Archie’s Law - Boon or Bane?  
An approach to estimate gas hydrate saturations
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Archie’s Law (1942)

$$\rho_f = a \rho_w \Phi^{-m} (1-S_h)^n$$

**The Boon:** A practical tool to relate resistivity to porosity, salinity, saturation estimates

**The Bane:** Often used standard coefficients may lead to over- / underestimated saturation estimates

Daube Delta, Western Black Sea
what is the gas hydrate saturation?
**Archie’s Law, parameters**

\[ \rho_f = \alpha \rho_w \Phi^{-m} \]

\[ \rho_f = \alpha \rho_w \Phi^{-m} \left(1-S_h\right)^n \]

\[ \rho_f = \rho_f \left(1-S_h\right)^n \]

\[ \frac{1}{R} = \phi^n \frac{S_h^n}{\alpha R_w \left(1-V_{cl}\right)} + V_{cl} \frac{S_w^{n-1}}{R_{cl}}. \]

- \( \rho_f \) = formation resistivity
- \( \alpha \) = constant, tortuosity factor
- \( \rho_w \) = pore water resistivity
- \( \Phi \) = porosity
- \( m \) = cementation factor
- \( S_h \) = gas hydrate saturation
- \( n \) = saturation exponent
- \( R_{cl} \) = clay resistivity
- \( V_{cl} \) = clay volume fraction


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**Archie coefficients :**

<table>
<thead>
<tr>
<th>( a )</th>
<th>( m )</th>
<th>( n )</th>
<th>Reference</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 for ( \Phi = 100% )</td>
<td>1.8 - 2.0</td>
<td>1.9386</td>
<td>0.5 - 4</td>
<td>Archie, 1942</td>
</tr>
<tr>
<td># 1</td>
<td>sand to shell</td>
<td>0.5 - 4</td>
<td>2.5 +/- 0.5</td>
<td>Pearson et al, 1983</td>
</tr>
<tr>
<td>Intercept of the log ( \rho / \log \Phi ) at ( \Phi = 100% )</td>
<td>clean sands</td>
<td>1.945</td>
<td>Archie, 1942</td>
<td>Depends on shape rather than grain size and sorting; Varies with clay content</td>
</tr>
<tr>
<td></td>
<td>compacted sandstone</td>
<td></td>
<td>Jackson et al., 1978</td>
<td>Salem &amp; Chillingaram, 1999</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Riedel et al., 2019</td>
<td>Archie, 1942</td>
</tr>
</tbody>
</table>

**a = 1, m = 2, n = 2** are often used standard Archie coefficients
**MeBo Drilling:**

\[ \rho_f = a \rho_w \Phi^{-m} \]

- porosity decrease from \( \sim 70\% \) at seafloor to \( \sim 45\% \) below 30m
- salinity decrease from 22 psu at seafloor to 2.5 (\( \sim 4.0 \)) psu below 30m
- pore water resistivity increase to 2.88 \( \Omega \)m (2.5 psu) below 30m (Fofonoff, 1983)

MeBo res. log is within the range of derived background resistivities

**Model selection:**

- SW
  - 0.9902
- MTD
  - higher resistivity
- SW
  - 0.9902
- CSEM sensitivity
- SW
  - 0.9902
- eastern levee
**MeBo resistivity compared to inverted resistivity:**

![Graphs showing resistivity comparison](image)

**Saturation Models**

- Average porosity: 75% → 45% below 40 m
- Average pore water resistivity: 0.4 Ωm → 2.86 Ωm below 40 m
- \(a=1\), \(n=2.5\)

**Does the same set of Archie coefficients hold for the entire model?**
**Stochastic Approach, 1D**

Input parameter ranges:
- Log \( \rho \) ranges (Gaussian)
- Average porosity +/- 20% (Gaussian)
- Average pore water resistivity, salinity +/- 10% (uniform)
- \( a = [0.9 \ 1.1]; m = [1.8 \ 2.5]; n = [2.0 \ 2.5] \) (uniform)

\[ S_h = 1 - \left[ \frac{a \ \rho_w \ \phi^{-m}}{\rho_f} \right]^{1/n} \]

After Sava and Hardage, 2007

68% confidence intervals

**Stochastic Approach, 2D**

Saturation with maximum probability

Schwalenberg et al., soon submitted to SI MarPotGeo
Conclusions

Archie’s Law (1942)

\[
\rho_f = a \rho_w \Phi^{-m} (1-S_h)^n
\]

- **The Boon:** A practical tool to relate resistivity to porosity, salinity, saturation estimates
- **The Bane:** Often used standard coefficients may lead to over- / underestimated saturation estimates
- **The stochastic approach** can help to define saturation ranges based on probability and credibility intervals

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