Distributed viscosity and flow velocity measurements using a fiber-optic shear stress sensor

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**Article Info**

**Keywords:**
Fiber-optic distributed sensing
Viscosity sensor
Optical time domain reflectometry (OTDR)

**Abstract**

The understanding and precise prediction of fluid and solid displacement is of great interest in many technical applications. Density and viscosity are two key parameters that govern process mechanisms. The possibility to measure transient processes over longer distances is desirable. We present a novel distributed shear stress sensor that allows to derive fluid rheological parameters such as the viscosity along a fiber-optic cable being exposed to a moving medium. This works because flow velocity and fluid viscosity directly translate to a shear stress and consequently to a tensile strain on the fiber optic cable. Using the technology of fiber-optic distributed strain sensing, strain changes (and temperatures) are detected in real-time at any location along the fiber. Given the cable mechanical properties and geometry of the flow path, the strain translates to a shear stress which can be correlated to either the flow velocity or fluid viscosity. We derive a theoretical characterization of the sensor based on the principles of fluid mechanics. Also, we perform laboratory experiments with the sensor and demonstrate that we can distinguish differences of 1 mPa s dynamic viscosities in a range as low as 1 – 7 mPa s. In the next phase, we are implementing this sensor into a real working environment in a wellbore application to investigate the applicability of this novel sensor technology.

1. Introduction

Understanding and precise prediction of liquid rheological parameters and fluid displacement processes is vital for many different applications and industries. In downhole/borehole application, the determination of fluid viscosities is important for operation, whether for oilfield applications [1], geothermal [2,3], carbon capture and sequestration operations [4]. In biomedical engineering, the blood viscosity is an important parameter for the analysis of cardiovascular systems [5–7]. Human whole blood has a viscosity range of 3.5 mPa s [8] to some 20–30 mPa s [9]. In practice, the viscosity of a fluid is often determined in laboratory set-ups and the results are transferred to field applications. However, deducing parameters such as the fluid viscosity from correlations introduces uncertainties due to alteration of the chemical composition of the process fluid (in open systems), changes in temperature and pressure or long-term rheological changes.

In good to reach surface facilities, there are numerous options presented in literature that enable to measure the fluid viscosity in real-time. Ricco et al. presents an acoustic wave point viscosity sensor with sensing ranges from 0.46 to 21.6 mPa s with best results for water at 1–13.5 mPa s (error <5 %, above ca. 40 %) [10]. Jakoby et al. presents a micro-acoustic point sensor for oil engines [11]. Haidekker et al. presents a radiometric fluid viscosity point sensor for a viscosity range of 1–400 mPa s [12]. Riesch et al. shows the potential of a microsensor plate viscosity point sensor with a piezoresistive readout [13]. Heinisch et al., show that a tunable resonator in low kHz range can be used for viscosity sensing [14]. There are a few fiber-optic based point viscosity measurement devices presented in literature that utilize standard telecommunication fibers, plastic fibers and fiber bragg gratings (FBG) [15–17]. Another micro viscosity point sensor for medical purposes was recently proposed [18]. A broader overview on general physical sensors of fluid properties [19] and specified for downhole fluid analysis in boreholes of the oil industry [20] are presented in literature.

In the context of borehole applications (and other fields of applications that are hard to reach and include long flow distances), viscosity point sensors are not necessarily sufficient to obtain an understanding of
varying fluid properties and displacement processes. Over these longer
distances, fluid mixing, temperature and pressure changes, chemical
alterations of the process fluid and varying cross sectional flow areas
are present which can all vary the fluid pumping and fluid displacement
efficiency. Analytical and numerical models available in literature to
model the fluid displacement are not capable to access the full
complexity of downhole conditions [21]. Also, modeling approaches are
difficult to validate if data from the subsurface is sparse.

Mishra et al. developed a downhole viscosity sensor for wire-line
operations [22]. It includes a miniaturized wire sensor which resonates
after electromagnetic excitation dependent on the viscosity of the
surrounding fluid. That sensor can resolve the viscosity at a downhole
location in the ranges from 0.2 mPas to 300 mPas with uncertainties
stated in the range of 10%. Another device for downhole viscosity
measurements which operates autonomously without wire-line
connection is proposed by a tuning fork device [23]. Gonzalez et al.
present two further fluid viscosity and density measurement platforms
for downhole applications [24]. These examples are good for tubing/-
casing measurements, but certainly not for annular measurements. Also,
these systems are point sensors.

