peaks sites, as shown in Table 2. Under a more robust peak definition, $f_{01}$ initially identified could be rejected, in which case the significant peak at a higher frequency would be picked up as the first peak frequency ($f_{03}$), as shown in Figure 7a with $f_{03} > f_{01}$ at many sites.

In comparison, Figure 7b indicates that, at 158 out of 169 (or 93%) sites, the highest peak frequencies ($f_p$) remain the same regardless of the change in peak-definition. This is exemplified in Figure 7c, which illustrates the HVSR($f$)s at site FKSH10. When a stricter peak definition is applied, the highest peak frequency, i.e., 6.19 Hz, remains unchanged. However, the first peak frequency changes from 0.42 to 6.19 Hz. Thus, Figure 7 implies that the $f_p$ is less susceptible to the defining criteria of significant peaks than $f_0$. 

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Unsmoothed FAS is spiky, and often smoothing is required. Konno-Ohmachi weighting function (Eq. 4, Konno and Ohmachi, 1998) is one of the most widely applied smoothing functions with a constant window width and symmetric shape around central frequency (on logarithmic scales):

\[ W_B(f, f_c) = \left( \frac{\sin(\log_{10} \frac{f}{f_c})^b}{(\log_{10} f_c)^b} \right)^4 \]  

(4)

where \( f_c \) is the central frequency at which the smoothing is performed, and \( b \) is a coefficient inversely proportional to bandwidth. A smaller \( b \) value results in a stronger smoothing. The degree of smoothing affects the amplitudes of HVSR and subsequently the qualification of significant peaks (e.g., Konno and Ohmachi, 1998; Bard, 1995).

Figure 8 compares the site resonant frequencies identified using different (Konno-Ohmachi smoothing) \( b \) values. At 25% of the sites (or 42 out of 169 sites), the first peak frequency \( f_{03} \) (Fig. 8a) identified using \( b=20 \) deviates significantly from that determined using \( b=40 \), namely \( |\log_{10}[f_{03} (b = 20)/f_{03} (b = 40)]| > 0.3 \). In contrast, the highest peak frequency \( f_{p3} \) (Fig. 8b) does not vary much with the smoothing coefficient; only 5 out of 169 (or 3%) sites are outside the range \( |\log_{10}[f_{p3} (b = 20)/f_{p3} (b = 40)]| \leq 0.3 \). This comparison implies that \( f_0 \) is more sensitive to the smoothing coefficient than \( f_p \).

When a stronger smoothing (a lower \( b \) value) is applied, the amplitudes of all peaks will decrease and, as a consequence, some significant peaks identified under a weaker smoothing may fail the same selection criteria of significant peaks. With the increase in \( b \) value, there are more sites with multiple significant peaks and fewer sites having no or a single peak (Table 2). Thus, for a site with multiple significant peaks, a stronger smoothing could result in a significant peak at
a higher frequency being identified as $f_0$ (Fig. 8a). However, $f_p$ (Fig. 8b) corresponds to the highest peak and thus is less sensitive to the degree of smoothing than $f_0$.

It is worth noting that, for KiK-net sites with both $f_0$ and $f_p$ assigned, $f_0$ equals $f_p$ for the majority of sites. The percentage of sites with $f_0=f_p$ is contingent on the peak screening criteria. For instance, there are 82% of sites where $f_{02}$ coincides with $f_{p2}$, and the percentage is 85% for $f_{03}=f_{p3}$. For those with different $f_0$ and $f_p$, the former is smaller than the latter, so are their amplitudes, hence, $f_p$ is associated with a larger velocity contrast at a shallower depth than $f_0$.

Comparison with Resonant Frequencies from SBSR

In a preceding study (Zhu et al., 2019a), we computed SBSRs at the 207 KiK-net stations, as exemplified in Figure 9. Also shown are surface-to-borehole cross-spectral ratios of horizontal components ($c$-SBSR), horizontal-to-vertical spectral ratios at borehole station ($HVSR_b$) and surface-to-borehole spectral ratios of vertical components ($SBSR_v$). All spectral ratios presented in this section, “Comparison with Resonant Frequencies from SBSR”, are calculated on smoothed FAS using a Konno-Ohmachi window ($b=20$) and are provided as an electronic supplement to this article (Ⓔ File Spectral Ratios, see Data and Resources). Criterion $C1$ and $C2$ are used to select significant peaks. Following the same procedure, we obtain the first ($f_{02_{SB}}$) and the highest ($f_{p2_{SB}}$) peak frequencies from each $SBSR(f)$ curve and compare them with those determined from $HVSR(f)$, i.e., $f_{02_{HV}}$ and $f_{p2_{HV}}$.

There are 20 sites for which $HVSR(f)$ exhibits a significant peak in a rather low-frequency band ($< \sim 1.0$ Hz), but a peak is completely absent in this frequency range on its corresponding $SBSR(f)$ curve (Fig. 9a). This clearly indicates the existence of an abrupt velocity contrast below the borehole, even though all these 207 KiK-net stations have downhole sensors installed in layers
at least 100 m below the ground surface and with shear-wave velocity larger than 800 m/s. For the remaining 187 sites, we take the \( SBSR(f) \) of each site as its empirical transfer function. Among them, there are 11 sites with resonant frequencies being identified from neither \( HVS\bar{R}(f) \) nor \( SBSR(f) \) (Fig. 9b), 29 sites with resonant frequencies from either \( HVS\bar{R}(f) \) or \( SBSR(f) \) (Fig. 9c), and 147 sites with resonant frequencies from both \( HVS\bar{R}(f) \) and \( SBSR(f) \) (Fig. 9d).

We only utilize the 147 sites at which resonant frequencies can be identified from both \( SBSR(f) \) and \( HVS\bar{R}(f) \). Though the effects of downgoing waves on \( SBSR(f) \) could be mitigated due to our careful data selection, we could not completely rule out the possibility of pseudo resonances, which could affect the determination of \( f_{02,SB} \) and \( f_{p2,SB} \) using \( SBSR(f) \) through our fully automatic procedure. Thus we carefully check the presence of pseudo resonances on \( SBSR(f) \) by comparing the theoretical transfer functions using within and outcrop boundary conditions. Wherever pseudo resonant frequencies are found to be mistakenly picked up from \( SBSR(f) \) by our automatic procedure, we modify the resonant frequencies to true resonant frequencies (only 13.6% of the sites need such modification). Then \( f_{02,SB} \) and \( f_{p2,SB} \) can be confidently taken as site fundamental and predominant frequencies and be used to gauge the resonant frequencies from \( HVS\bar{R}(f) \) in Figure 10.

To quantify the comparison, we adopt different measures of goodness-of-fit (GoF), including the Nash-Sutcliffe model Efficiency coefficient \( E \) (Nash and Sutcliffe, 1970), index of agreement \( d \) (Willmott, 1981), Pearson correlation coefficient \( r \) and mean absolute error \( MAE \) as listed in Table 3. \( E \) can range from \(-\infty\) to 1 whereas both \( d \) and \( r \) vary between 0 and 1. For all these three GoF measures, the higher the value, the better the match. For instance, a value of 1 indicates a perfect match. Also listed in Table 3 is the number of sites with \( f_{HV}/f_{SB} \) out of a certain range.
In terms of approximating the fundamental frequency $f_{02, SB}$, Table 3 demonstrates that there are a slightly less number of sites out of the range $|\log_{10}(f_{02, SB}/f_{02, HV})| < 0.1$ in Figure 10b than in Figure 10a, which is also true for the range $|\log_{10}(f_{02, SB}/f_{02, HV})| < 0.3$. GoF indicators in Table 3 also, in general, suggest that $f_{p2, HV}$ is better than $f_{02, HV}$ in approximating $f_{02, SB}$. The success rates of the HVSR method in detecting site fundamental frequency are approximately 70% for the benchmark $0.8 \leq f_{SB}/f_{HV} \leq 1.25$ and more than 90% for $0.5 \leq f_{SB}/f_{HV} \leq 2.0$.