In this paper, we present a novel technology to measure fluid vis-
cosity based on fiber optic distributed strain sensing. This sensing
approach has the potential to be used as a distributed viscosity sensor
over kilometer long sensing ranges with spatial viscosity resolutions in
the meter scale and a viscosity resolution in the range of 1 mPas.

2. Methods

The distributed viscosity sensor is based on the idea that a fluid in
motion generates fluid rheology depending shear stresses on surfaces in
its flow path which consequently translates to tensile strain on the fiber.
We analyzed the behavior of the sensor only for a laminar flow regime.
We use a standard telecommunication single-mode fiber as the sensing
element in the flow path on which a fluid flow induced force \( F \) translates
into a strain \( \varepsilon \) (Fig. 1). The schematic drawing shows a strain sensitive
fiber with diameter \( d_c \) in the center of a pipe. The fiber is mechanically
confined at the bottom “\( p_1 \)” and top “\( p_2 \)” of the flow path. The fiber is
exposed to a moving fluid that flows in an upward direction. The fluid in
motion drags both on the pipe tubes inner wall as well as the fiber in the
flow path resulting in tensile straining. For each length increment \( x \) of
the fiber, an area increment \( A_x \) is exposed to wall shear stresses which
result in a force \( F_x \) and a relative tensile strain of \( \Delta \varepsilon_x \). Going from top to
bottom, a strain gradient (Fig. 1, right) forms along the sensing fiber
because subsequent length increments add to the tensile stresses in the
fiber. A thorough analytical approach is presented in the Section 2.2.

First, we introduce the fiber optic sensing technology that we use to
measure the strain on the fiber. Then, we present the physical working
principle of our sensing approach using a model based on basic empir
ical equations from fluid mechanics. After that, we introduce our labo
ratory setup and experimental procedure. The sensitivity of this sensor is
highly depended on the fiber-optic cable type used for sensing. There
fore, we also analyze different cable types and discuss their applicability
for this monitoring approach.

2.1. Fiber optic distributed strain sensing

For the validation of our sensing system, we use the OTDR system
OBR 4400 from LUNA Technologies for distributed temperature and
strain sensing. A detailed description of the technology can be found in
literature [25]. The OBR system uses swept-wavelength interferometry
(SWI) to generate a spectral fingerprint of a sensing fiber at a given
temperature and strain state. Changing the temperature and/or strain

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**Fig. 1.** Example figure to show the working principle of the sensor.

**Fig. 2.** Workflow to determine forces on a fiber by flowing fluid in a forward
procedure for a known fluid (from top to bottom). During the real sensor
application, the procedure would be vise versa from force to viscosity
(green arrow).
state with respect to the reference measurement leads to a shift of the local spectrum of the fiber. This means in practice, that an OBR can only be used to measure changes in temperature and strain with respect to a given baseline measurement by performing a cross correlation of fiber signal segments. The spectral shift $\Delta \nu$ due to a change in temperature $\Delta T$ and/or strain $\Delta \varepsilon$ is given by the equation [26]:

$$\frac{\Delta \lambda}{\lambda_c} = \frac{\Delta \nu}{\nu_c} = C_T \Delta T + C_E \Delta \varepsilon$$ (1)

where $\lambda_c$ and $\nu_c$ are the central optical wavelength and frequency ($\lambda_c = 1550 \text{ nm}$), $C_T$ and $C_E$ are the temperature and strain calibration constants. For germanosilicate core fibers, which are used in this study, these constants are typically: $C_T = 6.45 \times 10^{-6} \text{ } ^\circ \text{C}^{-1}$ and $C_E = 0.780 \mu \text{e}^{-1}$ [27]. Tosi et al. provide a review of fiber-optic based technologies that enable strain and temperature sensing with high spatial resolution (sub-millimeter to centimeter-level) and gives an insight how these technologies find their application in small-scale biomedical applications [28]. Beisenova et al. performed experiments with the OBR technology to measure small-scale strain changes along a medical needle to detect events related to the needle penetration path [29]. The OBR technology was also used to create a 3D shape sensor for small-scale biomedical devices [30].