When it comes to matching the predominant frequency $f_{p2, SB}$ (Fig. 10c and d), all metrics in Table 3 consistently suggest that $f_{p2, HV}$ is favourable compared to $f_{02, HV}$. In addition, Figure 10c and d also illustrate that, at most sites, $f_{02, HV}$ and $f_{p2, HV}$ are either close to or lower than $f_{p2, SB}$, implying that the HVSR technique tends to underestimate the predominant frequency obtained using the SBSR method at some sites. This is consistent with previous findings based on the comparison of peak frequencies derived using HVSR and SSR approaches (e.g., Duval et al., 2001).

The underestimation of the HVSR approach to predominant frequency at some sites is mainly due to site amplification in the vertical component. For a pair of surface and downhole stations with no major velocity contrast below the borehole, we have

$$HVSR(f) = \frac{H(f)}{H_b(f)} \cdot \frac{V_b(f)}{V(f)} = \frac{HVSR_b(f)}{SBSR_v(f)} \cdot SBSR(f)$$  \hspace{1cm} (5)

where $H(f)$ and $V(f)$ - Fourier spectra of horizontal and vertical components, respectively, of a surface recording; $H_b(f)$ and $V_b(f)$ - Fourier spectra of horizontal and vertical components, respectively, of a downhole recording. For the 187 sites with no major velocity contrast below borehole, $\overline{HVSR}_b(f)$s and $\overline{SBSR}_v(f)$s are depicted in Figure 11.

For earthquake recordings, $\overline{HVSR}_b(f)$ (Fig. 11a) is close to 1.0 and is rather stable over the whole frequency band, namely a constant around unity, as also reported by Rong et al., (2017).
The vertical amplification $SBSR(f)$ (Fig. 11b) is near 1.0 only at low frequencies ($< \sim 1.0$ Hz), followed by significant amplifications at higher frequencies. Thus, according to Eq. 5, $HVSR$ amplitudes approximate site amplifications at low frequencies but underestimates them at relatively high frequencies mainly due to vertical site response (e.g., Lachet et al., 1996; Duval et al., 2001; Bindi et al., 2011). This underestimation could lead to the significant peak on $HVSR(f)$ at a lower frequency being picked up as the highest peak, underestimating the site predominant frequency at some sites (Fig. 10c and d).

Efficacy in Modelling Linear Site Effects

The comparisons of peak frequencies from $HVSR$ and $SBSR$ (Fig. 10 and Table 3) do not directly reveal which peak frequency, the first or the highest $HVSR$ peak frequency, should be utilized as a site indicator. To provide a direct answer to this question, the performance of a site proxy in depicting site effects should serve as a benchmark since one of the primary usages of site parameters is to characterize site amplification. To this end, we take the $SBSR(f)$ as the empirical amplification function of each site. Though this amplification is not identical to that derived from $SSR(f)$, the difference does not affect us gauging the efficacies of site proxies.

Konno-Ohmachi smoothing function with $b=20$ is utilized in this section. To increase the population size of resonant frequencies, we only apply a less restrictive criterion ($C1$) to determine peak frequencies from $HVSR(f)$. Besides $f_{p1,FAS}$ and $f_{01,FAS}$, $f_{p1,PSA}$ is also considered as a comparison, as well as the conventional site proxy $V_{S30}$. To compare with our previous study (Zhu et al., 2019a), we utilize resonant periods $T_{p1,FAS}$, $T_{01,FAS}$ and $T_{p1,PSA}$ which are reciprocals of
corresponding resonant frequencies. Amplification factors \((AF)\) at an oscillator period \(T=0.4\) s against \(T_{pl\_FAS}\) and \(V_{S30}\) are exemplified in Figure 12a and b, respectively.

Based on visual inspection, the relationship between \(AF\) and site period (Fig. 12a) can be described by a polynomial function. The function with fewest terms that are statistically significant is a quadratic function. Higher-order polynomials were also tested but only resulted in an insignificant increase in the coefficient of determination \((R^2)\). Thus, a second-order polynomial (Eq. 6) is adopted to depict the variation of \(AF\) with site resonant period. Site amplification is modelled as a linear function of \(V_{S30}\) (Eq. 7). We calculate the amplification residual \((Res)\) of observation about the regression line and the standard deviation of residuals \((\sigma_{Res})\) according to Eq. 8 and 9, respectively. The performance of each site indicator is evaluated based on \(\sigma_{Res}\). A lower \(\sigma_{Res}\) suggests a better modelling performance. The comparison is presented in Figure 13a for amplification models using a single proxy. Figure 13b depicts the relative change in \(\sigma_{Res}\) compared with that of model \(AF(T_{pl\_FAS})\).