### 2.2. Analytical approach

Basic equations from fluid mechanics are used to determine the tensile straining exerted from shear stresses on the cable by the flowing fluid. Fig. 2 shows an overview of the workflow. Input parameters are specified for both the geometry (such as hydraulic diameter, cable diameter, cable stiffness) and the fluid (such as density, flow rate).

Fluid flow rates are used to calculate fluid shear rate and fluid viscosity which is then used to estimate the flow regime according to the dimensionless Reynolds number ($Re$):

$$Re = \frac{qD_h}{\eta}$$ (2)

where $q$ is the average velocity of the flow, $\rho$ the density of the fluid, $D_h$ the hydraulic diameter of the pipe and $\eta$ the dynamic viscosity of the fluid. In the hydraulics of pipe flows, the Darcy-Weisbach equation is an important empirical formula to calculate pressure friction losses over a given length of a cylindrical pipe [31]. The pressure loss over length is a function of the density of the fluid, its velocity, the area of the pipe and the characteristic friction factor. The pressure friction loss can be approximated with the equation [31]:

$$\frac{\Delta p}{L} = f_0 \frac{\rho}{2 \eta} \frac{q^2}{D_h}$$ (3)

where $\Delta p$ is pressure loss over a length $L$ of a pipe. $f_0$ is the Darcy friction factor which in literature is also referred to as the flow coefficient $\lambda$. Solving Eq. (3) for the Darcy friction factor results in

$$f_0 = 2 \frac{\Delta p}{p_0} \frac{D_h}{q^2} L$$ (4)

Independently and almost simultaneously, Jean Poiseuille (1799–1869) and Gotthilf Hagen (1797–1884) defined the pressure drop in an incompressible, Newtonian fluid in laminar flow through a long cylindrical pipe of constant cross section [32,33]. In contrast to Darcy-Weisbach, it does not include a friction factor but the viscosity of the fluid. The Hagen-Poiseuille formula is defined as [34]:

$$Q = \frac{dV}{dt} = \frac{\pi}{4} \frac{D_h^4}{\eta}$$ (5)

Solving Eq. (5) for $\Delta p$ and replacing the volumetric flow rate $Q$ with the average flow speed $q$ gives:

$$\Delta p = 8qD_h \frac{\pi L}{D_h^4} = 32qL$$ (6)

Inserting Eq. (6) into Eq. (4) one obtains the Darcy friction factor for laminar flow in a circular pipe. Together with Eq. (2), the Darcy friction factor $f_0$ is [35]:

$$f_0 = \frac{64}{Re}$$ (7)

The friction factor for laminar flow $f_0$ is used to calculate the average wall shear stress $\tau_w$. From literature it is known that $\tau_w$ can be expressed as [36]:

$$\tau_w = 4 \frac{Q}{r} = 8 \frac{q}{D_h} = \frac{\rho}{2} \frac{q^2}{D_h}$$ (8)

The wall shear stress $\tau_w$ will create a force on a cable which is exposed to the flowing fluid. The diameter of the cable $d_c$ defines the area over which $\tau_w$ acts. Hence the absolute force $F_{\text{total}}$ acting along the cable in response to a certain flow condition defined by $\tau_w$ over a cable length $L$ can be written as:

$$F_{\text{total}} = \tau_w d_c L$$ (9)

The total force on the cable can also be expressed as the sum of a
sequence of force increments per cable length intervals \( L \):

\[
F_{\text{total}} = \sum_{i=0}^{n} F_i \Delta d_i \Delta L_i
\]

(10)

This force will lead to a deformation of a fiber-optic sensor/cable according to Hooke’s law [37]:

\[
F_{\text{cable}} = k \varepsilon
\]

(11)

where \( k \) is the specific stiffness of the fiber sensor and \( \varepsilon \) is the strain. Combining Eqs. (2), (7)–(9) and (11) gives the following equation:

\[
\varepsilon = \frac{2 \pi \rho q_i d_i^2}{kD_H}
\]

(12)