\[
\ln[AF(Y)] = b_0 + b_1 \ln(Y) + b_2 [\ln(Y)]^2
\]  
(6)

\[
\ln[AF(V_{S30})] = a_0 + a_1 \ln(V_{S30})
\]  
(7)

\[
Res = \ln(AF_{obs}) - \ln(AF_{pre})
\]  
(8)

\[
\sigma_{Res} = \sqrt{\frac{\sum_i^n [\ln(AF_{obs}) - \ln(AF_{pre})]^2}{N}}
\]  
(9)

where

\(AF_{obs}\) - Observed site amplification at a certain period using \(SBSR_{FAS}\);

\(AF_{pre}\) - Predicted site amplification, including \(AF(V_{S30})\) and \(AF(Y)\);

\(AF(V_{S30})\) and \(AF(Y)\)- Site amplifications predicted using \(V_{S30}\) and \(Y\), respectively;
Various site resonant periods, e.g., $T_{p1\_FAS}$, $T_{01\_FAS}$ or $T_{p1\_PSA}$;

$Res$ - Amplification residual;

$\sigma_{Res}$ - Standard deviation of amplification residuals;

$N$ - Number of sites.

Figure 13a and b indicate that $T_{p1\_FAS}$ performs slightly better than $T_{p1\_PSA}$ in the investigated period range in characterizing linear site response. This is in good agreement with our preceding result that using response spectrum could lead to a biased estimate of the resonant period (or frequency) for sites with multiple comparable peaks on a $\overline{HVSR}(f)$ curve (Fig. 5). Meanwhile, Figure 13b also shows that $T_{p1\_FAS}$ has a better performance than $T_{01\_FAS}$ at relatively short periods ($< \sim 0.7$ s) in characterizing linear site effects, lowering $\sigma_{Res}$ by as much as 9%. However, at longer periods, $T_{01\_FAS}$ appears to be advantageous over $T_{p1\_FAS}$ by an insignificant margin (less than 2%). When only sites with $T_{p1\_FAS} \neq T_{01\_FAS}$ are used in the regression analysis, the difference in performance between $T_{p1\_FAS}$ and $T_{01\_FAS}$ is more evident. In general, the highest peak period (or frequency) exhibits a better overall performance than the first peak period in depicting linear site amplification, which can be explained by the fact that the former corresponds to a larger velocity contrast than the latter. In addition, Figure 13b also illustrates that adding $T_{01\_FAS}$ as a complementary proxy to $T_{p1\_FAS}$ can further lower the scatter to a small degree (less than 4%) over the examined period range.

In Figure 13a, the highest peak periods ($T_{p1\_PSA}$ and $T_{p1\_FAS}$) show a better performance than $V_{S30}$ in depicting linear site effects. However, the first peak period ($T_{01\_FAS}$) exhibits no evident advantage over $V_{S30}$, which is inconsistent with our previous results (Zhu et al., 2019a). In the preceding study, we adopted the site period ($T_0$) data provided by Wang et al., (2018) and showed that $T_0$ was a better site proxy than $V_{S30}$ in characterizing site effects. This inconsistency arises
from the different definitions of significant peaks. Wang et al. (2018) obtained $T_0$ data following a rather stringent procedure, which led to 265 (56%) out of 473 KiK-net sites having $T_0$. In contrast, resonant periods used in Figure 13 are picked up using only the first criterion $[C1: \log_{10}\overline{HVSR}(f_0) > 0.3])$, which results in identifying resonant period at a higher percentage of sites (202 or 98% out of 207 sites). A stricter peak screening would exclude more sites in the regression (no significant peak, as evidenced in Table 2) and retain only these at which site response is fundamentally governed by strong resonance. At these sites, resonant period has enhanced performance compared to $V_{S30}$. However, even under such a relaxed definition, the highest peak periods ($T_{p1,PSA}$ and $T_{p1,FAS}$) still outperform $V_{S30}$ (Fig. 13a) due to their insensitivities to peak definition (Fig. 7b).

Conclusions

Following an automatic procedure, we obtained the resonant frequencies at 207 KiK-net sites using the horizontal-to-vertical spectral ratio ($HVSR$) technique on earthquake ground-motions. We then investigated the factors contributing to the discrepancy in resonant frequencies determined from $HVSR$, e.g., using Fourier ($FAS$) or response ($PAS$) spectral ratio and using the first or the highest peak frequency. Our results indicate that, due to the scenario-dependence of response spectral ratio, utilizing response spectrum to compute the $HVSR$ could bias the estimates of resonant frequency for sites having multiple significant peaks with comparable peak values. Thus, the Fourier spectrum should be preferred in computing $HVSR$.

For most KiK-net sites (more than 80%) investigated in this research, the first peak on an average $HVSR$ curve (in the frequency domain) coincides with the highest peak. However, our results show that the highest peak frequency ($f_p$) is less susceptible than the first peak frequency
(f₀) to the selection criteria of significant peaks and the extent of smoothing to spectrum. Besides, comparing to the surface-to-borehole spectral ratio (SBSR), HVSR can only provide a lower-bound estimate of predominant frequency at which the strongest amplification occurs. However, fₚ tends to underestimate the predominant frequency less than f₀. More importantly, fₚ has a better overall performance than f₀ in characterizing linear site-effects. Thus, if the HVSR method is utilized to derive the resonant frequency of a seismic recording station, network operators should publish fₚ as a priority. Ideally, operators should also release the average HVSR curve from which end-users can extract the resonant frequency using self-defined criteria.

In this study, we also confirm that the HVSR amplitude underestimates the level of amplification at relatively high frequencies. This underestimation is site-specific, depending on the ratio between the HVSR at downhole and the SBSR in the vertical direction. Since the HVSR at downhole rock can be approximated by a constant close to unity over the examined frequency range (0.25~20.0 Hz), thus the underestimation is primarily controlled by the vertical transfer function of a site. Hence it is intriguing to explore the usage of HVSR amplitudes for frequencies at which vertical amplification is insignificant in future investigations.

Data and Resources

Velocity profiles of KiK-net recording stations were downloaded from the http://www.kyoshin.bosai.go.jp (last accessed on 05/06/2018). Matlab was used to automatically identify the first as well as the highest peak frequencies. Supplemental content includes a table and a file. The table tabulates site resonant frequencies and other site information (latitude, longitude,
$V_{S30}$ and $Z_{0.8}$) at the 207 KiK-net stations selected in this study. The file contains all average spectral ratios (e.g., HVSR and SBSR) at these 207 sites.

Acknowledgements

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References


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Figure 1. Consistency check on empirically-derived site resonant frequencies at selected KiK-net stations, Japan (Zhu et al., 2019). $f_{0,HV}$ (Wang et al.), $f_{0,SB}$ (Wang), $f_{0,RV}$ (Fujiiwara et al.) are site frequencies derived by Wang et al. (2018) using horizontal-to-vertical ($HV$) spectral ratio, by Wang (2017) and Pousse (2005) using surface-to-borehole ($SB$) spectral ratio, and by Fujiiwara et al. (2009) using radial-to-vertical ($RV$) spectral ratio approaches, respectively. The solid line represents the 1-to-1 line, and the dashed lines correspond to $f_{1} / f_{2} = 0.5$ and 2.0, respectively (after Zhu et al., 2019a).

Figure 2. (a) $V_{SB}$ vs. $Z_{0.8}$ distribution of the 207 selected KiK-net stations; and (b) $M_{eq}$-$R_{rup}$ distribution of 1840 selected ground motions.

Figure 3. (a) Influence of using FAS and PSA on the identification of the highest peak frequencies $f_{p1}$, where solid line and dashed lines represent $|\log_{10}(f_{P1,FAS} / f_{P1,PSA})| = 0$ and 0.3, respectively; and (b) $[HVS_{FAS}(f)] / HVS_{FAS}(f)$ for the 207 KiK-net sites and their average (± one standard deviation). FAS and PSA were smoothed using Konno-Ohmachi window ($b=20$).