The expression \( \varepsilon/L \) is the strain gradient (schematically introduced in Fig. 1) for a cable in an annular tube. The strain gradient is a function of the hydraulic diameter of the tube \( D_H \), the average flow velocity \( q \), the fluid dynamic viscosity \( \eta \) and the cable diameter to cable stiffness ration \( d_i/k \) [38,39]. Analytical assessments for the cumulative stiffness of different multilayer cables are presented in literature [38,39]. Fig. 3 shows a number of fiber types, including the tested bare fiber (acrylate coated single-mode fiber) and two more rigid cables. The strain sensitive cable formed an example of a tight-buffer cable with a sensing fiber, a multi-buffer single-mode fiber) and two more rigid cables. The strain sensitive cable number of fiber types, including the tested bare fiber (acrylate coated single-mode fiber) and two more rigid cables. The strain sensitive cable.

2.3. Experimental approach

An experimental setup was designed that enables to expose a strain sensing fiber to flowing fluids. A technical drawing of the experiment can be found in Fig. 4. The main component of the experiment is a 2 m long borosilicate glass pipe with an outer diameter of \( OD = 24 \text{ mm} \) (ID = 20.4 mm) and two custom-made 3-D printed crosspieces at both ends of the glass pipe. The sensing fiber (ID = 220 μm) is positioned in the straight path of the setup and fixed with Swagelog fitting at both in- and outlet of the crosspieces. During fixation, pretension was applied to the fiber. Fluids are circulated from a fluid container through the experimental setup. An impeller pump is used to form a relative continuous fluid flow velocity. The flow velocity is measured with a flowmeter with a measurement accuracy of 0.5 %. Temperature sensors are installed in the fluid reservoir and both next to the bottom and the top of the sensor fiber inlet (measurement accuracy \( \pm 0.15 \text{ K} \)). A differential pressure sensor (IDPT100) is installed over the length of the borosilicate pipe with a measurement accuracy of 0.25 %. Flow velocity, temperature and differential pressure are sampled with a temporal resolution of 1 s. The OBR 4400 data is sampled with a laser scan range from 1561.2 to 1571.9 nm and a gain of 6 dB. The data is processed with the official Luna OBR 4400 software to obtain spectral shift profiles relative to the baseline state at the start of each experimental run. The data processing is performed with a spatial resolution of 0.1 mm, an integration width of 10 mm and a shift resolution of 10 mm.

We prepared five fluid samples with different viscosities for the experiment. One fluid is demineralized water and the other four fluids are water + CaCl₂ solutions in different concentrations (see Table 2). We have chosen water + CaCl₂ because of its lower temperature sensitivity in comparison to e.g. viscous oils and the viscosity can be well adjusted via the CaCl₂ concentration. Density and viscosity measurements were performed with an Anton Paar AMVn automatic micro viscosimeter at different temperatures. The measurement accuracy is 0.5 % for the viscosity measurement and 0.05 °C for temperature. The temperature variation from Table 2 results from the temperatures measured with the temperature sensors in the experimental setup. The values for density and viscosity are interpolated for the given temperatures using the viscosimeter data from the reference measurement.

3. Results

Fig. 5 shows the results from the experimental run for the most viscous fluid sample “H₂O + CaCl₂ 4” (see Table 2). The upper subplot shows the flow meter reading translated to a flow velocity (in black) and the temperature recordings in the fluid container and at bottom and top of the fiber optic sensing length along the borosilicate pipe. The center subplot shows four profiles of static distributed strain profiles at the marked locations (FO1–FO4 in the upper subplot). The grey shaded...
interval marks the fiber length which is exposed to the moving fluid. With increasing flow velocity, the tensile straining from the shear stress along the fiber increases leading to a steeper strain gradient. There is an offset towards negative strains in the displayed strain profiles. The dotted lines in the center subplot depict a linear interpolation along each of the strain profiles over the center distance of 1.8 m in the pipe, meaning that the first and last 10 cm of the pipe are truncated. The gradient of each strain measurement of this experimental run is plotted as a dot in the lower subplot. The lower plot also shows the empirical model (Eq. (12)) with the input parameter from Tables 1 and 2. Each measured strain profile is depicted with a grey cross (before temperature correction). The green triangles show temperature corrected data using averaged temperature data from temperature gauge "bottom" and "top".