Figure 4. (a) Velocity profile of KiK-net site IWTH16, and (b) Source-site distance versus magnitude of 144 earthquakes recorded at IWTH16.

Figure 5. (a) $HVS_{FAS}(f)$ of 144 recordings at KiK-net site IWTH16. Dots correspond to the highest peak frequency ($f_{p1}$) of each individual curve; Bold solid curve and dashed curves in each plot denote the mean spectral ratio $HVS_{FAS}(f)$ and its variation (mean ± one standard deviation), respectively.

Figure 6. Identification of the highest peak frequency $f_{p1}$ at KiK-net site IWTH16 from the average (a) $HVS_{FAS}$, and (b) $HVS_{PSA}$ of $N$ sample recordings randomly selected from the 144 records (Fig. 5). For FAS, $N$=3, 5, 7 and 9; and for PSA, $N$=3, 5, 7, 9, 13, 21 and 59. For each $N$, the test was repeated 100 times.

Figure 7. Influence of peak definition on the identification of (a) the first peak frequency $f_{0}$, and (b) the highest peak frequency $f_{p3}$; and (c) $HVS_{FAS}$ at site FKSH10. Solid and dashed bold lines in (c) represent the mean $HVS_{FAS}$ (f) and mean ± one standard deviation, respectively. $HVS_{FAS}$s are computed on smoothed FAS with a Konno-Ohmachi window ($b=20$).

Figure 8. Influence of smoothing (Konno-Ohmachi) coefficient on the identification of (a) first peak frequency $f_{0}$, and (b) the highest peak frequency $f_{p3}$. Both $f_{0}$ and $f_{p3}$ are derived using $HVS_{FAS}$ on smoothed FAS. Solid line and dashed lines represent $|\log_{10}(f_{1} / f_{2})| = 0$ and 0.3, respectively.

Figure 9. Spectral ratios at exemplary site (a) AKTH19 (with a major velocity contrast below borehole sensor), (b) NGNH10 (neither $HVS_{FAS}$ nor $SBS_{FAS}$ has a significant peak), (c) KKWH14 (either $HVS_{FAS}$ or $SBS_{FAS}$ has a significant peak), and (d) FKSH10 (both $HVS_{FAS}$ and $SBS_{FAS}$ have a significant peak). Vertical lines mark the locations of resonant frequencies, $f_{0,HV}$ and $f_{p2,HV}$ are the first and the highest peak frequencies, respectively, identified from $HVS_{FAS}$; and $f_{0,SB}$ and $f_{p2,SB}$ are the first and the highest peak frequencies, respectively, identified from $SBS_{FAS}$. All spectral ratios are calculated on FAS smoothed using a Konno-Ohmachi window with $b=20$.

Figure 10. First ($f_{0,HV}$) and highest ($f_{p2,HV}$) peak frequencies derived from $HVS_{FAS}$ versus those ($f_{0,SB}$ and $f_{p2,SB}$) obtained from $SBS_{FAS}$ at 147 KiK-net sites. (a) $f_{0,HV}$ vs $f_{0,SB}$, (b) $f_{p2,HV}$ vs $f_{p2,SB}$, (c) $f_{0,HV}$ vs $f_{p2,SB}$, (d)
Solid line, dotted lines and dashed lines in each plot represent $|\log_{10}(f_1/f_2)| = 0, 0.1$ and 0.3, respectively. Other conditions are the same to those in Fig. 9.

**Figure 11.** (a) $HVS\bar{R}_{\delta}(f)$s and (b) $SB\bar{R}_{\delta}(f)$s of 147 KiK-net sites. Bold solid and dashed curves in each plot represent the mean and mean ± one standard deviation, respectively. Other conditions are the same to those in Fig. 9.

**Figure 12.** Site amplification at $T=0.4$ s vs. (a) $T_{p1,FAS}$, and (b) $V_{S30}$ (In-In units).

**Figure 13.** Standard deviation of residuals of amplification models using $T_{p1,FAS}$, $T_{01,FAS}$, $T_{p1,PSA}$ and $V_{S30}$ as the sole predictor variable; and (b) Relative change in standard deviation using $AF(T_{p1,FAS})$ as a reference with positive and negative values denoting decrease and increase in $\sigma_{Res}$, respectively.
Figures

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### Tables

#### Table 1

Determination of site resonant frequency using spectral ratio techniques

<table>
<thead>
<tr>
<th>Reference</th>
<th>Recording Stations</th>
<th>Approach</th>
<th>FAS or PSA*</th>
<th>Smoothing</th>
<th>Definition of Significant Peaks</th>
<th>( f_0 ) or ( f_p )</th>
<th>Percentage of sites assigned ( f_0 ) or ( f_p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hassani and Atkinson (2018)</td>
<td>California</td>
<td>HVSR(^{\dagger})</td>
<td>PSA</td>
<td>No</td>
<td>Amplitude larger than both 1.5 times ( HVSR ) and 2.0</td>
<td>( f_p )</td>
<td>554 (72%) out of 767 sites</td>
</tr>
<tr>
<td>Hassani and Atkinson (2016)</td>
<td>CENA(^{\dagger})</td>
<td>HVSR</td>
<td>PSA</td>
<td>No</td>
<td>Amplitude larger than both 1.5 times ( HVSR ) and 2.0</td>
<td>( f_p )</td>
<td>315 (59%) out of 535 sites</td>
</tr>
<tr>
<td>Wang et al. (2018)</td>
<td>KiK-net</td>
<td>HVSR</td>
<td>PSA</td>
<td>No</td>
<td>Amplitude larger than both 1.5 times ( HVSR ) and 2.0; and ( 0.5 \leq \frac{f_0}{f_R} \leq 2.0 )</td>
<td>( f_0 )</td>
<td>265 (56%) out of 473 sites</td>
</tr>
<tr>
<td>Wang (2017)</td>
<td>KiK-net</td>
<td>c-SBSR(^{#})</td>
<td>FAS</td>
<td></td>
<td>Amplitude larger than 20% of the highest peak</td>
<td>( f_0 )</td>
<td>249 (59%) out of 425 sites</td>
</tr>
<tr>
<td>Kwak et al. (2017)</td>
<td>K-NET &amp; KiK-NET</td>
<td>HVSR</td>
<td>FAS</td>
<td></td>
<td>Amplitude larger than both 1.5 times ( HVSR ) and 2.0</td>
<td>( f_0 )</td>
<td>1608 (97%) out of 1658 sites</td>
</tr>
<tr>
<td>Ghofrani et al. (2014)</td>
<td>KiK-net</td>
<td>HVSR</td>
<td>PSA</td>
<td></td>
<td>Amplitude larger than twice ( HVSR )</td>
<td>( f_0 )</td>
<td>n.a.</td>
</tr>
<tr>
<td>Ghofrani et al. (2013)</td>
<td>KiK-net</td>
<td>HVSR</td>
<td>FAS</td>
<td></td>
<td>Amplitude larger than twice ( HVSR )</td>
<td>( f_0 )</td>
<td>Single &quot;peak&quot; (60%); Two peaks (20%)</td>
</tr>
<tr>
<td>Fujiwara et al. (2009)</td>
<td>KiK-net</td>
<td>RVSR*</td>
<td>FAS</td>
<td>n.a.</td>
<td>Amplitude larger than 2 in a statistical sense; and within the frequency ranges ([0.5f_0, f_0 ) and ([f_0, 2f_0]), there exists a frequency at which amplitude is at least 10% lower than the amplitude at ( f_0 )</td>
<td>( f_0 )</td>
<td>All 674 (100%) sites</td>
</tr>
<tr>
<td>Pousse (2005)</td>
<td>KiK-net</td>
<td>SBSR(^{*})</td>
<td>FAS</td>
<td>n.a.</td>
<td>Amplitude larger than 2 in a statistical sense; and within the frequency ranges ([0.5f_0, f_0 ) and ([f_0, 2f_0]), there exists a frequency at which amplitude is at least 10% lower than the amplitude at ( f_0 )</td>
<td>( f_0 )</td>
<td>501 (or 93%) out of 538 sites</td>
</tr>
</tbody>
</table>

* FAS and PSA – Fourier and response spectra, respectively;