Fig. 6 shows the fiber optic strain gradient measurements for all tested fluids together with the expected results from the modeling approach. For demineralized water at 1 mPa s (first subplot), the gradient of the measurement data matches the model but with a systematic error of +0.5 με/m. For the fluids with a dynamic viscosity λ of 1.9 mPa s, 4.4 mPa s and 6.8 mPa s, the measured data shows a good fit with the modeled viscosity trend line. For the fluid with λ = 3.2 mPa s, the measured strain gradient ε/L is significantly lower at flow rates below ca. 0.2 m/s. Above that flow velocity, the measured data shows a good fit with the modeled viscosity trend line.

4. Discussion

In this paper, we investigate the possibility to use distributed fiber-optic strain sensing to derive fluid rheological parameters such as the fluid viscosity. This sensing approach is based on the idea that a fluid in motion generates shear stresses on surfaces in its flow path which consequently translates into a force. The resulting force which acts on a fiber/fiber-optic cable can be translated to strain given the cumulative cable material stiffness. By rearranging basic empirical equations from fluid mechanics, we can show that a strain on the sensing fiber is a function of the hydraulic diameter of the flow path D_H, the fluid flow velocity q, the fluid viscosity λ and the fiber/cable specific parameters such as the diameter d_c and the stiffness k (see Eq. (12)). A standard acrylate coated single-mode fiber was used in this study. As presented in Fig. 3, the tested bare fiber has a relatively high d_c/k ratio compared to other conventional cable types, which is favorable for the detection of fluid shear stress induced deformations. When using a fiber-optic cable with an elastomer mantle material, the sensor sensitivity can be improved by a factor 2 compared to the tested fiber.

To validate the theoretical approach based on empirical equations, an experimental setup is designed that allows for a controlled exposure of a strain sensing fiber to a moving fluid. We intentionally selected to test fluids with rather low dynamic viscosities in the range of 1–7 mPa s to limit test the capabilities of this sensor. From the theoretical approach we calculate that a fluid with 1 mPa s creates just enough force for a detectable strain with the given strain sensing technology. In total, the experimental results show a good fit with the theoretical approach. With respect to Fig. 6, the investigated fluid samples could be distinguished with confidence based on the resulting strain gradient ε/L which forms along the sensing fiber. The results are best in the flow velocity range from 0.1 m/s to 0.3 m/s. For lower flow rates, the strain gradient is rather inconsistent. The poorer data quality at q < 0.1 m/s can be partly attributed to the low tensile straining due to the slow fluid movement. In addition, it was observed during the experiment, that the impeller pump had difficulties to maintain a steady fluid flow at the low flow rates and the fluid moved rather periodically. Using a sensing system in the future...
with a higher repetition rate might result in higher data quality. For the fluid with a dynamic viscosity of $\eta = 3.2$ mPa s, the resulting strain gradient $\varepsilon/L$ shows a certain offset from the theoretical prediction (Fig. 6), especially for flowrates $q < 0.2$ m/s. One reason for this could be trapped air in the fluid pathway. In between each experimental run, the fluids are exchanged. In this process the fluid column is shortly interrupted and an air pocket enters the system which leads to a sticking of tiny air bubbles to all surfaces, including the sensing fiber. This irregularity alters the fiber boundary layer which can lead to an increased wall shear stress of the moving fluid.

The sensor can be used in two ways, either to derive the viscosity or the flow velocity of a medium. However, one of the two parameters need to be available to determine the other. This means that the sensor can be used to create a distributed viscosity profile along the sensing length by providing the flow velocity, or it can be used to create a distributed flow velocity profile by providing the fluid viscosity.

All the measured distributed strain profiles show a superimposed signal of different factors. Only a part is due to the fluid shearing along the fiber. Other main sources are: temperature effects, thermal and fluid shearing induced expansion from the experimental setup and the mechanical and thermal state of the fiber during the reference measurement. These other effects are partly visible in the strain profiles in the center subplot in Fig. 5. An offset towards negative strains is visible in the data. This effect is most likely attributed to a cooling and compression of the experimental setup and a subtle shortening of the sensing length in the fluid path. The advantage of our sensor is that these factors cancel out and have no consequence for the determination of the strain gradient in Eq. (12). An exception which is not canceled out is a temperature gradient over the sensing length. If the fluid flow velocity is high enough, isothermal conditions can be assumed. Otherwise, temperature effects need to be corrected by independent temperature sensors, ideally by using fiber-optic distributed temperature sensing in the sensor cable [41], or by using thermocouples as we did in our experiment.