† \( f_0 \) and \( f_p \) – the frequencies corresponding to the first and the predominant peaks, respectively;

\( \dagger \) \( HVSR \) - horizontal-to-vertical spectral ratio;

\( \ddagger \) \( HVSR \) - \( HVSR \) averaged over all usable frequencies at multiple stations;

\( \|$ CENA \) - Central & Eastern North-America;

\( \# \) SBSR and c-SBSR - Surface-to-borehole spectral ratio and cross spectral ratio, respectively;
+ RVSR - radial-to-vertical spectral ratio;

++ \( f_k \) - Analytical fundamental frequency calculated from an available 1D velocity profile using the Rayleigh method.

**Table 2**

Influences of peak definition and smoothing on the number of significant peaks

<table>
<thead>
<tr>
<th>Smoothing coefficient</th>
<th>Criteria</th>
<th>No peak</th>
<th>Single peak</th>
<th>Multiple peaks</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b = 20 )</td>
<td>( C1 )</td>
<td>6 (3%)</td>
<td>61 (29%)</td>
<td>140 (68%)</td>
</tr>
<tr>
<td>( C1, C2 )</td>
<td></td>
<td></td>
<td>113 (55%)</td>
<td>58 (28%)</td>
</tr>
<tr>
<td>( C1, C2, C3 )</td>
<td></td>
<td></td>
<td>116 (56%)</td>
<td>53 (26%)</td>
</tr>
<tr>
<td>( b = 30 )</td>
<td>( C1, C2, C3 )</td>
<td>22 (11%)</td>
<td>83 (40%)</td>
<td>102 (49%)</td>
</tr>
<tr>
<td>( b = 40 )</td>
<td>( C1, C2, C3 )</td>
<td>16 (8%)</td>
<td>55 (27%)</td>
<td>136 (66%)</td>
</tr>
</tbody>
</table>

**Table 3**

Goodness-of-fit between resonant frequencies derived from \( \overline{SBSR}(f) \) * and \( \overline{HVS}(f) \) †

<table>
<thead>
<tr>
<th></th>
<th>( f_{02,SB} )</th>
<th>( f_{p2,SB} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( f_{02,HV} )</td>
<td>( f_{p2,HV} )</td>
</tr>
<tr>
<td></td>
<td>(Fig. 10a)</td>
<td>(Fig. 10b)</td>
</tr>
<tr>
<td></td>
<td>(Fig. 10c)</td>
<td>(Fig. 10d)</td>
</tr>
<tr>
<td>Nash-Sutcliffe model</td>
<td>0.706</td>
<td>0.751</td>
</tr>
<tr>
<td>efficiency coefficient ( E )</td>
<td></td>
<td>-0.022</td>
</tr>
<tr>
<td></td>
<td>0.216</td>
<td></td>
</tr>
<tr>
<td>Index of agreement</td>
<td>0.910</td>
<td>0.923</td>
</tr>
<tr>
<td></td>
<td>0.675</td>
<td>0.745</td>
</tr>
<tr>
<td>Pearson’s correlation</td>
<td>0.889</td>
<td>0.878</td>
</tr>
<tr>
<td>coefficient ( r )</td>
<td>0.655</td>
<td>0.700</td>
</tr>
<tr>
<td>Mean absolute error</td>
<td>1.297</td>
<td>1.226</td>
</tr>
<tr>
<td>( MAE )</td>
<td>2.749</td>
<td>2.280</td>
</tr>
<tr>
<td>Sites with ( f_{HV}/f_{SB} ) out of range ([0.5, 2.0])</td>
<td>8%</td>
<td>6%</td>
</tr>
<tr>
<td></td>
<td>22%</td>
<td>16%</td>
</tr>
<tr>
<td>Sites with ( f_{HV}/f_{SB} ) out of range ([0.8, 1.25])</td>
<td>31%</td>
<td>29%</td>
</tr>
<tr>
<td></td>
<td>50%</td>
<td>40%</td>
</tr>
</tbody>
</table>

* \( SBSR(f) \) – Average surface-to-borehole spectral ratio over several recordings;

† \( HVS(f) \) – Average horizontal-to-vertical spectral ratio over several recordings;