With ongoing time of circulation of a fluid sample, the temperature of the tested fluid sample increases. The higher the viscosity of the fluid sample, the higher the rate of temperature increase. Also, the temperature at the top of the experiment increases at a slightly higher rate than the temperature at the bottom. The temperature increase can be partly attributed to heat generated by friction of the fluid in the narrow flow path. This means, that there is not only a strain gradient on the fiber, but also a temperature gradient. With respect to the strain gradient, we have a high strain at the bottom and a low strain at the top, and for the temperature gradient, we have low temperatures at the bottom and high
temperatures at the top. According to Eq. (1) this means, that both strain and temperature are opposite and potentially cancel each other out. We take the assumption that the temperature is linearly distributed between the thermocouple at the top and the bottom of the experiment and performed the temperature correction on the data to obtain the final strain profile.

The testing environment of the presented experiment provides ideal laminar flow conditions, a uniform flow path, homogeneous single-phase fluid, and ideal central placement of the strain sensing fiber. Such idealized conditions cannot be expected with regards to a real industrial working environment. Taking boreholes as a potential key application as an example, the empirical approach needs to be carefully adapted to provide a reliable theoretical translation of a given measured strain to the fluid rheological parameters of a passing fluid. With respect to Eq. (12), this would require adapting the hydraulic diameter according to the annular space in the well path, to assess the annular flow regime. When using a better sensing approach in the future with a higher repetition rate, we would expect to see turbulence in the form of a high noise level due to the complex and irregular fluid particle movement. Further investigations need to be performed to verify whether fluid rheological parameters can be derived from the strain sensing fiber in a turbulent flow regime.

5. Conclusion

We demonstrate that fiber optic distributed strain sensing can be used to derive fluid velocities and fluid rheological parameters such as viscosity. By using basic empirical equations from fluid mechanics, we derived a model that connects fluid rheological properties with a strain along a (fiber-optic) cable in its flow path. The model predicted that we can detect viscosity differences in the range of ±1 mPa s. With our experiment, we were able to validate the functionality of the sensor. We intentionally performed measurements at the lower end of the sensor resolution for watery solutions which only create a strain close to the resolution limit of around 1 µε/m of the sensing system. We expect to be able to measure higher viscosities than presented in this study (>7 mPa s). In narrow, extensive, and difficult to reach technical environments, this novel sensor technology provides new possibilities to monitor dynamic fluid rheological properties in a distributed manner over long sensing length in real-time.

Declaration of Competing Interest

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: Martin Peter Lipus, Thomas Reinsch, Stefan Kranz, Jan Henninges have patent #EP 3 730 926 A1 licensed.

Data Availability

Data will be made available on request.

Acknowledgements

The development of the sensor was partly financed by the GFZ Innovation Fund. The authors would like to thank Ronny Giese and Matthias Poser from GFZ Potsdam for their technical support during the installation of the experimental setup. The authors would also like to thank Philipp Wella from Technical University Freiberg for the implementation of the field prototype of the experimental setup. The authors are grateful to Massimo Faccini for the discussion of suitable fiber-optic cables and for providing a cable sample for testing.

Appendix. : Error propagation

The calculation of the uncertainty of the theoretical approach includes the parameters given in Eq. (12). The parameters include the uncertainty of the flow rate q of the pump, the uncertainty of the apparent viscosity η of the fluid, the fiber outer diameter d_f and the fiber stiffness k. The uncertainty in hydraulic diameter is neglected here. A Gaussian error is determined according to

$$
\Delta f(q, \eta, d_f, k) = \sqrt{\left(\frac{\partial f}{\partial q} \Delta q\right)^2 + \left(\frac{\partial f}{\partial \eta} \Delta \eta\right)^2 + \left(\frac{\partial f}{\partial d_f} \Delta d_f\right)^2 + \left(\frac{\partial f}{\partial k} \Delta k\right)^2}
$$  \hspace{1cm} (A1)

where f is Eq. 12, Δq the maximum deviation of the mean flow rate q, Δη the maximum difference of the apparent viscosity due to temperature differences from the mean viscosity, Δd_f an estimate for the uncertainty of the measurement of the outer diameter of the fiber, and Δk the maximum error for the fiber stiffness which was measured in a laboratory test by stretching the respective fiber with increasing weight pieces in the range of 0 – 240 mN. An overview of the error data is given in table A1. The partial derivatives of Eq. (A1) are

$$
\frac{\partial f}{\partial q} = \frac{32\pi d_f}{D_H} \frac{1}{k} \eta
$$  \hspace{1cm} (A2)

$$
\frac{\partial f}{\partial \eta} = \frac{32\pi d_f}{D_H} \frac{1}{k} q
$$  \hspace{1cm} (A3)

$$
\frac{\partial f}{\partial d_f} = \frac{32\pi 1}{D_H} \frac{1}{k} \eta
$$  \hspace{1cm} (A4)

$$
\frac{\partial f}{\partial k} = \frac{32\pi d_f}{D_H} \frac{1}{k^2} \eta
$$  \hspace{1cm} (A5)

A second uncertainty estimate is performed on the temperature control measurements at the top and the bottom of the flow path. The input parameter for the data validation is given in Table 3. The uncertainty arising from the temperature uncertainty of the thermocouples (1.08 µε/m) is nearly 7 times higher than the Gaussian error calculated from the uncertainty of the input parameter in Eq. (12) (0.158 µε/m).
Table 3
Input parameter for data validation. A Gaussian error is calculated for the input parameters of Eq. (12). A secondary error is calculated for the uncertainty of the temperature readings at the thermocouples.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Uncertainty</th>
<th>Error from input parameters Eq. (12)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hydraulic diameter $D_h$</td>
<td>2.02E-02 m</td>
<td>m</td>
</tr>
<tr>
<td>Flow velocity $q$</td>
<td>3.00E-01 m/s</td>
<td>m/s</td>
</tr>
<tr>
<td>Dynamic viscosity $\eta$</td>
<td>6.80E + 00 mPa s</td>
<td>mPa s</td>
</tr>
<tr>
<td>Cable diameter $dc$</td>
<td>2.00E-04 m</td>
<td>m</td>
</tr>
<tr>
<td>Stiffness $k$</td>
<td>1.11E + 03 N</td>
<td>N</td>
</tr>
</tbody>
</table>

Partial derivatives

| Eq. (A2) | $\frac{\delta f(q)}{\delta q}$ | 3.67E-08 |
| Eq. (A3) | $\frac{\delta f(\eta)}{\delta \eta}$ | 9.16E-08 |
| Eq. (A4) | $\frac{\delta f(d_c)}{\delta d_c}$ | 9.16E-08 |
| Eq. (A5) | $\frac{\delta f(k)}{\delta k}$ | 8.27E-08 |

Error $(c/L)$ $\Delta f(q,dc,k)$

<table>
<thead>
<tr>
<th>Error $(c/L)$</th>
<th>$\Delta f(q,dc,k)$</th>
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<tbody>
<tr>
<td>Error from Thermocouples (TC)</td>
<td></td>
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<tr>
<td>TC accuracy $\Delta T_1$</td>
<td>1.50E-01 °C</td>
</tr>
<tr>
<td>Gaus. E. for 2 TC $\Delta T_2$</td>
<td>2.12E-01 °C</td>
</tr>
<tr>
<td>$\delta C / \delta kHz$</td>
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<td>$\delta L / \delta C$</td>
<td>1.02E + 01 μC/C</td>
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<tr>
<td>$\delta L / \delta m$</td>
<td>5.08E + 00 μm</td>
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Error from TC $\delta f_T(c/L)$ $\Delta T_2$

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<th>Error from TC</th>
<th>$\Delta T_2$</th>
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<tbody>
<tr>
<td>TC accuracy</td>
<td>1.08E + 00 μm</td>
</tr>
</tbody>
</table>

References


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